



# The physics and mathematics of wound healing



Tanniemola B Liverpool

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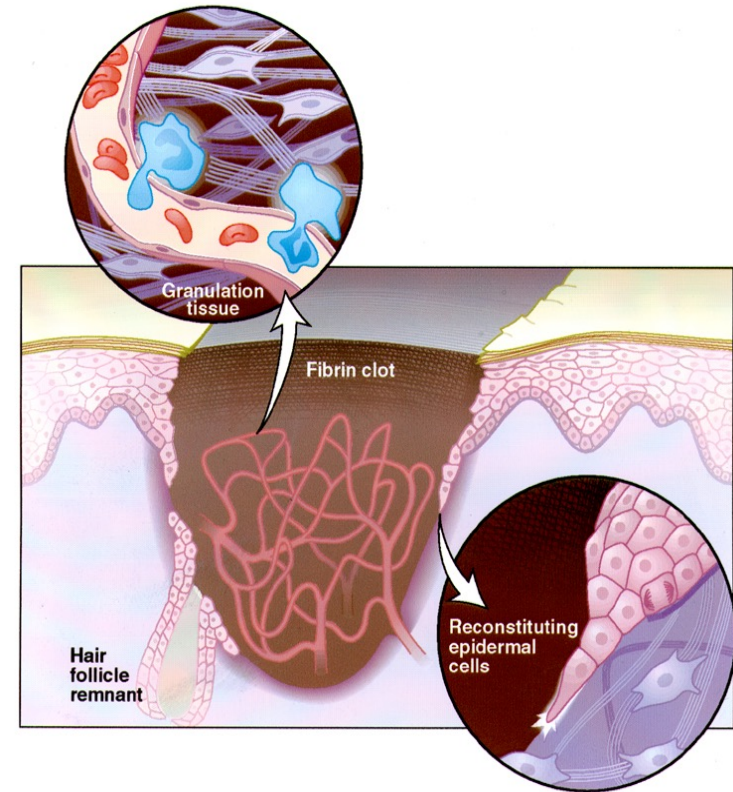
Erlangen

6 Nov 2024



# Wound healing

A complex process



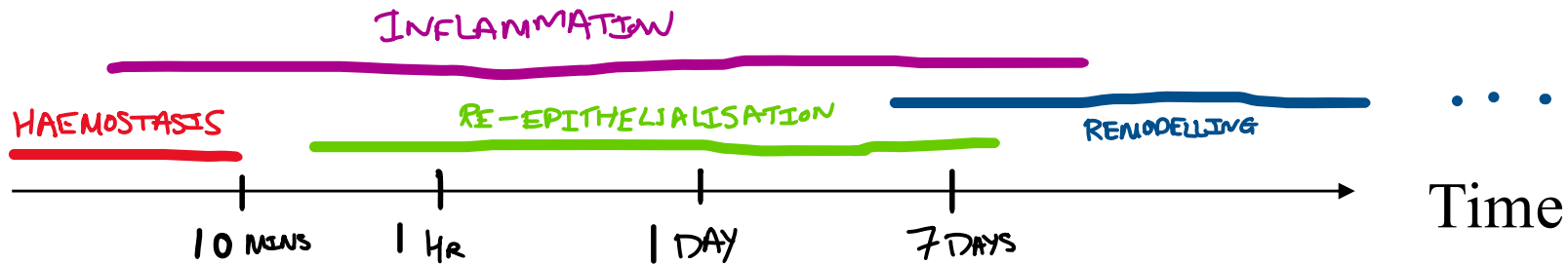
Martin (1997) Science

Blood clotting

Inflammation

Tissue growth

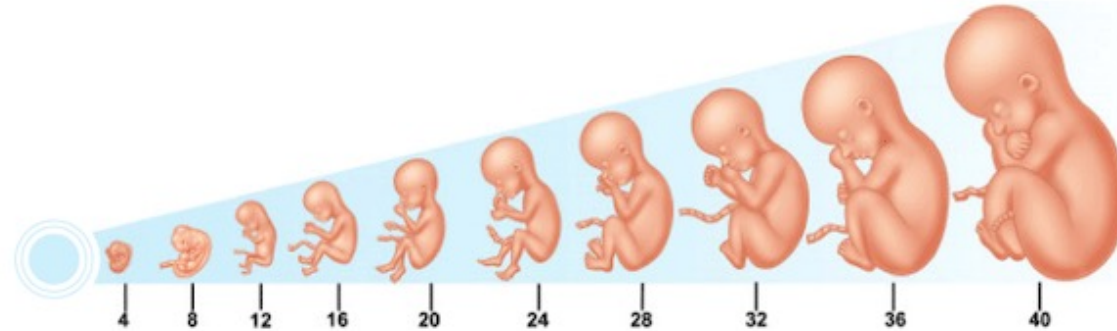
Tissue remodelling





# Wound healing

Wound healing gives us a window into fundamental processes of **development**



Better understanding of **chronic wounds** which are wounds that do not heal – sometimes for years

Chronic wounds are a huge burden on the health system



Falanga et al, *Nat Rev Dis Primers* **8**, 50 (2022)



# Wound healing in model organisms



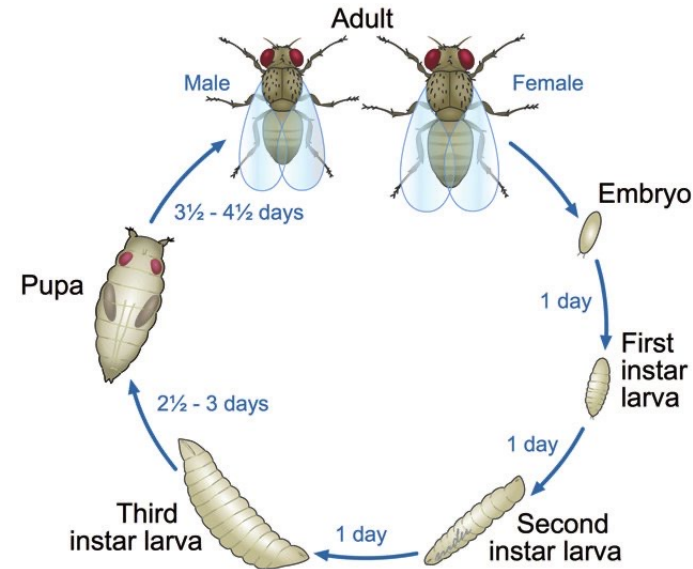
LIVING !

*Drosophila melanogaster*

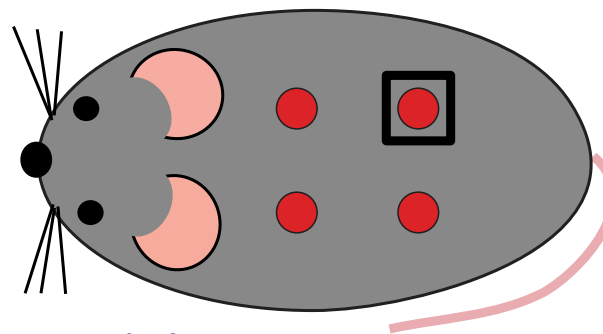
Wood, Jacinto et al (2002) *Nat. Cell Biol.*



Zebrafish



Ong et al (2014) *Nanotoxicology*

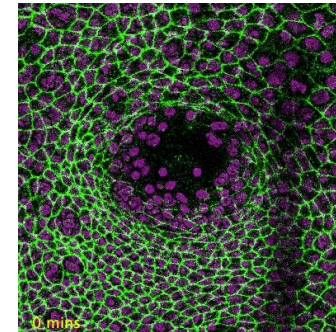


Mouse

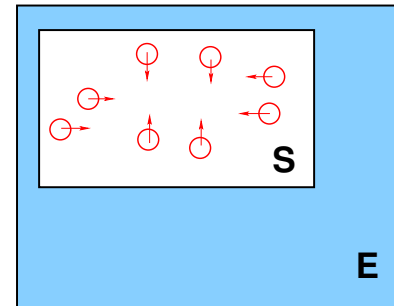
<https://www.mpg.de/10973406/mice>



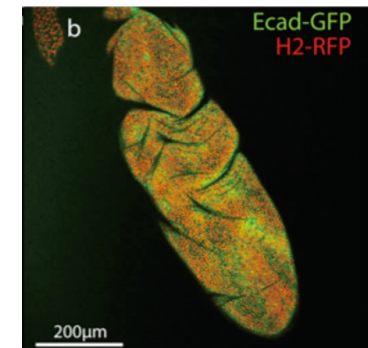
## 1. Quantifying wound healing at cellular scales



## 2. From statistical mechanics to active processes



## 3. Analysis of fluctuating tissue growth and repair



# Acknowledgements

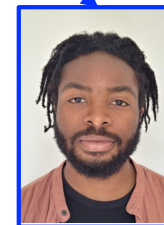
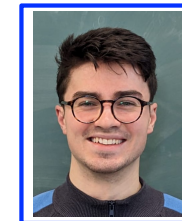
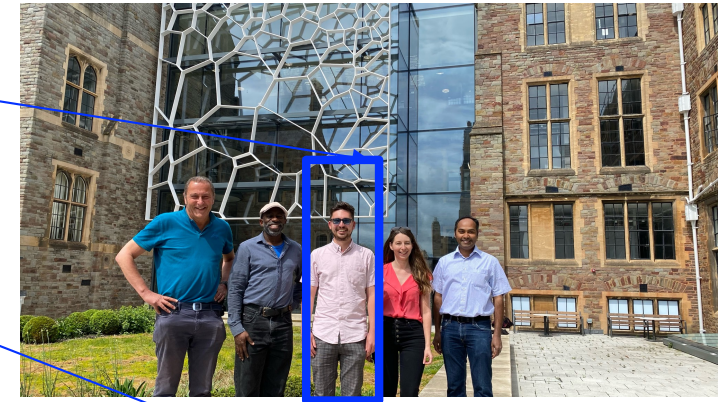


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Engineering and Physical Sciences  
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Medical  
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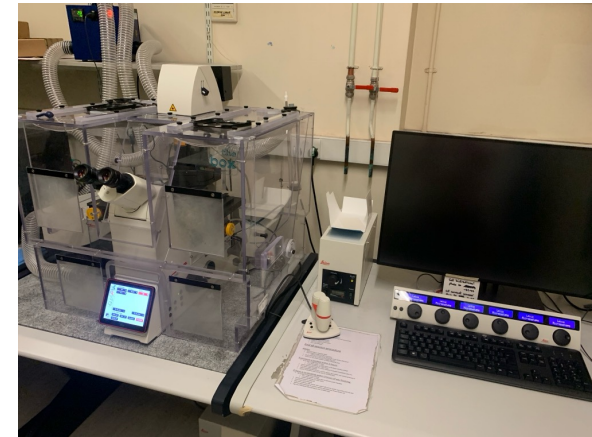
The Leverhulme Trust

Nov 2024

# Confocal microscopy



Leica sp8 confocal



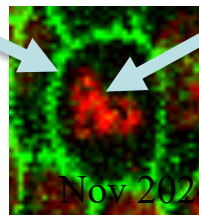
## Fluorescent protein

GFP (green fluorescent protein)



- First isolated from the jellyfish
- Flies can be genetically modified to have fluorescent protein fused on to other proteins we would like to image

RFP (red fluorescent protein)

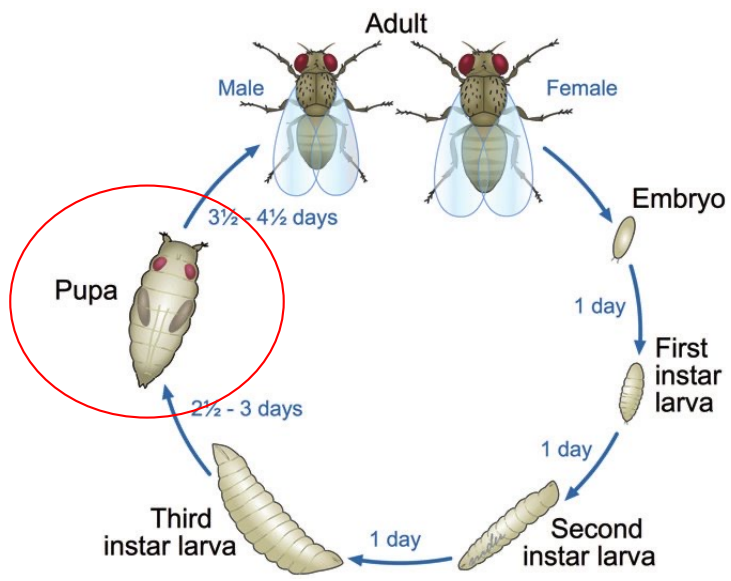
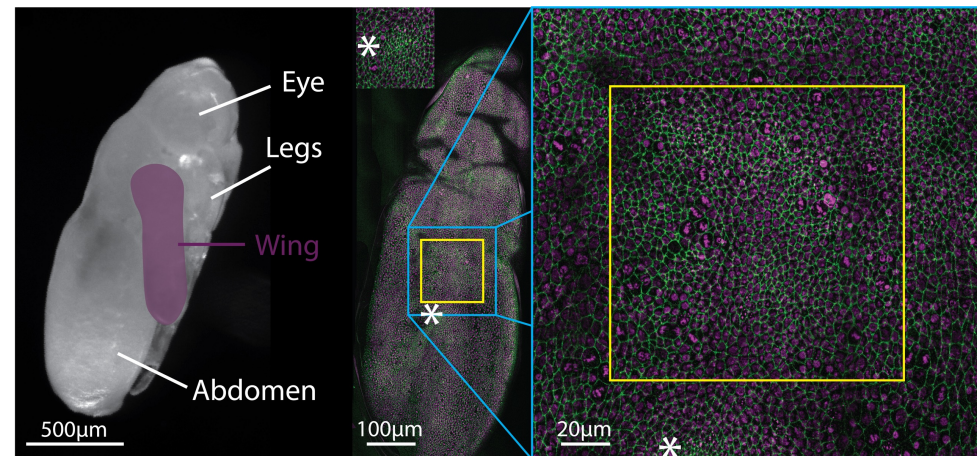


# Imaging drosophila pupa

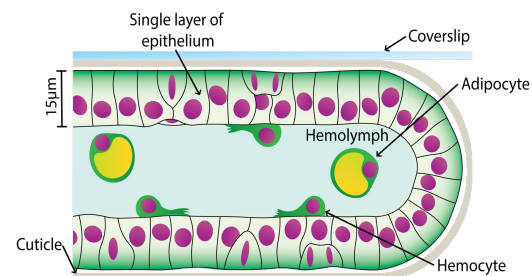
## Long-term *In Vivo* Tracking of Inflammatory Cell Dynamics Within *Drosophila* Pupae

Helen Weavers<sup>1,2</sup>, Anna Franz<sup>1</sup>, Will Wood<sup>3</sup>, Paul Martin<sup>1,4</sup>

<sup>1</sup>School of Biochemistry, Biomedical Sciences, **University of Bristol**, <sup>2</sup>School of Cellular and Molecular Medicine, Biomedical Sciences, **University of Bristol**, <sup>3</sup>MRC Centre for Inflammation Research, **University of Edinburgh**, **Queens Medical Research Institute**, <sup>4</sup>School of Physiology, Pharmacology, and Neuroscience, Biomedical Sciences, **University of Bristol**



