FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

Constructing a consistent and computable ‘quantum spacetime’

New tensor-network algorithms

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Emmy Noether-Programm

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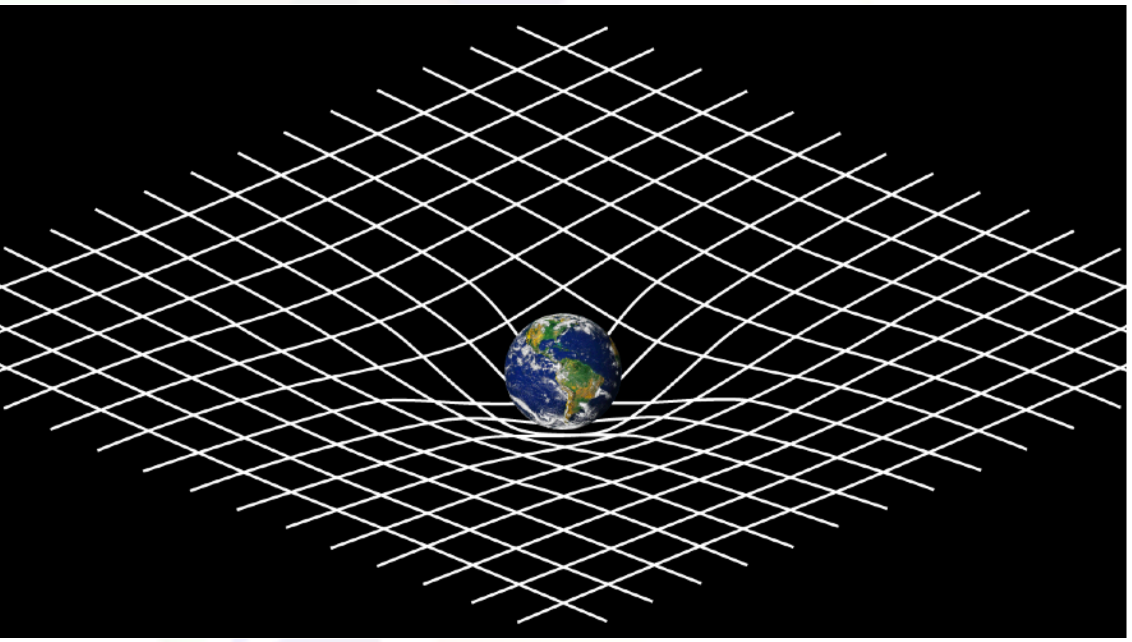
Outline

- **Constructing quantum space-time**
  - Path Integral formulation
- **Quantum geometry**
  - Spin networks
- **Tensor network algorithms**
  - Computational challenges and opportunities
- **Diversity experiences**
  - The good and the bad