## Cross Section of the Wing

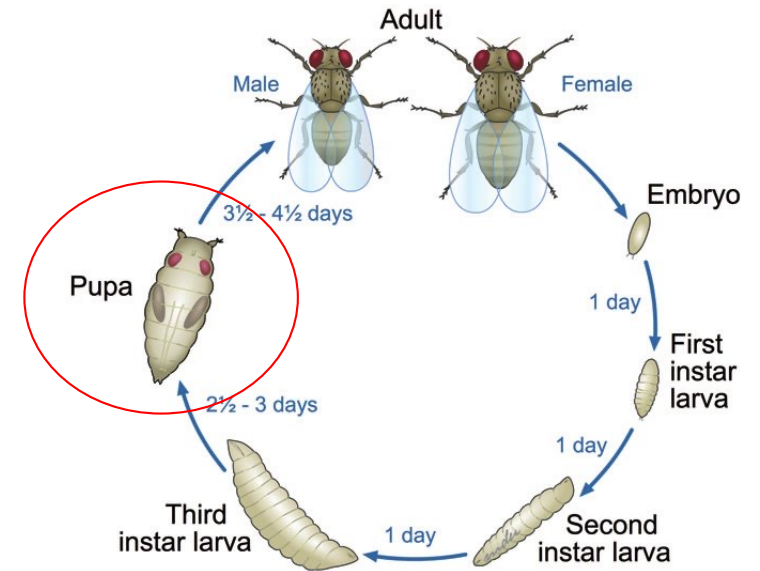
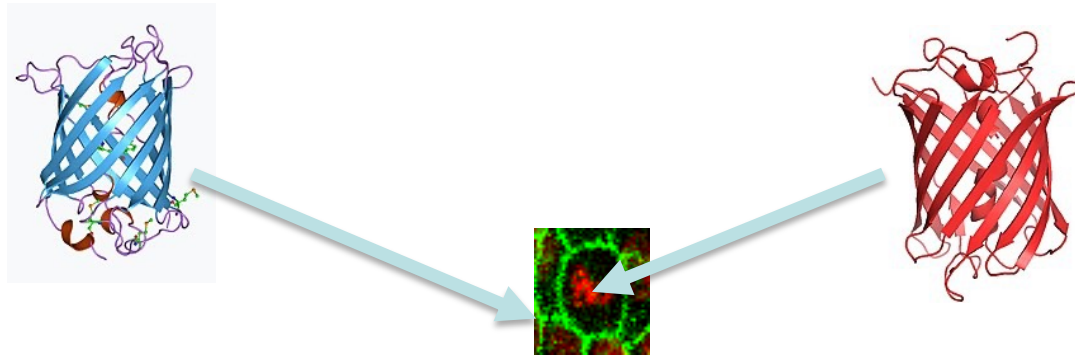
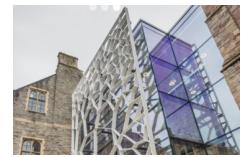


## Laser wound

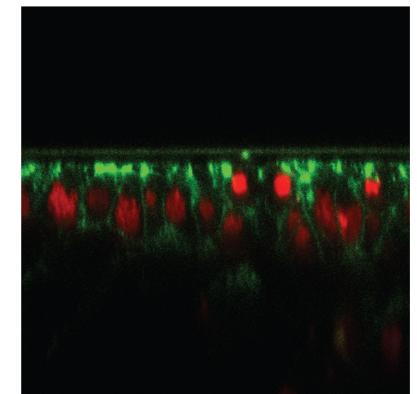
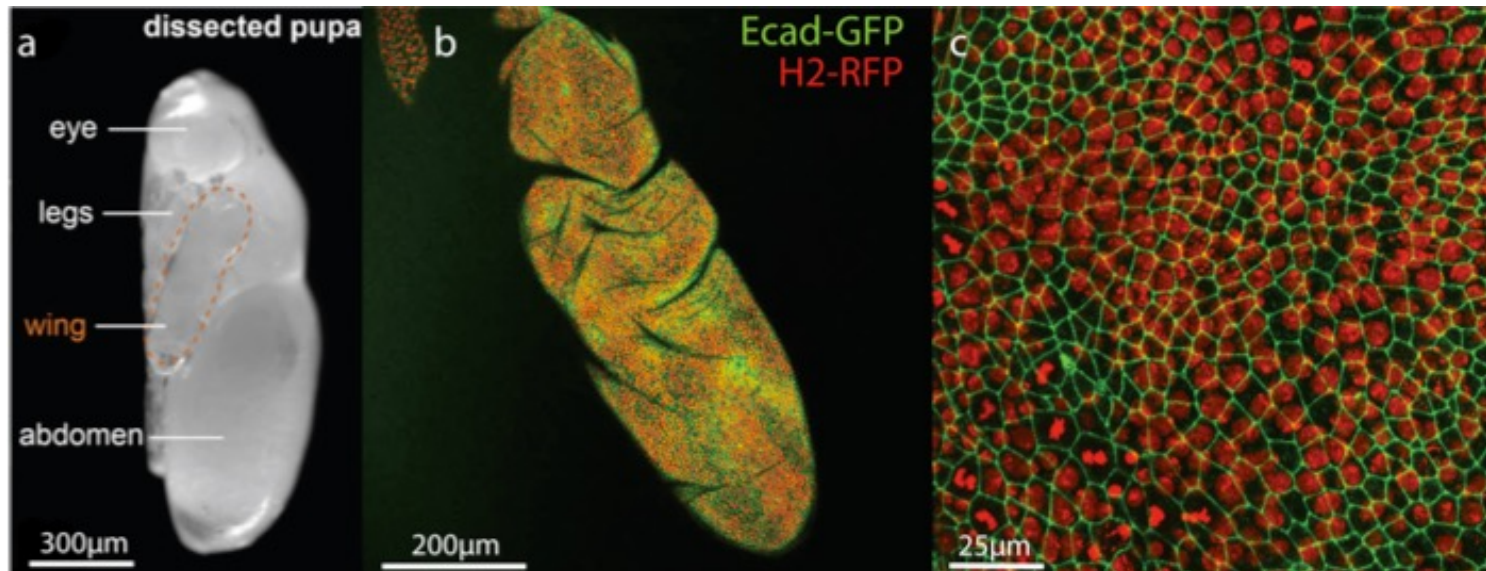




# Imaging drosophila pupa



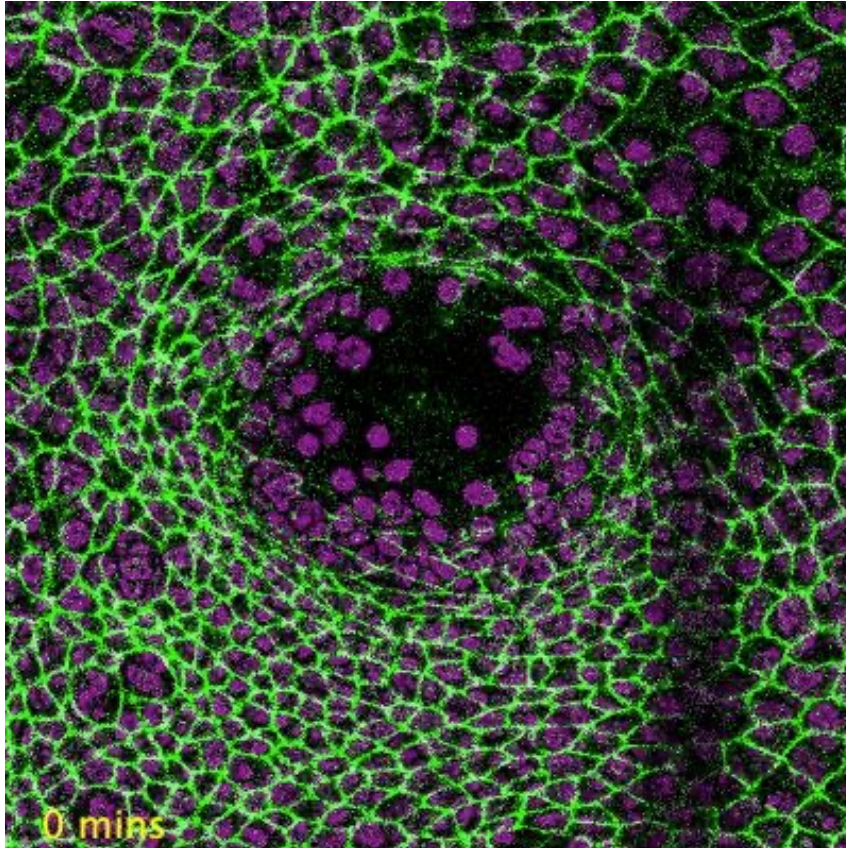
- Collect confocal videos



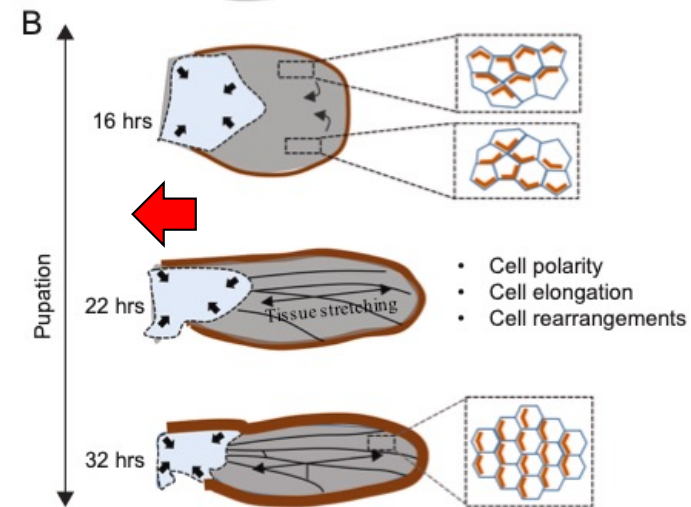
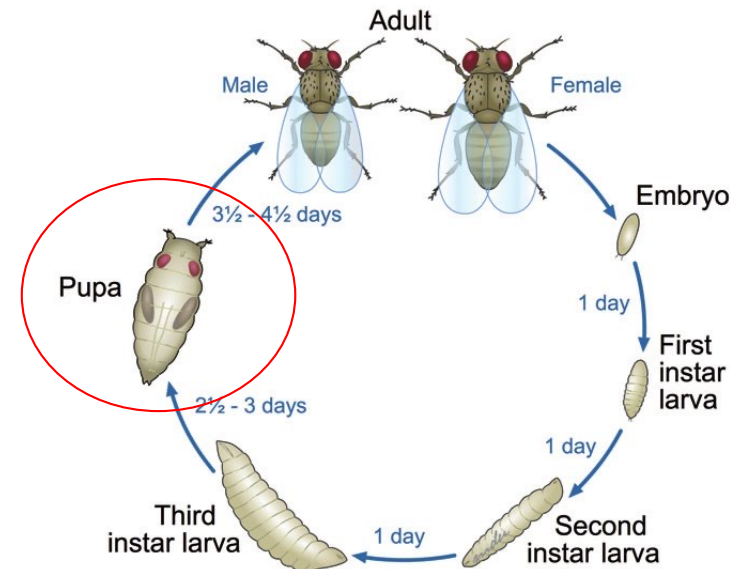
# Imaging drosophila pupa



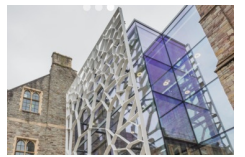
## Laser ablation wound



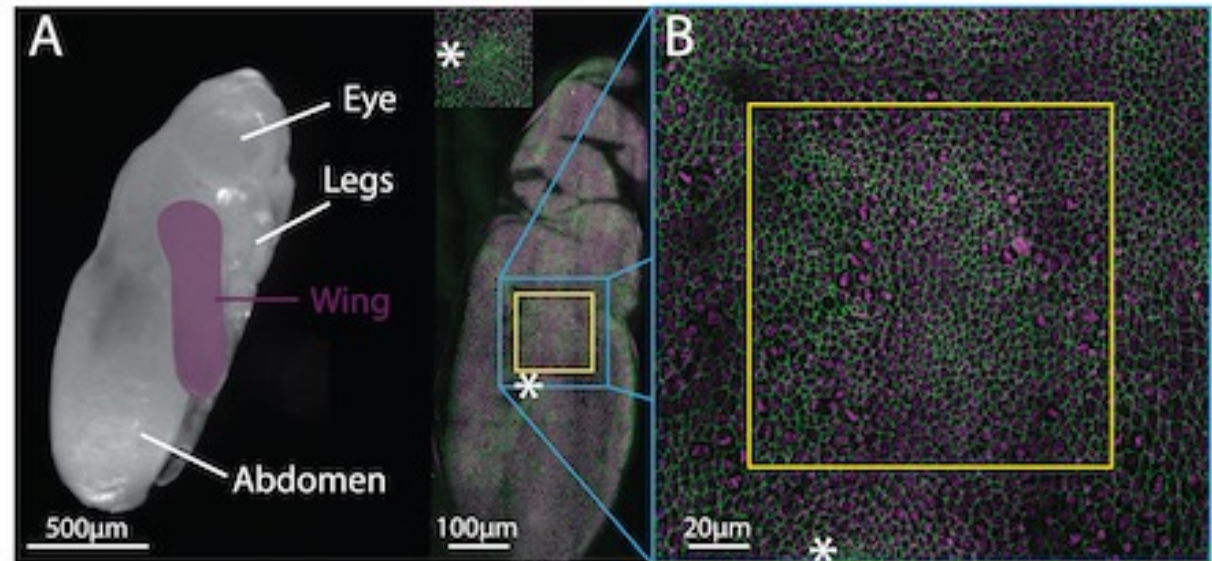
18 hours after puparium formation



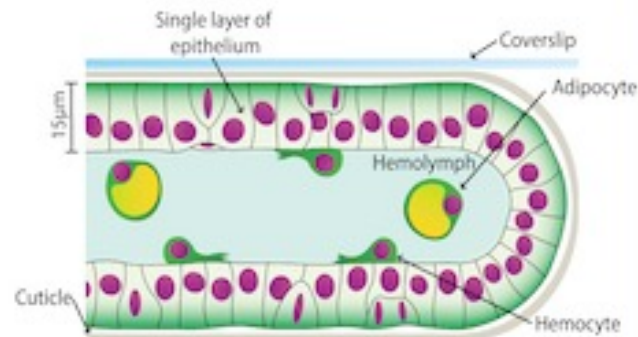
# Imaging drosophila pupa



How **does** the  
epithelium heal ?



C Cross Section of the Wing



Compare **wounded** to **healthy** tissue

# Imaging drosophila pupa



How do

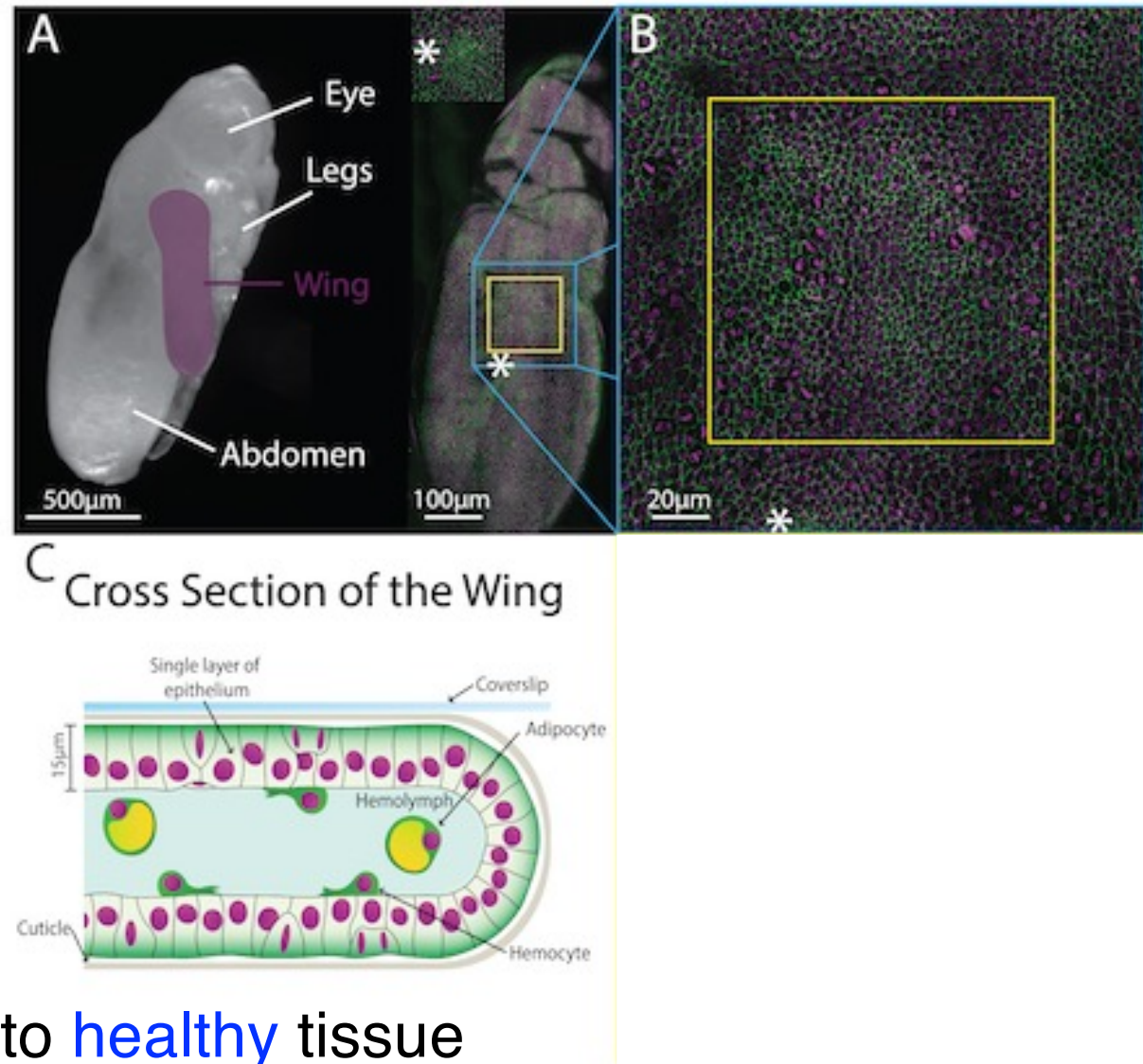
cell shape

cell motion

cell division

evolve during  
reformation of  
epithelia?

Compare wounded to healthy tissue

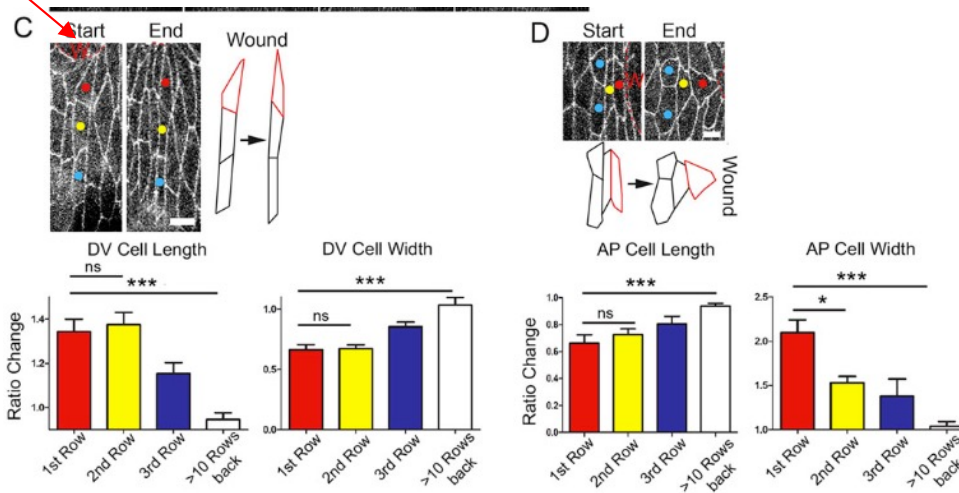




# Why mathematics is needed

- BIG data is difficult to collect and fuzzy (noisy)

wound



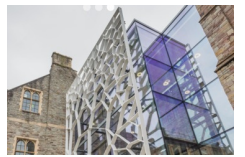
Razzell et al, *Development* (2014)

cell shape  
cell motion  
cell division

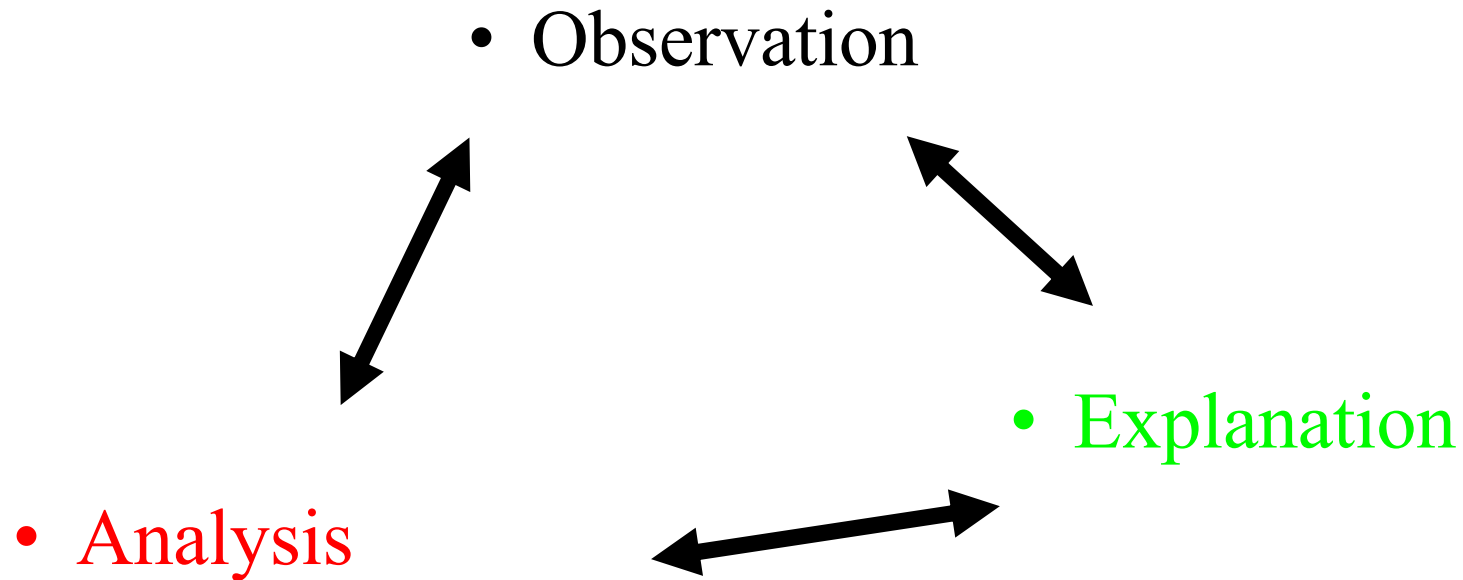
Each video is data of dimension  $\mathbb{R}^{2 \times 93 \times 42 \times 512 \times 512}$

- How do you extract the “signal” from the “noise” ?

Q: What (else) should one measure ?

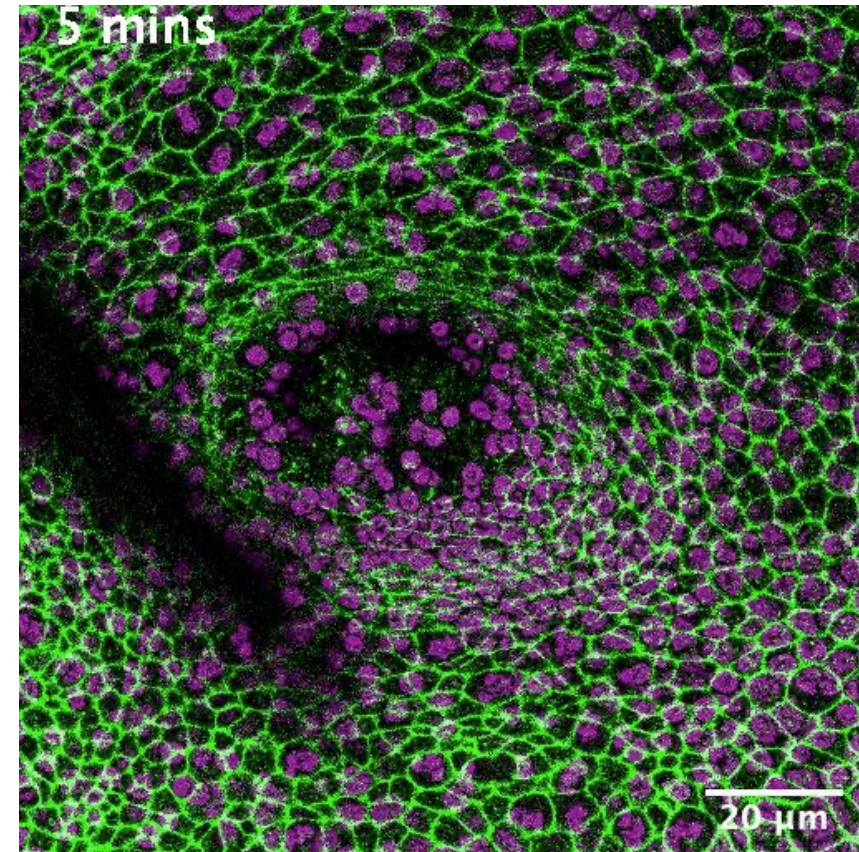
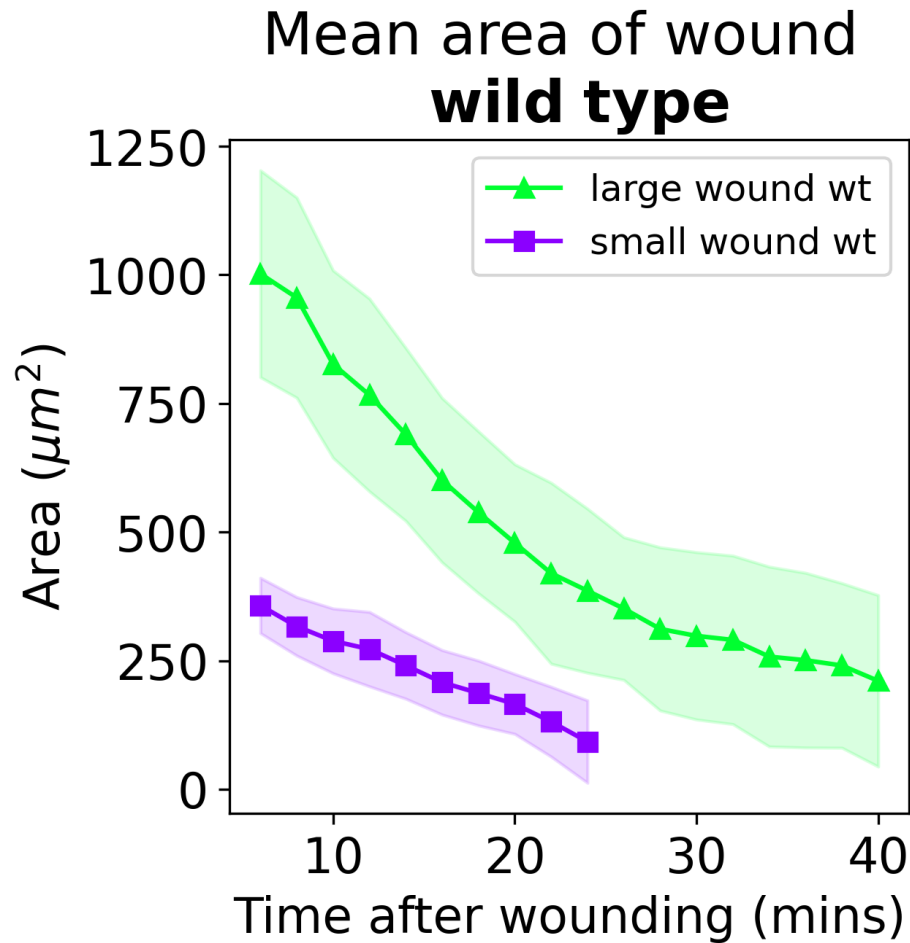


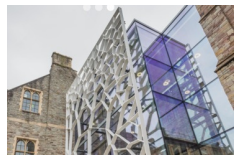
# Physical models are needed for ...