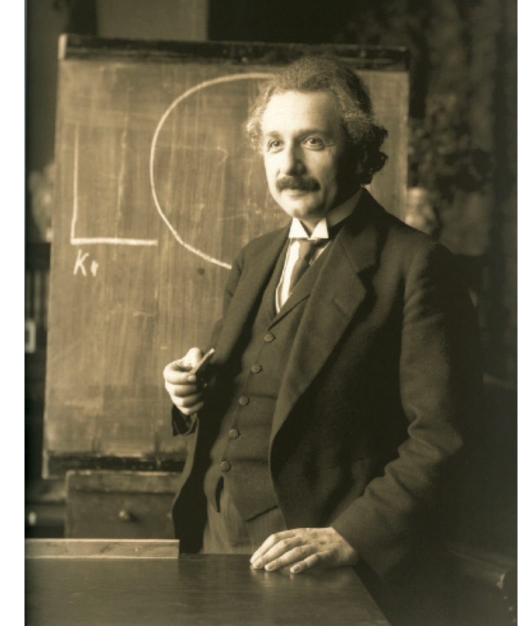
Gravity

Gravity = geometry of spacetime

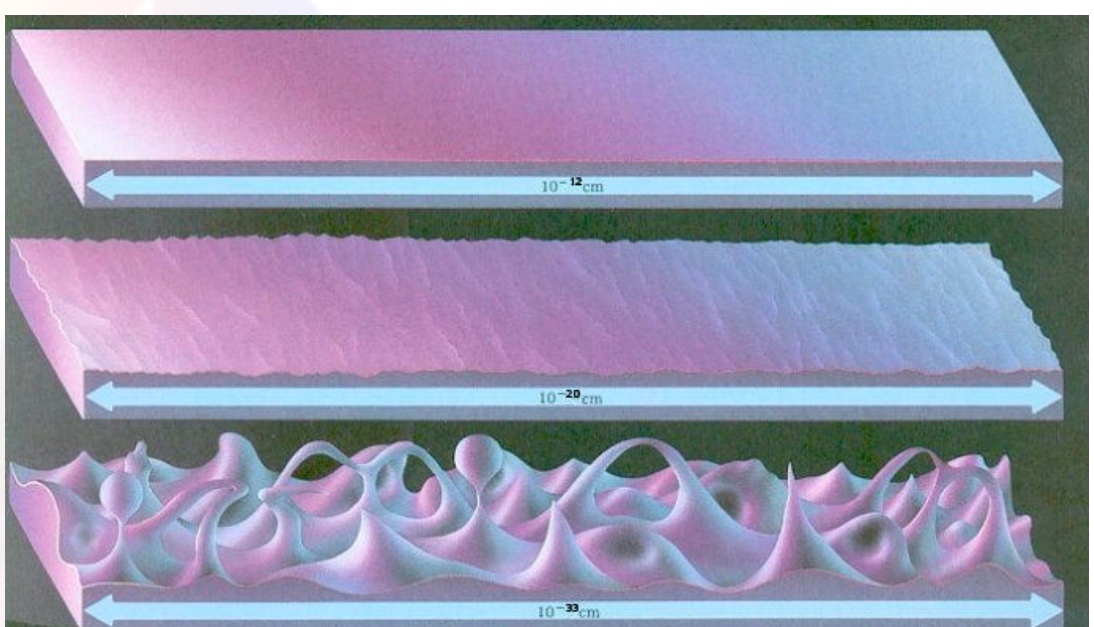


Credit: NASA Astrophysics

- Bending of light
- Gravitational waves
- Black holes



[A. Einstein]