# Dynamics of wound healing

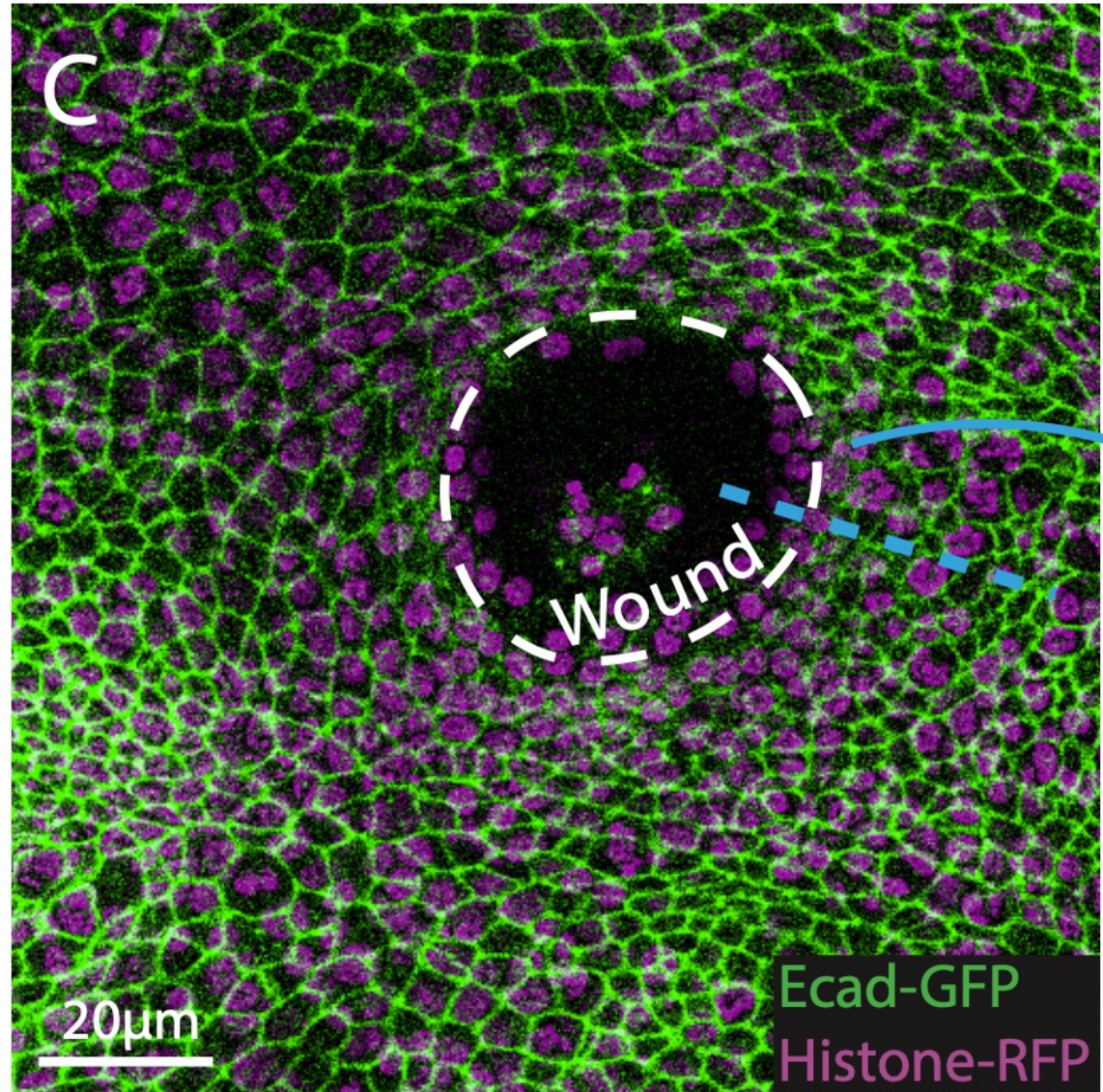




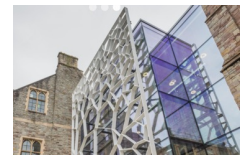
shape

motion

**divisions**

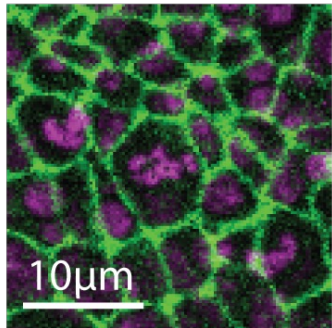




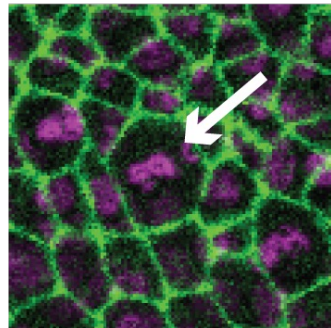


# Cell divisions in unwounded and wounded tissue

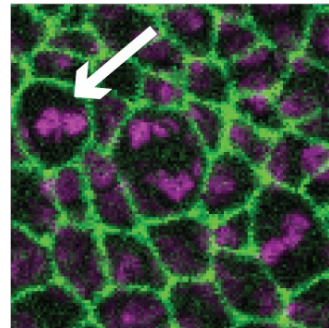
0 mins



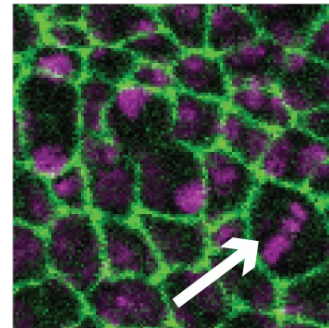
2 mins



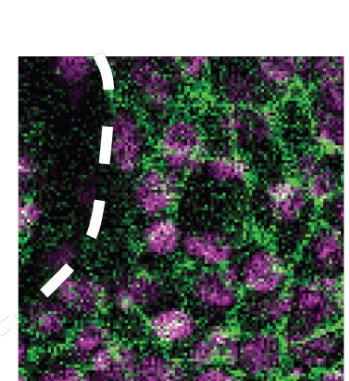
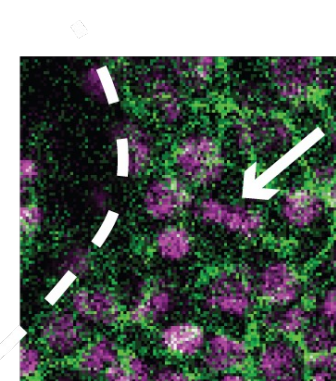
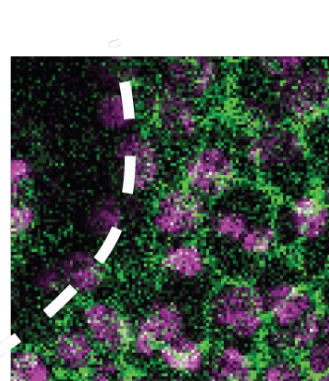
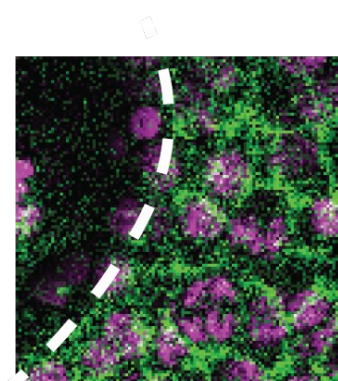
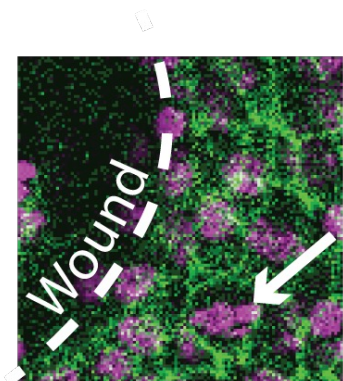
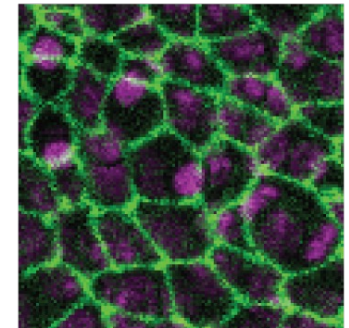
4 mins



6 mins



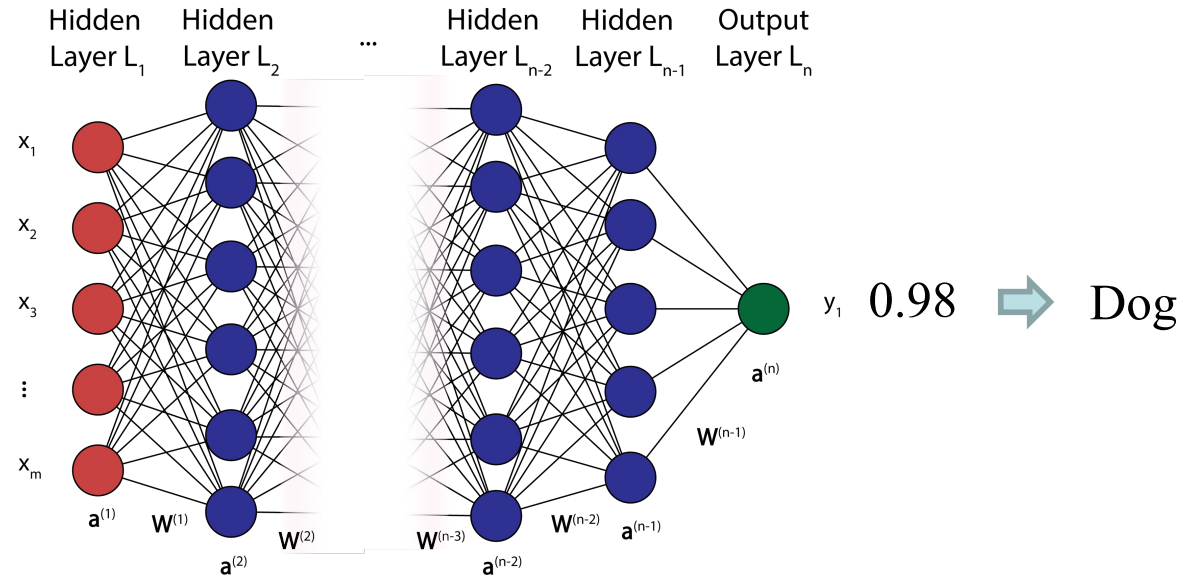
8 mins



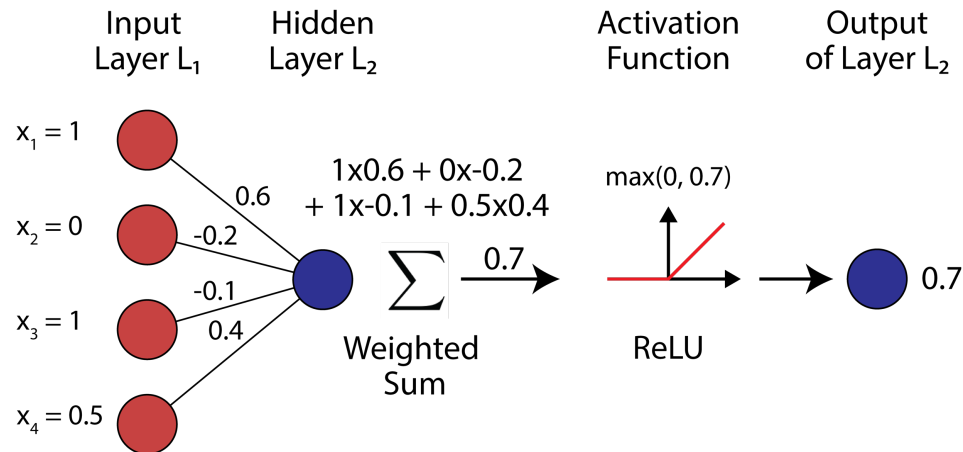


# Deep learning

## Deep Learning



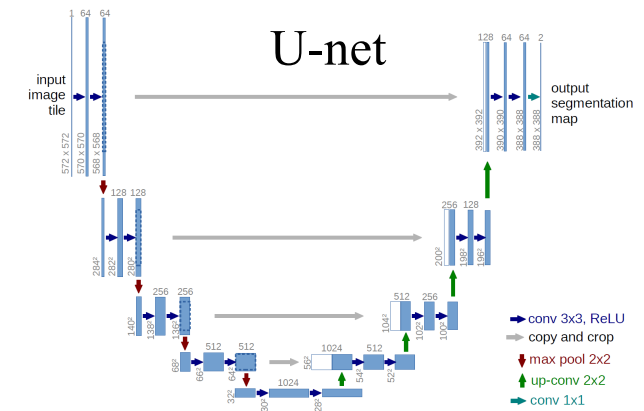
## Example Layer:



# Image segmentation models



Use U-net architecture to segment images into categories

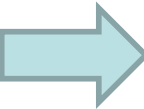


Falk, T. et al. U-Net: deep learning for cell counting, detection, and morphometry. *Nat. Methods* **16**, 67–70 (2019).

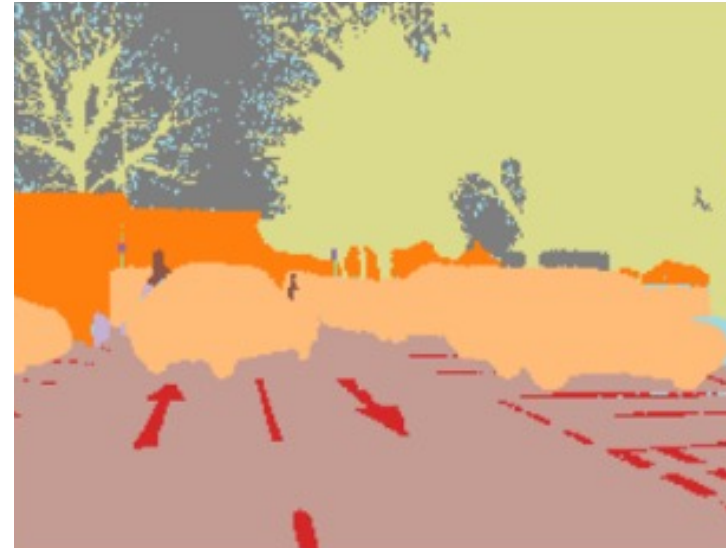
Input



U-net  
model



Output





# Training models – finding best weights

$$Y = M(\mathbf{W}, X)$$

Where  $M$  is the model,  $\mathbf{W}$  is a vector of the weights and  $X, Y$  the in and outputs.

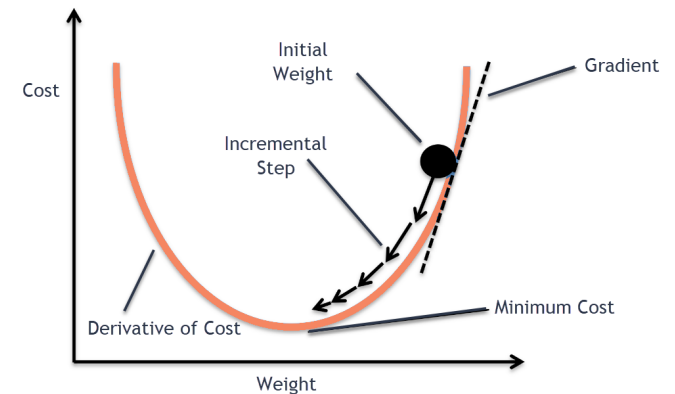
Loss function

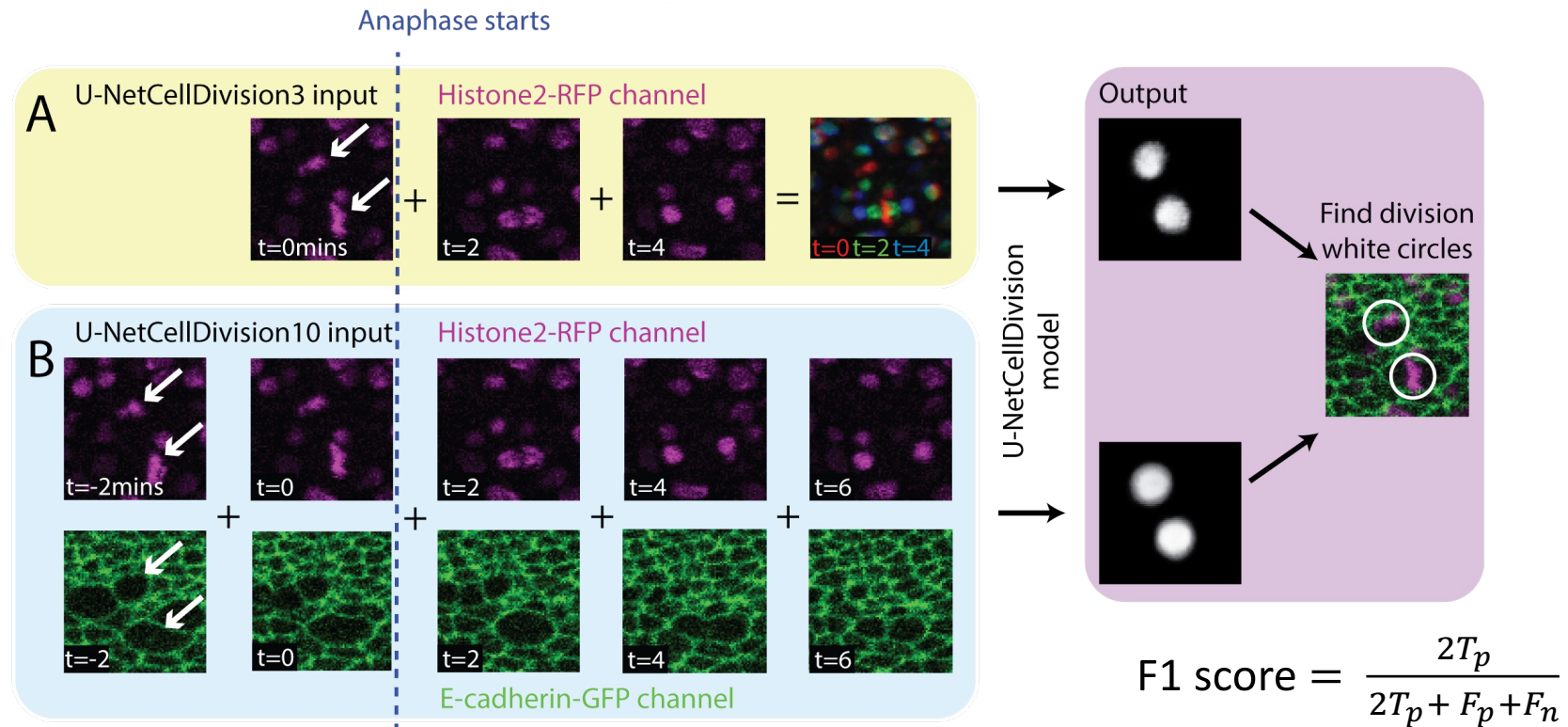
$$L(\mathbf{W}, X) = (\hat{Y} - M(\mathbf{W}, X))^2$$

$\hat{Y}$  is the ground truth. To optimize the loss function we use stochastic gradient descent.

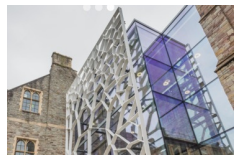
$$\mathbf{W} = \mathbf{W} - l_r \nabla L(\mathbf{W}, X_i)$$

Where  $l_r$  is the learning rate.

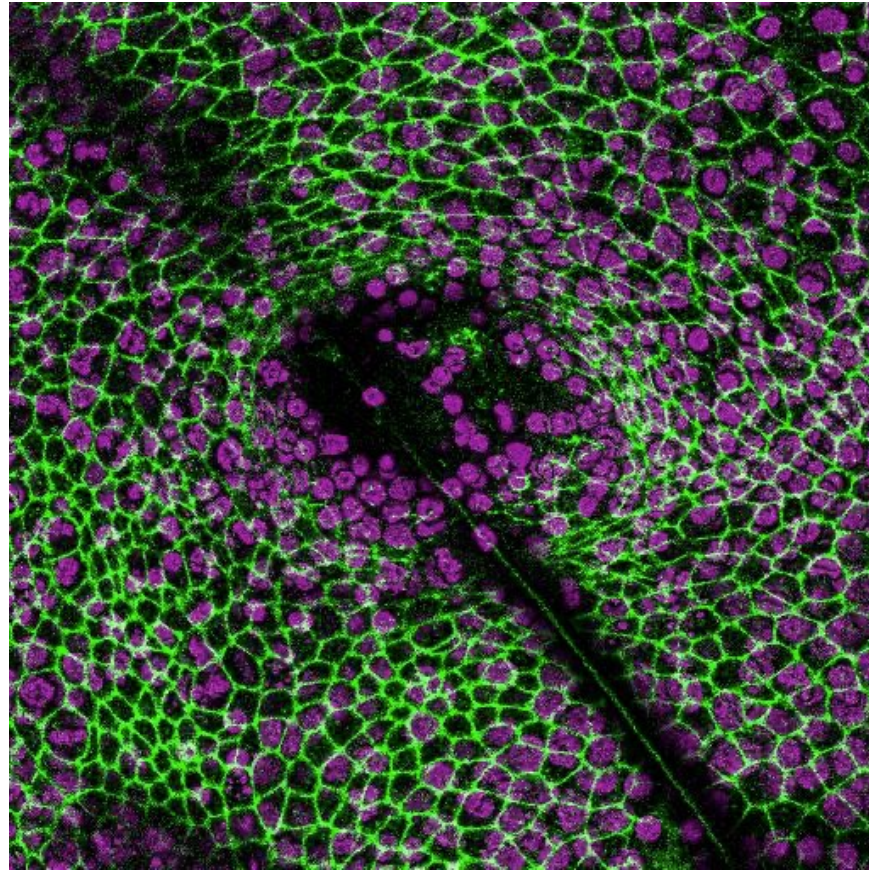




Model	True Positives	False Positive	False Negative	F1 score
U-NetCellDivision3	797	216	310	0.752
U-NetCellDivision10	1057	28	50	0.964

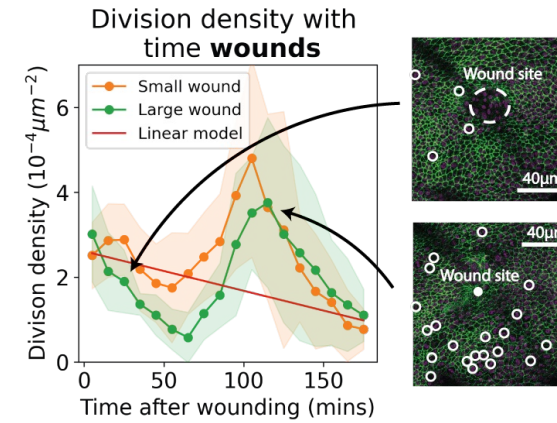
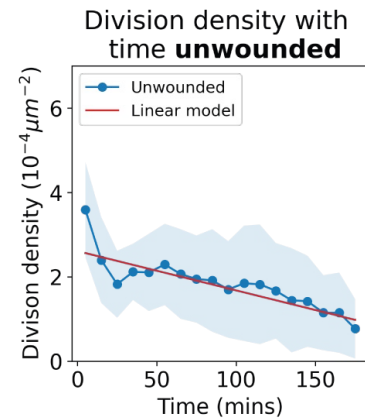


# Detected cell divisions

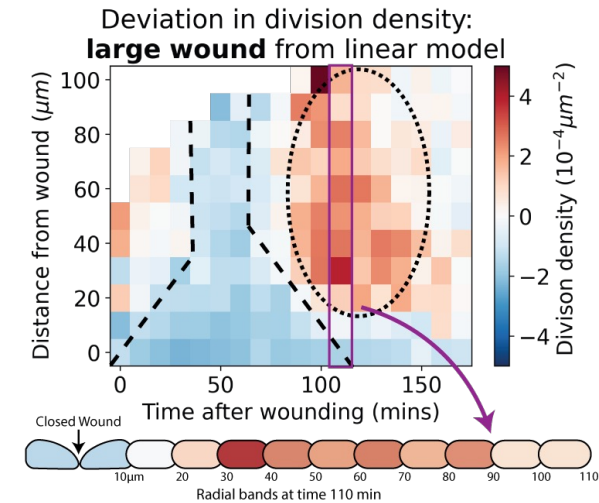
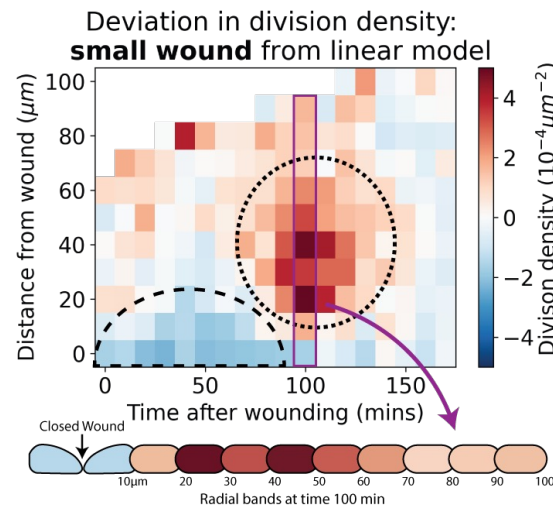
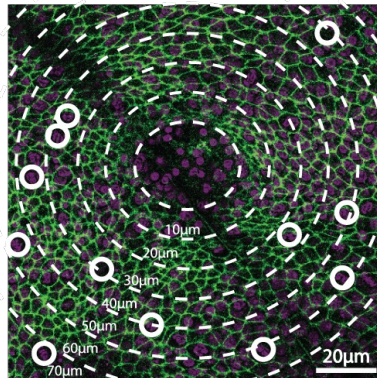




# Division density in living epithelial tissue *in vivo*

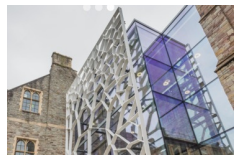


Schematic of radial bands from wound



J Turley et al, [biorXiv: 10.1101/2023.03.20.533343](https://doi.org/10.1101/2023.03.20.533343)

Nov 2024

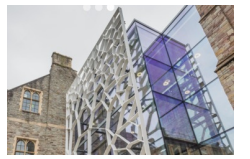


# Division density in living epithelial tissue *in vivo*

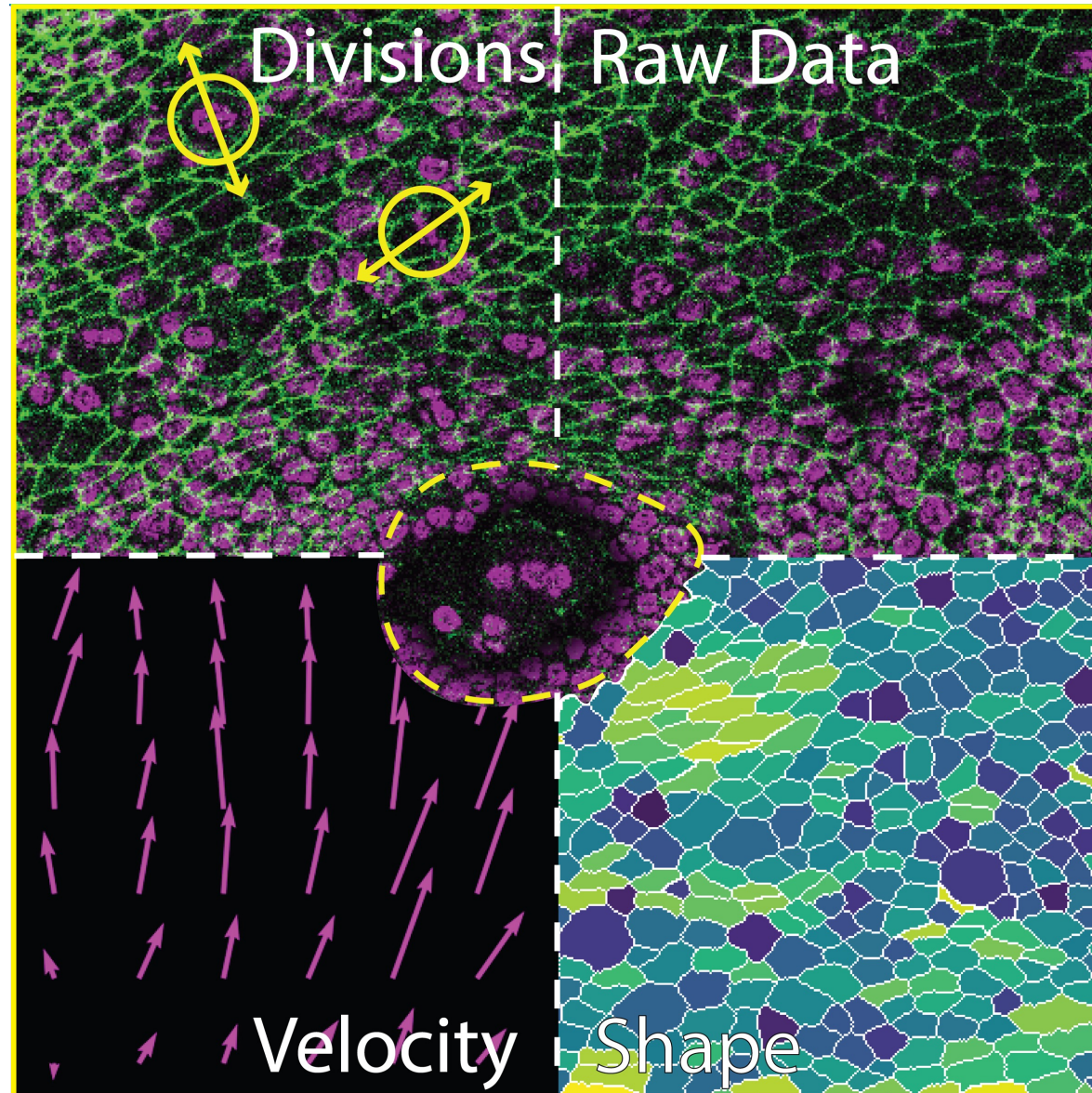
- Epithelial cell divisions are oriented according to lines of tissue tension
- Wounding triggers a delayed and synchronised (whose orientations are not affected by wound) wave of cell divisions back from the leading edge
- Spatio-temporal cell division analyses following wounding reveal spatial synchronicity that scales with wound size
- Additional deep learning tools enable rapid analysis of cell division orientation

**J Turley et al, *eLife* 12 : RP87949 (2023)**



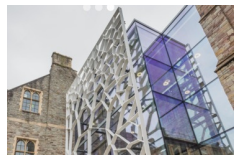


divisions



shape

# Overview



**1. Quantifying wound healing  
at cellular scales**

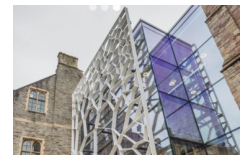
**2. Statistical mechanics of  
active processes**

**3. Analysis of fluctuating  
tissue growth and repair**



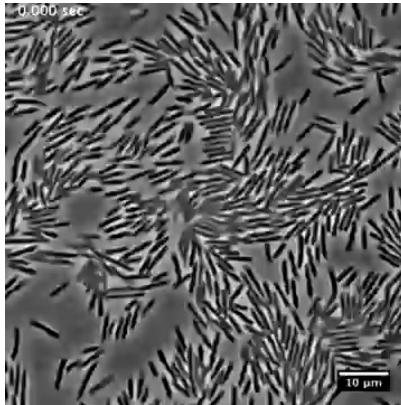
# Tissue is “active matter”

	Traditional (Soft) Condensed Matter	Active Matter
Many “particles”	✓	✓
Interactions	✓	✓
$k_B T$	✓	✗
<u>Local</u> energy conservation	✓	✗