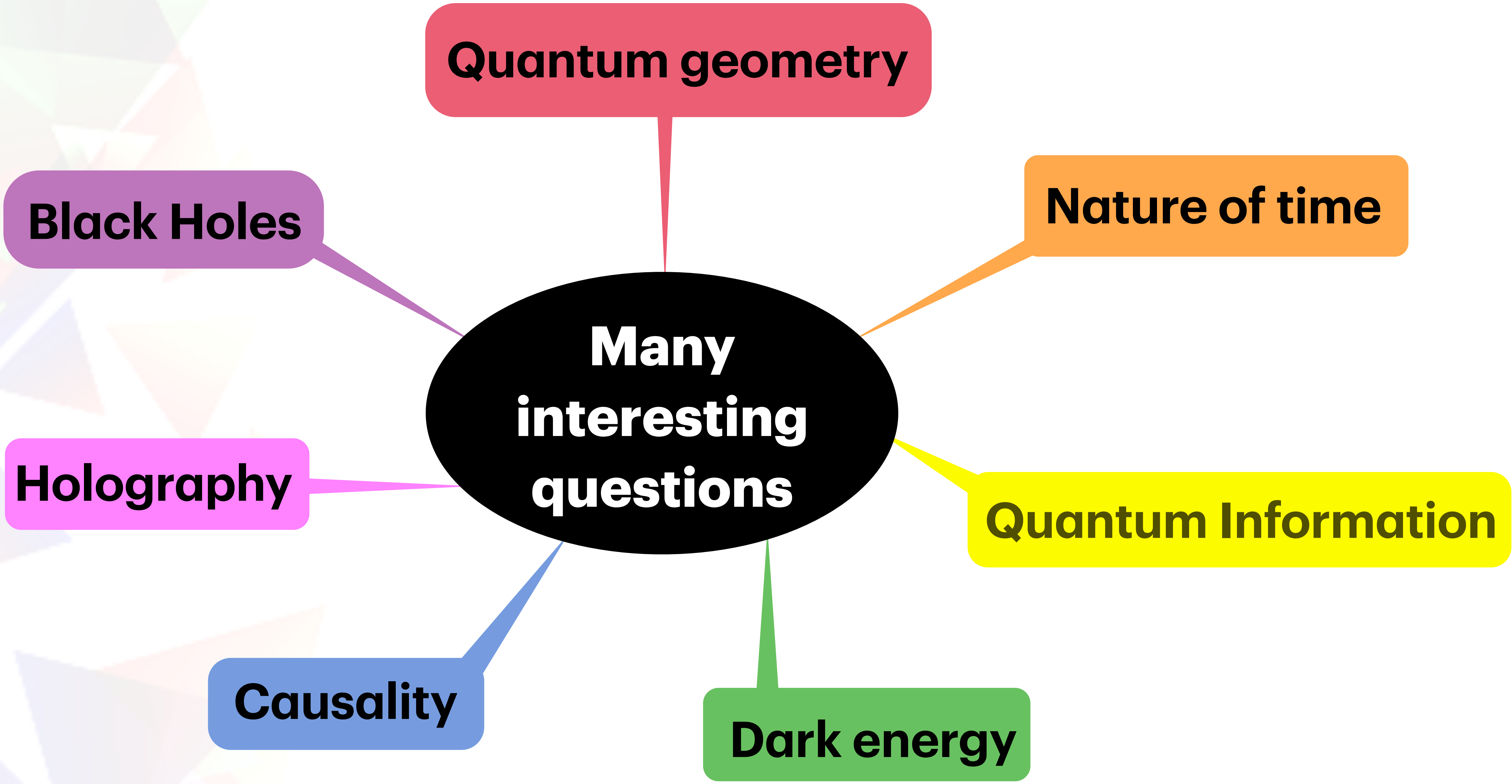
Credit: universe-review.ca

Breakdown of at very short distances

Quantum gravity:

Structure of quantum spacetime

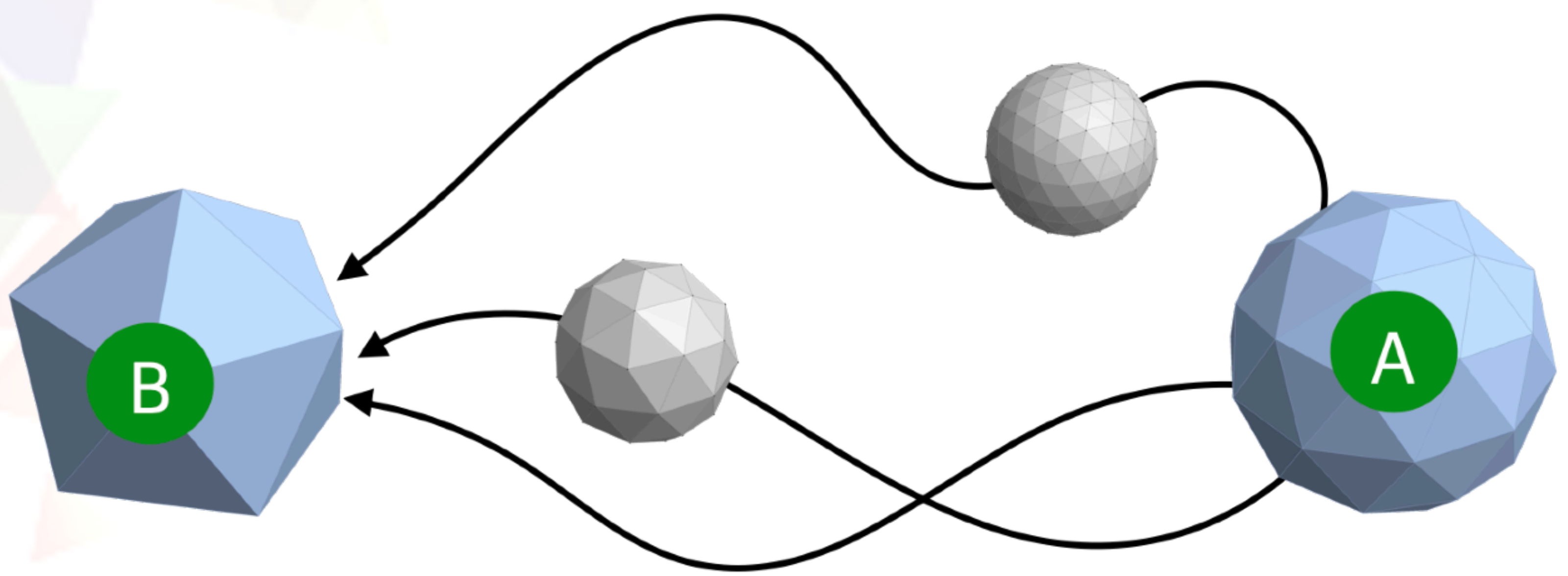
A change of perspective



How do we define a quantum spacetime?