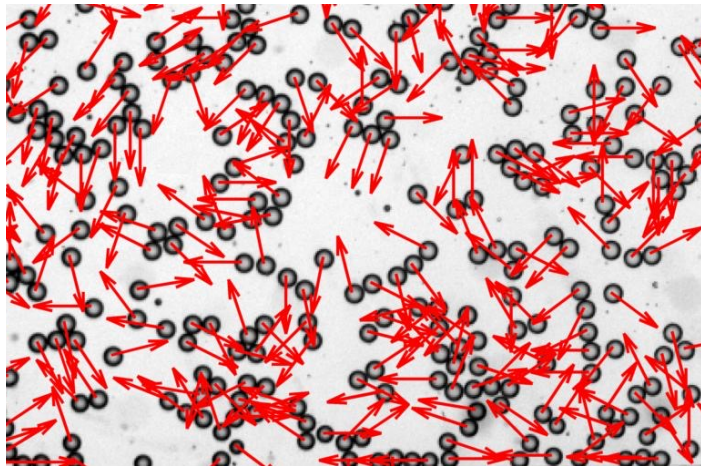


# active matter

## E-coli bacteria swarm

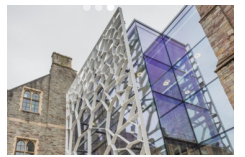


## Starling murmurations



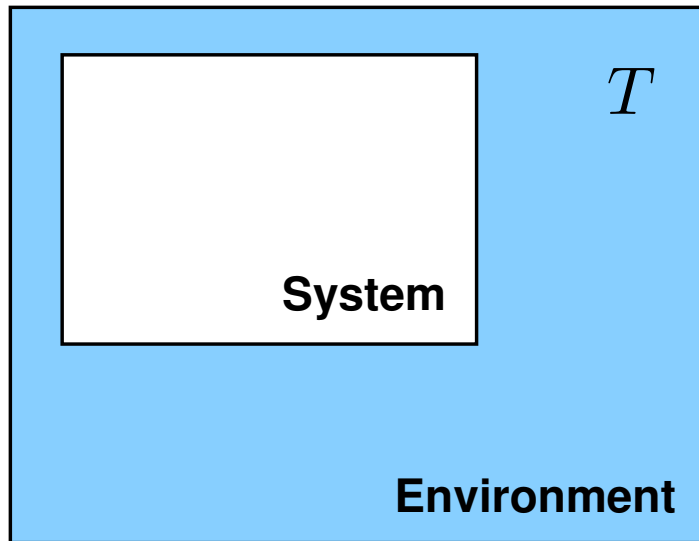
## Self-propelled colloids

S. Thutupalli et al., NJP 13, 073021 (2011)



# Thermodynamic equilibrium

“coarse-grained” system with “many” degrees of freedom



$$\{r_i\} \quad i \in \{1, \dots, N\} \quad N \gg 1$$

$$\vec{r} = (r_1, \dots, r_N)$$

“Hamiltonian”  $\mathcal{H}(\vec{r})$

Equilibrium = steady state with Gibbs-Boltzmann distribution

$$\rho(\vec{r}) = \frac{1}{Z} \exp[-\mathcal{H}/T]$$

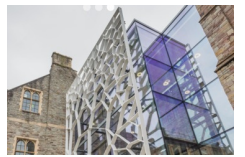
which maximises **entropy**

Canonical Ensemble

$$\text{Tr}(\rho) = 1 \quad \text{Tr} \equiv \int_{\vec{r}}$$

$$S[\rho] = - \int_{\vec{r}} \rho(\vec{r}) \ln \rho(\vec{r})$$

$$\rho(\vec{r}) \Rightarrow \text{Macroscopic properties} \quad A = \langle A(\vec{r}) \rangle = \text{Tr}(A\rho)$$



# Thermodynamic equilibrium

Example : stabilised colloidal suspension

Interaction potential

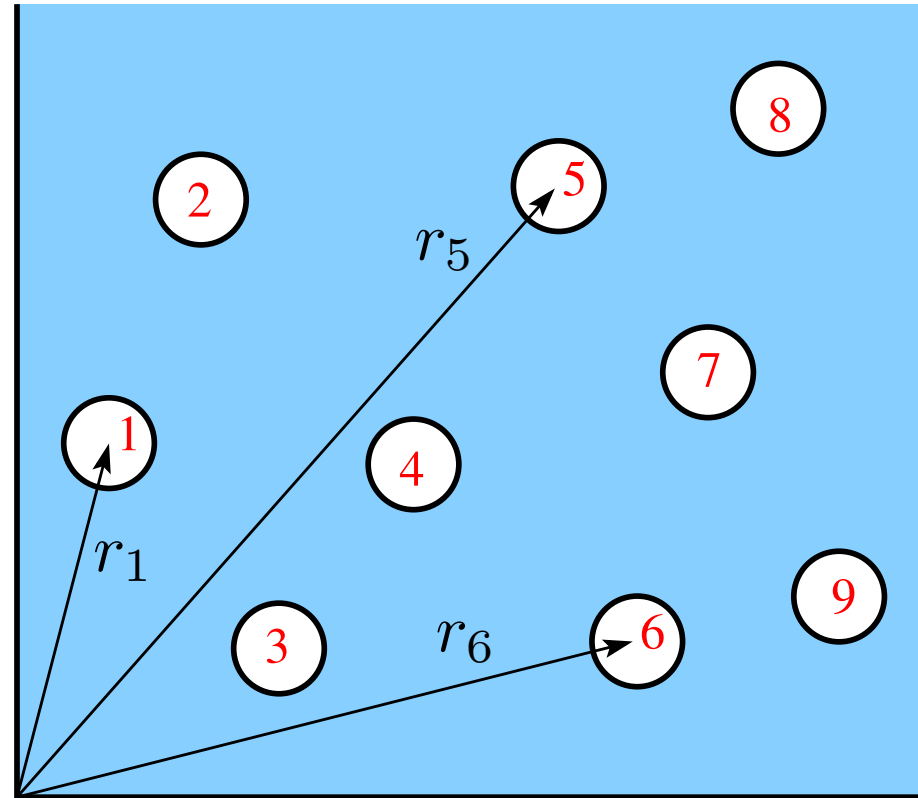
$$V_{ij} = U(|\mathbf{r}_i - \mathbf{r}_j|)$$

“Hamiltonian”

$$\mathcal{H}(\vec{r}) = \frac{1}{2} \sum_{i \neq j} V_{ij}$$

Phase diagram

(colloidal fluids, gels, glasses, crystals)



microscopic



MACROSCOPIC



# Driven systems

“slow” d.o.f.  $r_i(t)$  Langevin equation  $i \in \{1, \dots, N\}$   $N \gg 1$

addition of  $\Rightarrow$  non-equilibrium driving

$$T = \zeta = 1$$

$$\dot{r}_i = -\nabla_i \mathcal{H} + v_i(\vec{r}) + f_i(t) \quad v_i \neq \nabla_i \Phi(\vec{r})$$

$$\langle f_i(t) \rangle = 0$$

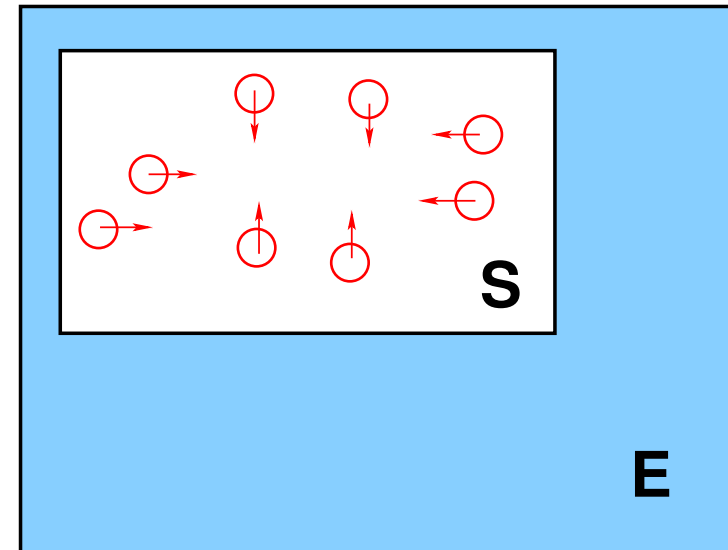
$$\langle f_i(t) f_j(t') \rangle = 2\theta \delta_{ij} \delta(t - t')$$

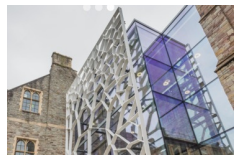
Driving in bulk

e.g. active matter

$$\int_S |v_i| \propto S$$

Classical





# Driven systems

Langevin equation  $\rightsquigarrow$  Fokker-Planck

Many particle density  $P(x_1, \dots, x_N; t) = \left\langle \prod_{i=1}^N \delta(x_i - r_i(t)) \right\rangle$

$$\frac{\partial}{\partial t} P(\vec{x}; t) + \sum_{i=1}^N \nabla_i J_i = 0$$



$$J_i = -\theta \nabla_i P - (\nabla_i \mathcal{H} - v_i) P$$

Q: Is there an equivalent to equilibrium for these systems?

A: Yes, but we must generalize the idea of a steady state

## Non-equilibrium steady states





# A theorem

Active matter has non-equilibrium steady states characterized by 2 quantities

- 1) A **stable** many particle distribution (like equilibrium)
- 2) A **deterministic** dynamical system (unlike equilibrium)

$$\rho_{ss}(\vec{x}) \equiv \frac{1}{Z} \exp[-h(\vec{x})]$$

Bakry, Emery, Guionnet, ...

The system follows the **typical** trajectories

$$\vec{X}(t) = (X_1(t), \dots, X_N(t))$$

$$\frac{d}{dt} \vec{X} = \vec{V}[h(\vec{X})]$$

$$V_i = v_i - \nabla_i \mathcal{H} + \theta \nabla_i h$$

The stationarity condition

$$\sum_i \nabla_i (\rho_{ss} V_i) = 0 \Rightarrow h(\vec{x}) \quad (*)$$



# Summary

equilibrium

$$\rho(\vec{r}) = \frac{1}{Z} \exp[-\mathcal{H}/T]$$

$$\vec{J}_{eq} = 0 \quad \Rightarrow \quad \frac{d\vec{X}}{dt} = 0$$

ness

$$\rho_{ss}(\vec{x}) \equiv \frac{1}{Z} \exp[-h(\vec{x})]$$

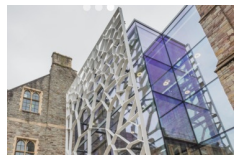
$$\sum_i \nabla_i (\rho_{ss} V_i) = 0 \quad \Rightarrow \quad h(\vec{x})$$

$$\vec{J}_{ss} = \rho_{ss} \vec{V} \quad \Rightarrow \quad \frac{d\vec{X}}{dt} = \vec{V}$$

$$\vec{V} = \vec{v} - \vec{\nabla} \mathcal{H} + \theta \vec{\nabla} h$$

**T.B. Liverpool**, PRE 101, 042107 (2020)

sufficient but not necessary



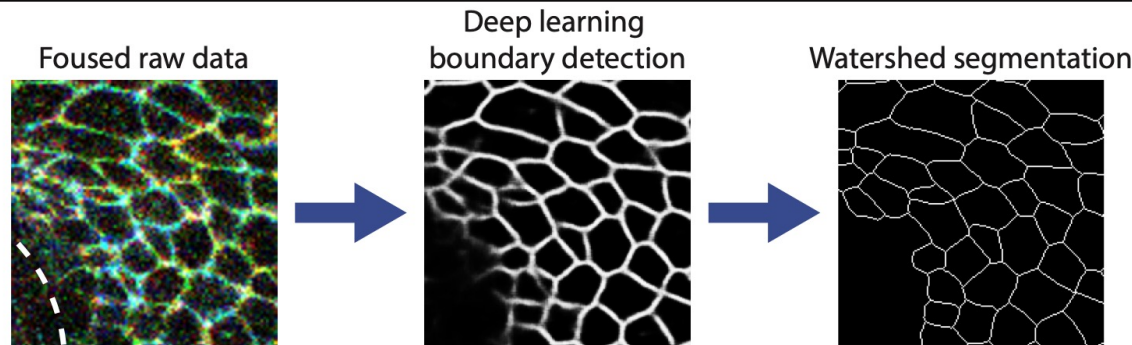
# Quantifying the tissue dynamics on a cellular scale



# Analysis of shapes

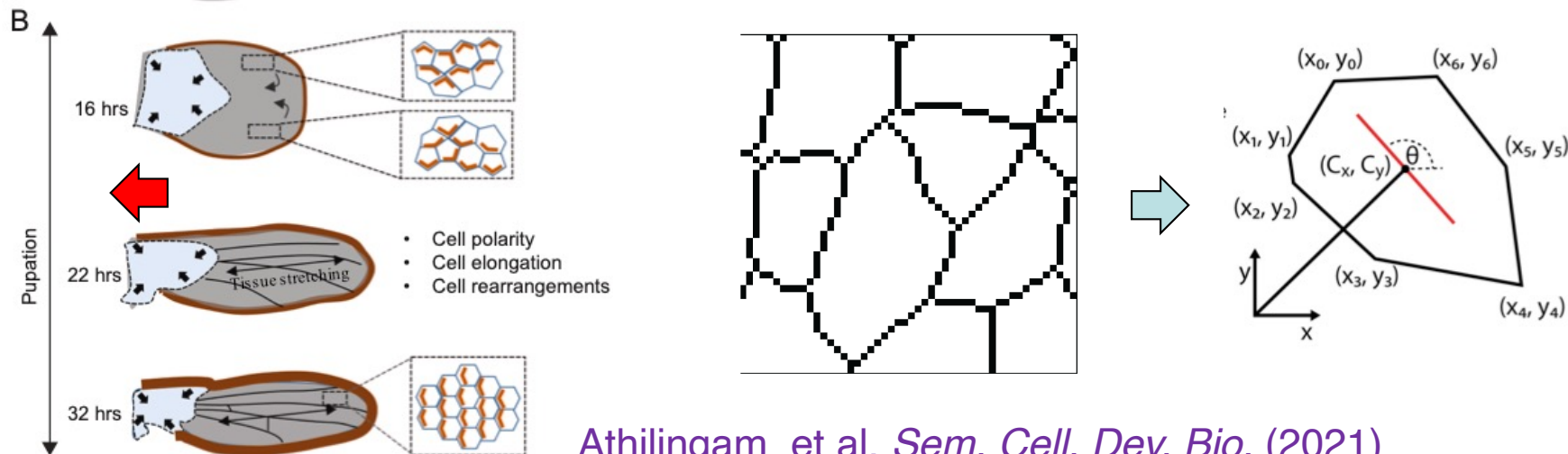
Let's look at cells of healthy unwounded tissue in detail

## Binary Shape Detection



Polygon  
5-16 sides

## Stage of development

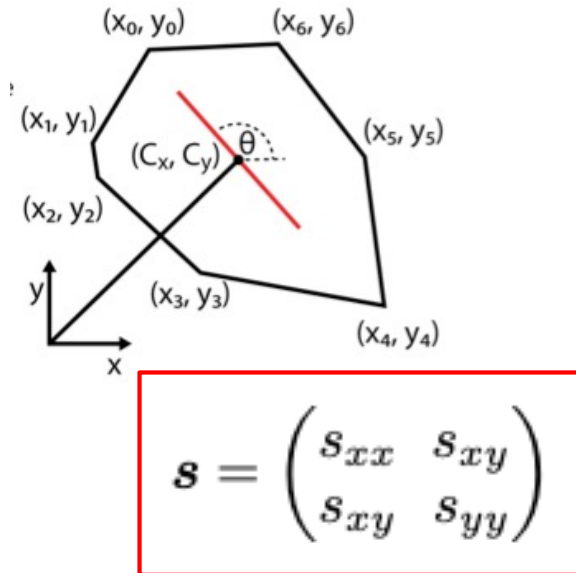


Athilingam et al, *Sem. Cell. Dev. Bio.*, (2021)



# Analysis of shapes

## Cell shapes & protein distributions



Area

$$A = \iint_A dx dy$$

Centre

$$C_x = \frac{1}{A} \iint_A x dx dy$$

$$C_y = \frac{1}{A} \iint_A y dx dy$$

Shape tensor  $f(x, y) = 1$

Protein distribution tensor

$$f(x, y) = \text{prot. conc.}$$

$$y' = y - C_y \quad , \quad x' = x - C_x$$

$$s_{xx} = -\frac{1}{A^2} \iint_A f(x', y') y'^2 dx' dy'$$

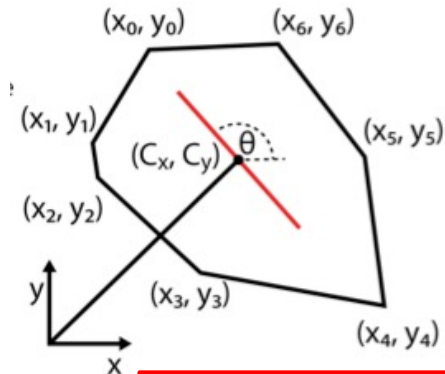
$$s_{xy} = \frac{1}{A^2} \iint_A f(x', y') x' y' dx' dy'$$

$$s_{yy} = -\frac{1}{A^2} \iint_A f(x', y') x'^2 dx' dy'$$



# Analysis of shapes

## Cell shapes & protein distributions



$$\mathbf{s} = \begin{pmatrix} s_{xx} & s_{xy} \\ s_{xy} & s_{yy} \end{pmatrix}$$

Area

$$A = \iint_A dx dy$$

Centre

$$C_x = \frac{1}{A} \iint_A x dx dy$$

$$C_y = \frac{1}{A} \iint_A y dx dy$$

Shape tensor  $f(x, y) = 1$

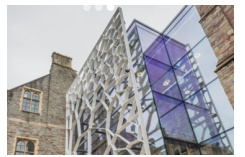
Protein distribution tensor

$$f(x, y) = \text{prot. conc.}$$

$$s_{xx} = -\frac{1}{A^2} \iint_A f(x', y') y'^2 dx' dy'$$

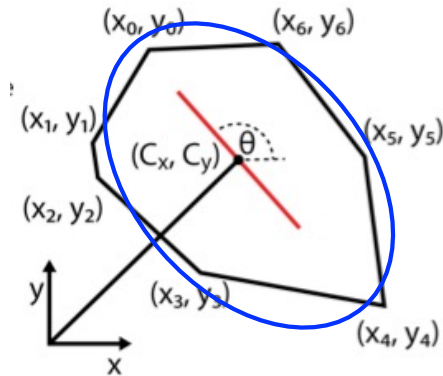
$$s_{xy} = \frac{1}{A^2} \iint_A f(x', y') x' y' dx' dy'$$

$$s_{yy} = -\frac{1}{A^2} \iint_A f(x', y') x'^2 dx' dy'$$

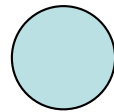


# Analysis of shapes

## Cell shapes & protein distributions



Eigenvalues of shape tensor  $\lambda_1, \lambda_2$

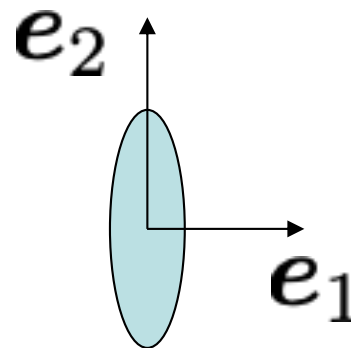


$$\lambda_1 = \lambda_2$$



$$|\lambda_1 - \lambda_2| > 0$$

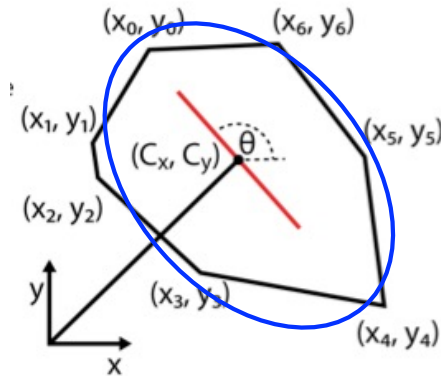
Eigenvectors  $\Rightarrow$  orientation





# Analysis of shapes

## Cell shapes & protein distributions



Q-tensor  $\mathbf{q} = \mathbf{s} - \text{Tr}(\mathbf{s})\mathbf{I}$

$$\mathbf{q} = \begin{pmatrix} q_1 & q_2 \\ q_2 & -q_1 \end{pmatrix} \neq 0 \quad \rightarrow \text{elongated cell}$$

$$\mathbf{q} = \begin{pmatrix} q_1 & q_2 \\ q_2 & -q_1 \end{pmatrix} = \frac{q_0}{\sqrt{2}} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = q_0 \hat{\mathbf{q}} \quad q_0^2 = \|\mathbf{q}\|^2 = \frac{1}{2} \text{Tr}(\mathbf{q}^2)$$

Average over N cells

$$\alpha \in [1, 2, \dots, N]$$

Mean

$$\mathbf{Q} \hat{\mathbf{Q}} = \frac{1}{N} \sum_{\alpha=1}^N \hat{\mathbf{q}}_{\alpha}$$

Variance

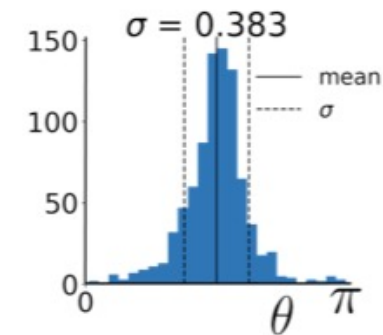
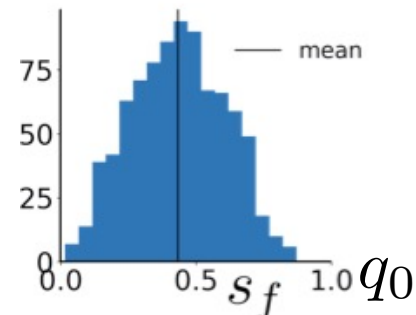
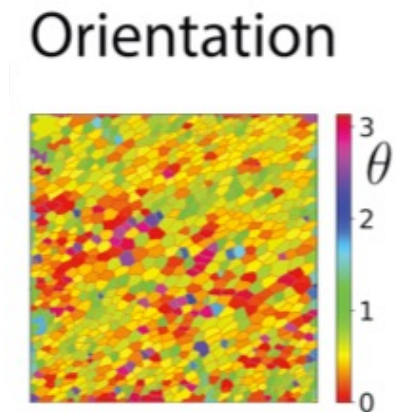
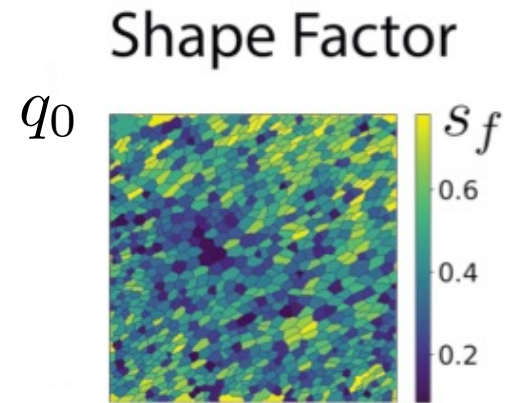
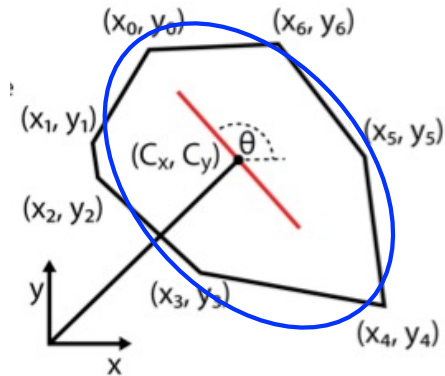
$$\sigma_q^2 = \frac{1}{N} \sum_{\alpha=1}^N \left\| \hat{\mathbf{q}}_{\alpha} - \hat{\mathbf{Q}} \right\|^2$$





# Analysis of shapes

## Cell shapes & protein distributions



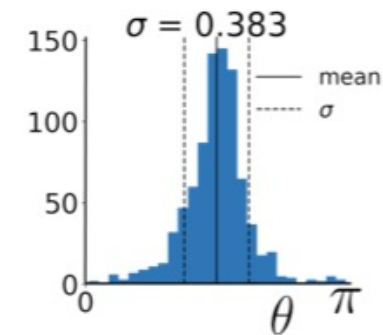
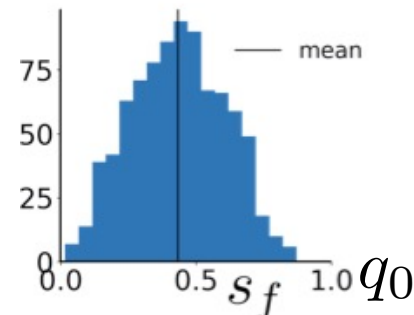
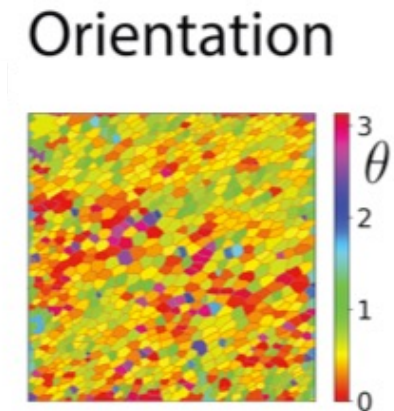
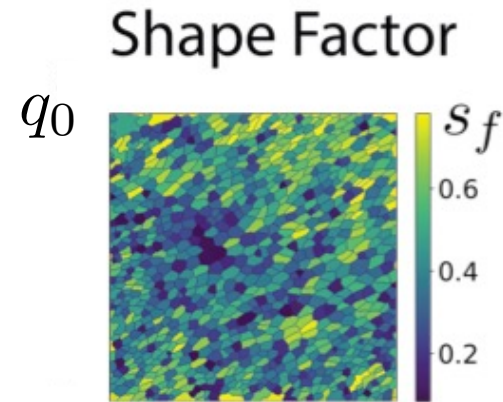
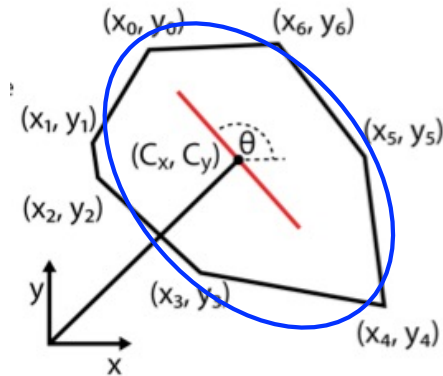
Average over cells

mean and variance



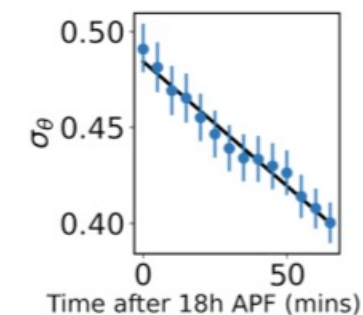
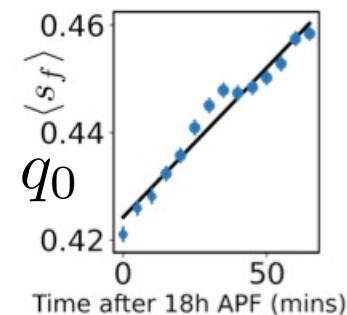
# Analysis of shapes

## Cell shapes & protein distributions



Average over cells

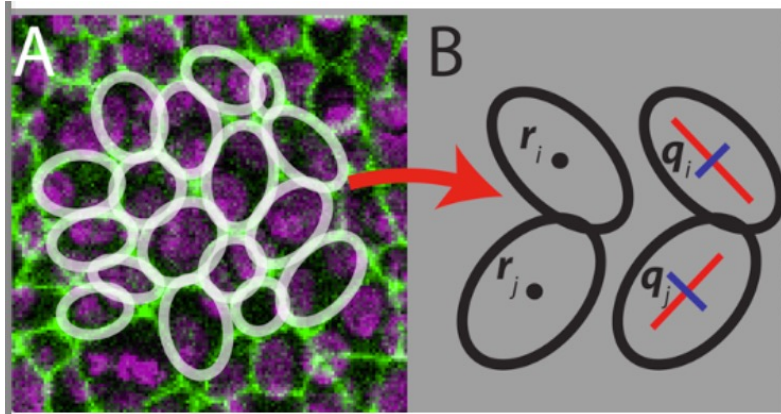
mean and variance







# “Noisy” dynamics of tissue

Microscopic degrees of freedom, “atoms”  $\vec{X} = (\mathbf{r}_1, \mathbf{q}_1, \dots, \mathbf{r}_N, \mathbf{q}_N)$



Positions  $\mathbf{r}_i(t)$    
Shapes  $\mathbf{q}_i(t)$  

$$U = \sum_{i \neq j} [W_0(\mathbf{r}_i - \mathbf{r}_j) + W_2(\mathbf{r}_i - \mathbf{r}_j) \text{Tr}(\mathbf{q}_i \mathbf{q}_j)]$$

$$\zeta_q \partial_t q_i^{\alpha\beta} = -b \left( q_i^{\alpha\beta} - \bar{q}^{\alpha\beta} \right) - \frac{\partial U}{\partial q_i^{\alpha\beta}} + \xi_i^{\alpha\beta}$$

$$\zeta \partial_t r_i^\alpha = \bar{V}^\alpha(t) - \frac{\partial U}{\partial r_i^\alpha} - b' \sum_j \left( q_i^{\alpha\beta} - q_j^{\alpha\beta} \right) \cdot \left( r_i^\beta - r_j^\beta \right) f(|\mathbf{r}_i - \mathbf{r}_j|) + \eta_i^\alpha$$

$$\langle \xi_i^{\alpha\beta}(t) \rangle = 0$$

$$\langle \eta_i^\alpha(t) \rangle = 0$$

$$\langle \xi_i^{\alpha\beta}(t) \xi_j^{\alpha'\beta'}(t') \rangle = C_q \delta_{\alpha\alpha'} \delta_{\beta\beta'} \delta_{ij} \delta(t - t') \quad \langle \eta_i^\alpha(t) \eta_j^{\alpha'}(t') \rangle = C_v \delta_{\alpha\alpha'} \delta_{ij} \delta(t - t')$$



# “Noisy” dynamics of tissue

Collective degrees of freedom

Density  $\left( \frac{1}{\text{Area}} \right)$

$$\rho(\mathbf{r}, t) = \sum_i \langle \delta(\mathbf{r} - \mathbf{r}_i(t)) \rangle$$

Shape

$$\rho(\mathbf{r}, t) Q_{\alpha\beta}(\mathbf{r}, t) = \sum_i \langle q_i^{\alpha\beta}(t) \delta(\mathbf{r} - \mathbf{r}_i(t)) \rangle$$

Velocity

$$\rho(\mathbf{r}, t) V_\alpha(\mathbf{r}, t) = \sum_i \langle \partial_t r_i^\alpha(t) \delta(\mathbf{r} - \mathbf{r}_i(t)) \rangle$$

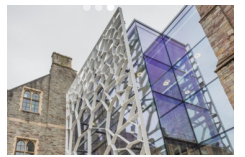
Coarse grain  $\Rightarrow$  Hydrodynamic equations

$$\partial_t \rho(\mathbf{r}, t) + \nabla_\alpha J_\alpha[\rho, \mathbf{Q}, \mathbf{V}] = d - a$$

$$\partial_t Q_{\alpha\beta}(\mathbf{r}, t) = \Theta^{\alpha\beta}[\rho, \mathbf{Q}, \mathbf{V}]$$

$$V_\alpha(\mathbf{r}, t) = \nu^\alpha[\rho, \mathbf{Q}, \mathbf{V}]$$

+ other fields ...