[Feynman, Schwinger,..]

Path Integral A formalism adopted by many approaches



$$Z[A;B] = \int_{A}^{B} D\mu(\text{geom}) \;\; \exp(\mathrm{i}\, S[\text{geom}])$$

<u>Statistics of geometry</u>

Transition amplitude between states of geometry

- Sum over histories of '<u>all possible</u>' geometries

What are the fundamental constituents of quantum geometry ?

Quantum geometry

Diversity of representations of geometry

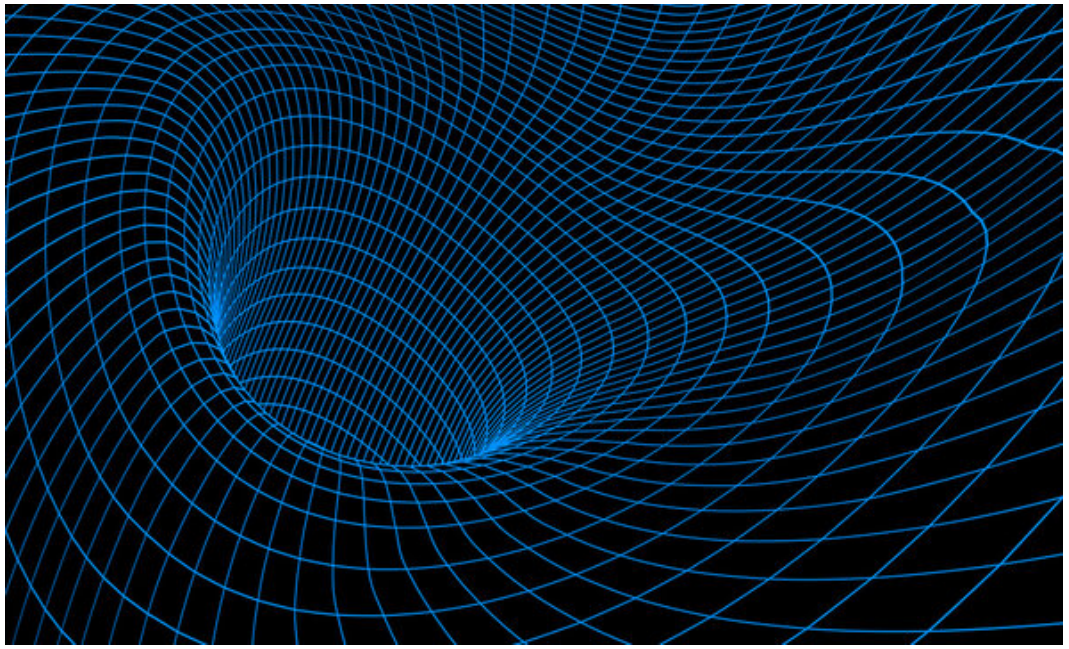
Metric geometry

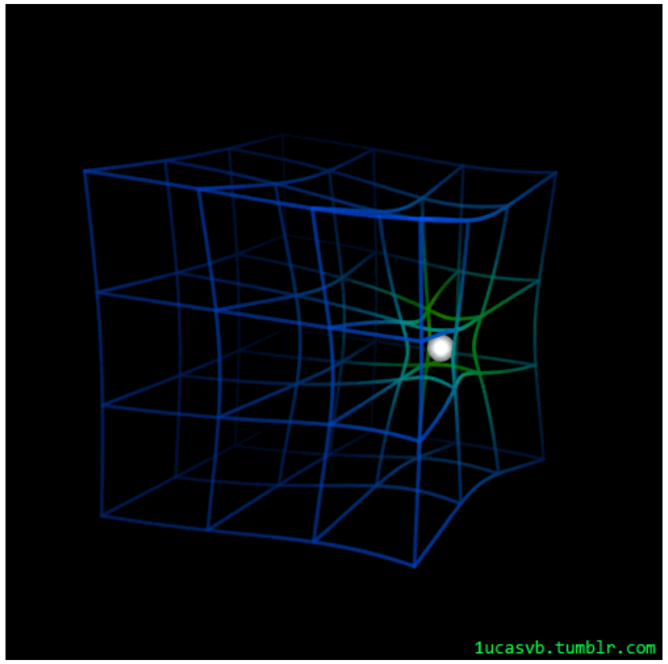
Discrete structures

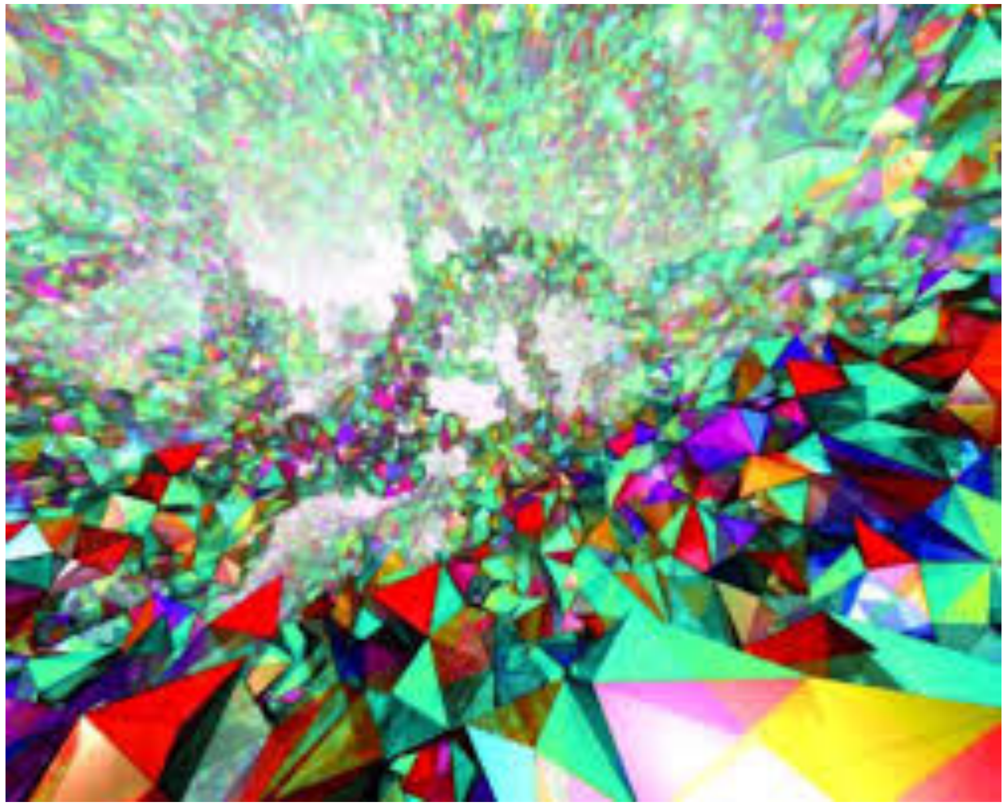
Spin networks

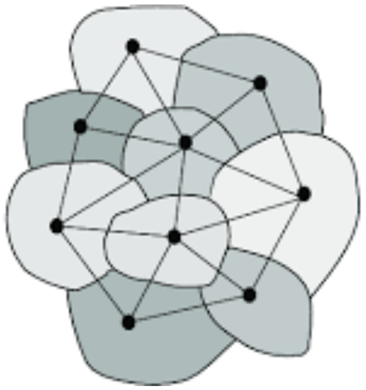
Higher-gauge theories

Emergent geometry









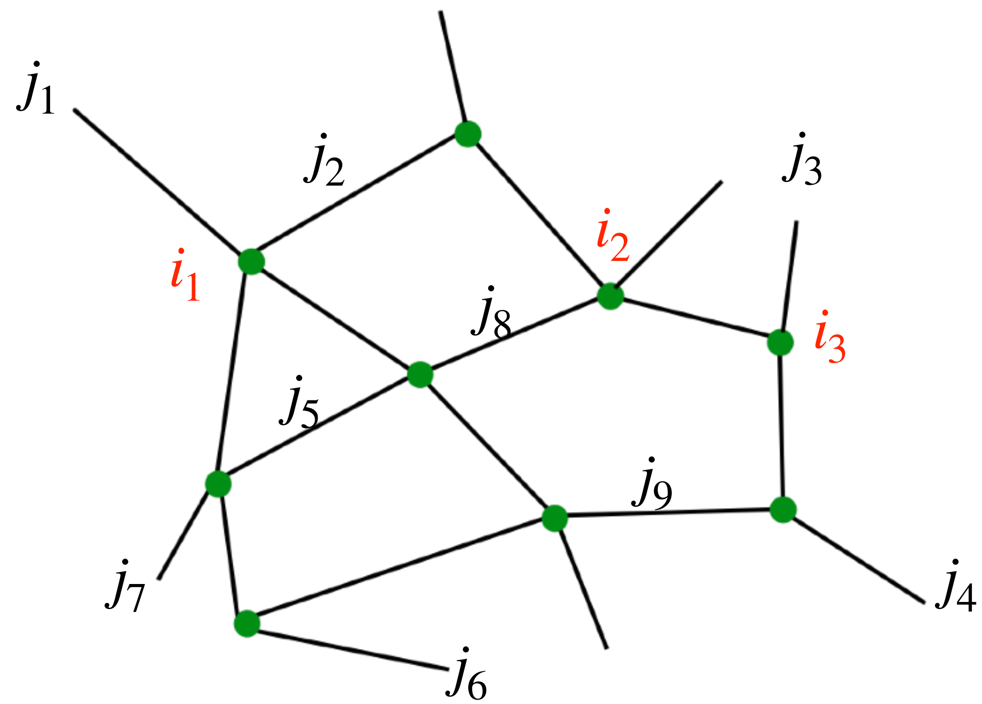
Quantum geometry

[R. Penrose..]