# “Noisy” dynamics of tissue

Linearised fluctuating hydrodynamics

Density  $\rho(\mathbf{r}, t) = \rho^*(t) + \bar{\rho}(\mathbf{r}) + \delta\rho(\mathbf{r}, t)$

Shape  $Q_{\alpha\beta}(\mathbf{r}, t) = Q_{\alpha\beta}^*(t) + \bar{Q}_{\alpha\beta}(\mathbf{r}) + \delta Q_{\alpha\beta}(\mathbf{r}, t)$

Velocity  $V_{\alpha}(\mathbf{r}, t) = V_{\alpha}^*(t) + \bar{V}_{\alpha}(\mathbf{r}) + \delta V_{\alpha}(\mathbf{r}, t)$

Deterministic + **Fluctuations**

$$\partial_t \delta\rho(\mathbf{r}, t) = D\nabla^2 \delta\rho + \nabla_{\alpha} \xi_{\alpha}^{\rho}(\mathbf{r}, t)$$

$$\partial_t \delta Q_{\alpha\beta}(\mathbf{r}, t) = [-b - 2B\rho^* + \rho^* L_{\gamma\epsilon} \nabla_{\gamma} \nabla_{\epsilon}] \delta Q_{\alpha\beta} + \xi_{\alpha\beta}^Q(\mathbf{r}, t)$$

$$\delta V_{\alpha}(\mathbf{r}, t) = -A\nabla_{\alpha} \delta\rho + \xi_{\alpha}^V(\mathbf{r}, t)$$

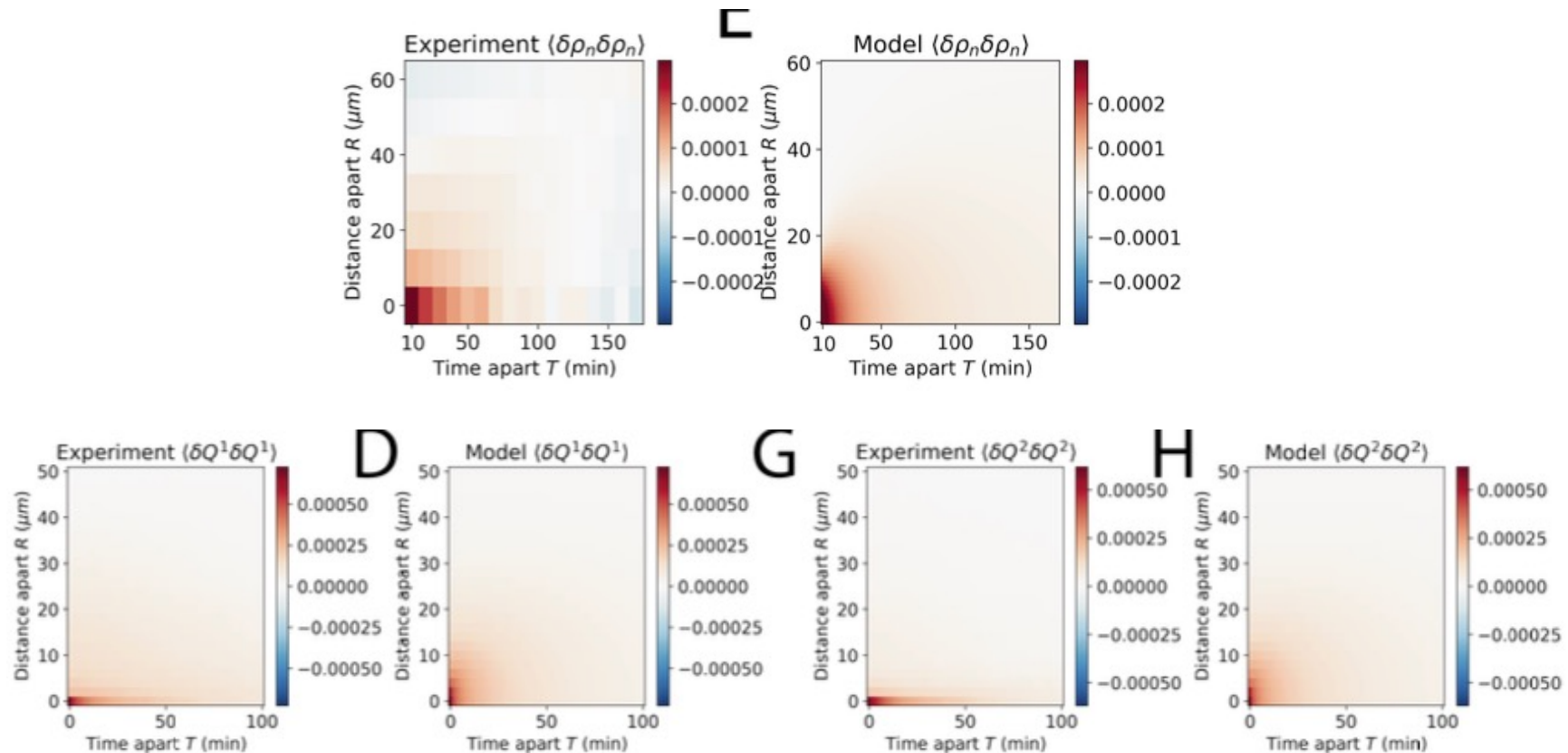
$$D, B, L, A \Leftrightarrow W_0, W_2, b, b'$$



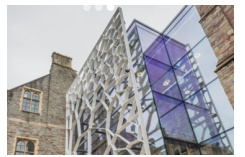
# “Noisy” dynamics of tissue

## Correlation functions

$$\langle \delta\rho(\mathbf{r}, t)\delta\rho(\mathbf{r}', t') \rangle, \quad \langle \delta Q_{\alpha\beta}(\mathbf{r}, t)\delta Q_{\alpha'\beta'}(\mathbf{r}', t') \rangle$$



$$D, B, L, A \Leftrightarrow W_0, W_2, b, b'$$

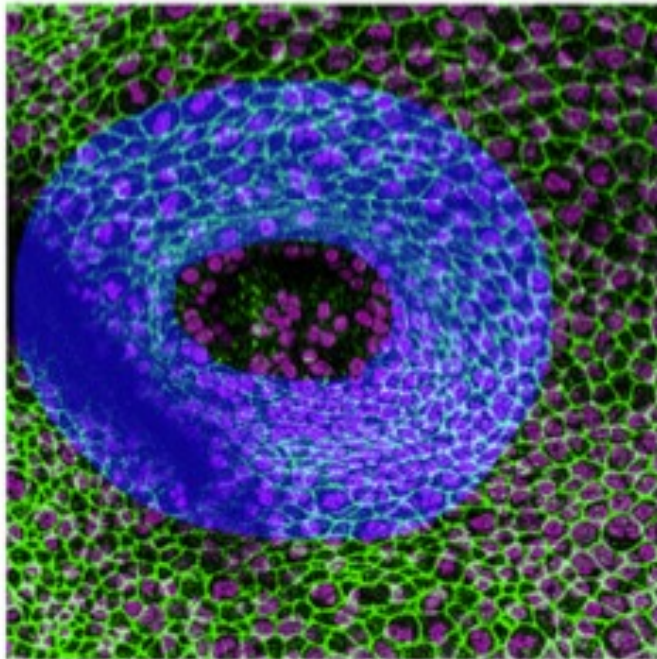


# “Noisy” dynamics of tissue

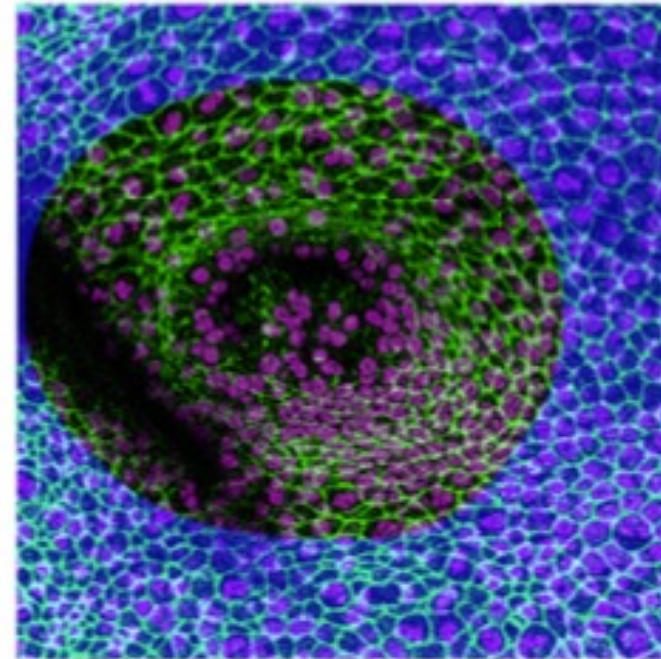
Example

$$\langle \delta\rho(\mathbf{r}, t)\delta\rho(\mathbf{r}', t') \rangle \quad , \quad \langle \delta Q_{\alpha\beta}(\mathbf{r}, t)\delta Q_{\alpha'\beta'}(\mathbf{r}', t') \rangle$$

Close to wound



Far from wound



Compare to unwounded - major change only in  $b$  close to wound !!

# Calcium wave, JNK signalling and inflammation

## Wound-induced epithelial signals

Calcium wave – one of the first signals that occurs within seconds of wounding (using *act* > *trpm-RNAi*). Does it drive these cell behaviours?

JNK signalling – Associated in literature with shape changes in development (using *act* > *bsk<sup>DN</sup>*)

## Inflammation

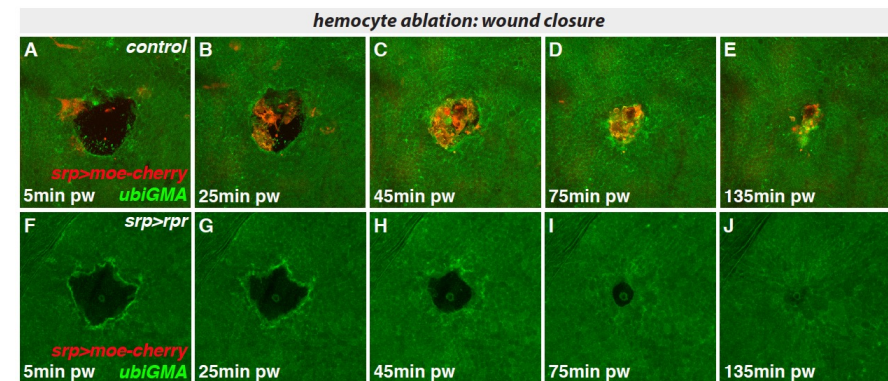
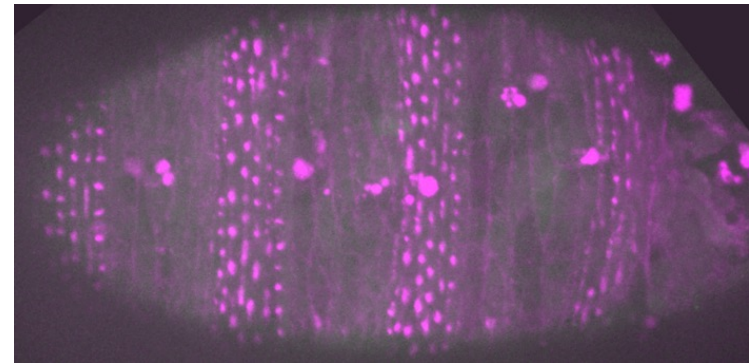
Test wound healing after genetic macrophage ablation (using *srp* > *reaper*)

**Report**

**Calcium Flashes Orchestra the Wound Inflammatory Response through DUOX Activation and Hydrogen Peroxide Release**

William Razzell,<sup>1,3</sup> Iwan Robert Evans,<sup>2,3</sup> Paul Martin,<sup>1,4,\*</sup> and Will Wood<sup>2,4,\*</sup> via the GAL4-UAS system [15] by using the e22c-Gal4 driver [16]. We coexpressed mCherry-moesin to visualize cortical

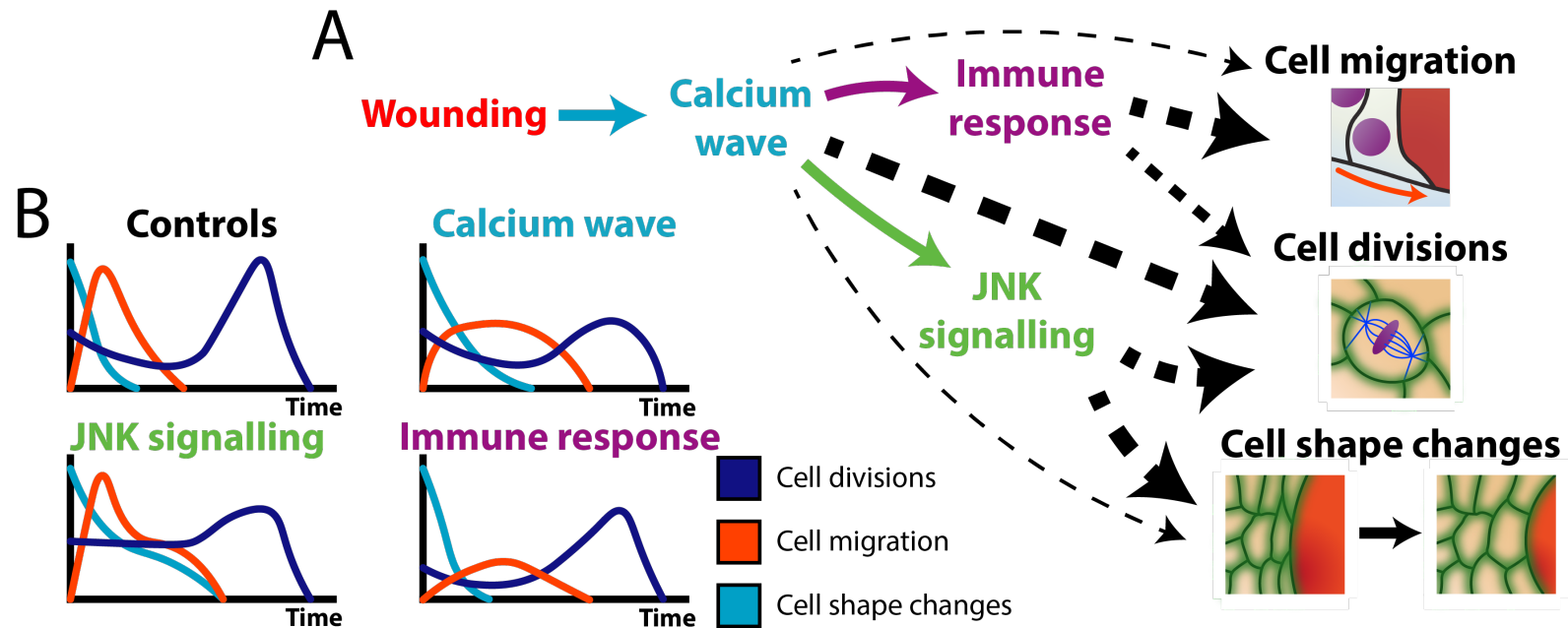
CellPress





# Calcium wave, JNK signalling and inflammation

- Quantify how perturbations change parameters



J Turley et al, bioarxiv (2024)

# conclusions and perspectives



machine learning tools have been developed and are being used to **observe** fruitfly pupa wounds in wing tissue

We can thus **analyse** the tissue behaviour at the cellular scale with unprecedented detail (in particular fluctuations!)

We are beginning to come up with mathematical models that give **explanations** of some of the emergent behaviour that we see ....

... but we are definitely closer to the beginning of the story than the end

**M. Olenik et al, PRE, 107, 014403 (2023)**

**J Turley et al, eLife 12 : RP87949 (2023)**

**J Turley et al, Development 151 : dev202943 (2024)**