Spin Networks Mathematically well-defined structures

Decorated graphs

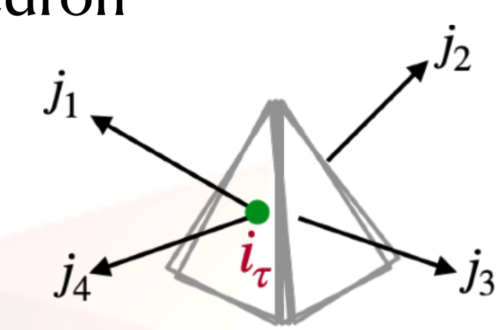
Encode quantum geometries



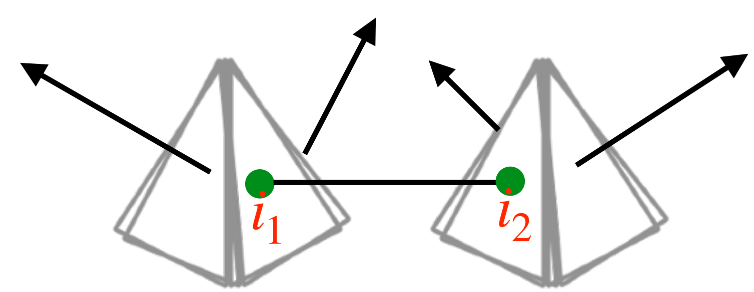
Labels: i_e, j_f

quantum numbers

quantum tetrahedron



gluing geometries

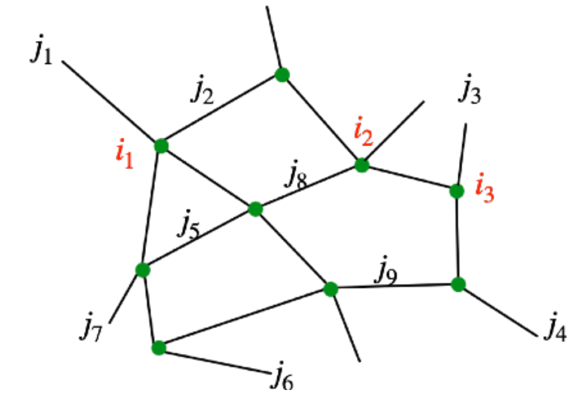


Functional space of connections invariant under local gauge transformations

Quantum geometry

Spin Networks

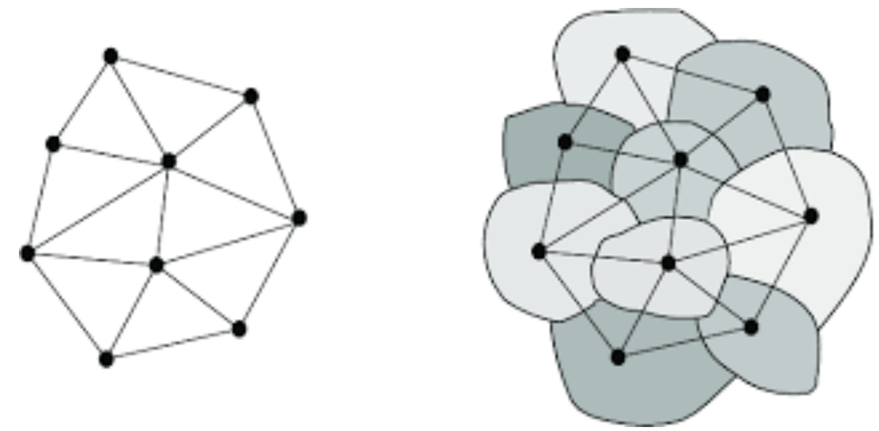
Some properties of quantum geometry



Discrete quanta of space

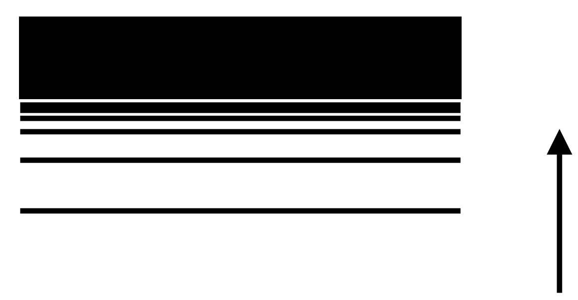
- well-defined ‘operators’ that measure:

  lengths, areas, volumes



- non-commuting geometric operators

- eigenvalues are discrete



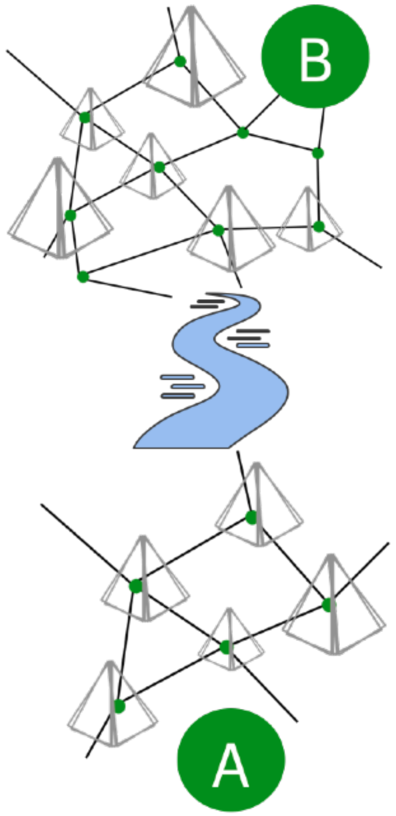
area gap:

$\sim \ell^2_{\text{Planck}}$

spectrum

$j = 0$

Dynamics of quantum geometry

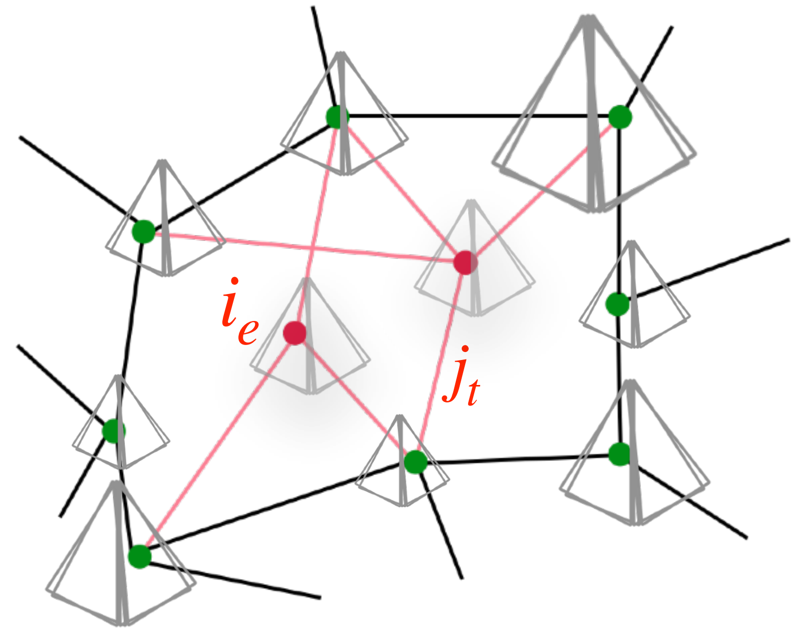


Spin foam models

Path integral over 'discrete' quantum geometries

- sum over histories of spin-networks

$$Z[A;B] = \sum_{\{i_e, j_t\}} f\left(\right.$$



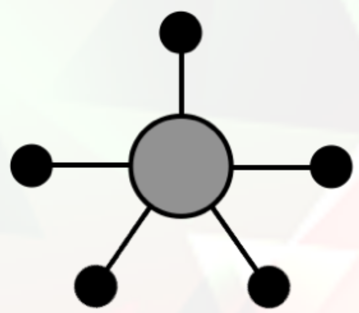
$$\left. \right)$$

- defined on a fixed graph or lattice

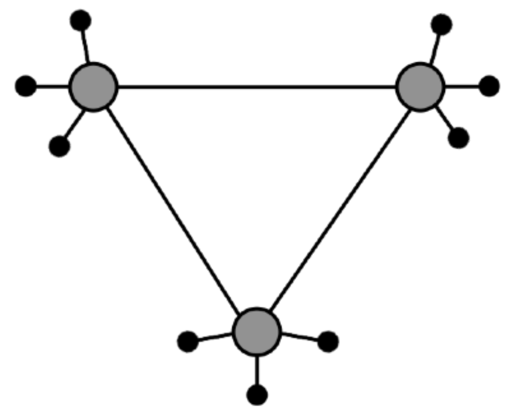
[Perez, Livine, Bianchi, Riello, Oriti, Haggard, Geiller, Dittrich, Bahr, Ryan, Steinhaus, Delcamp, Goeller, Dupius, Girelli, Engle, Pereira, Rovelli, Friedel, Ashtekar, Smolin, Fairbain, Barett, Meusburger, Speziale, Vidotto, Dona, Gozzini, Sarno, Thiemann, Han, Lui, Lewandowski, Corichi, Kaminski, Ricardo, Oliveira, Krasnov, ..., SKA, ...]

Examples

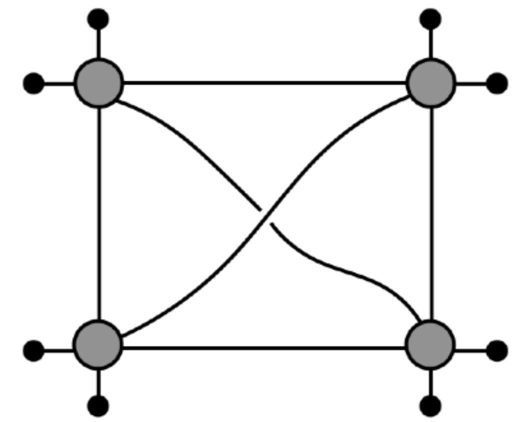
Triangulations



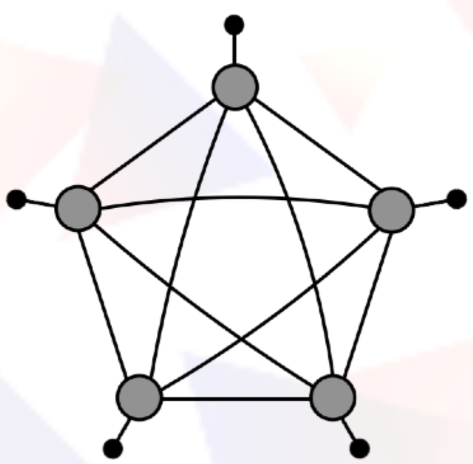
4-simplex



3-3 triangulation

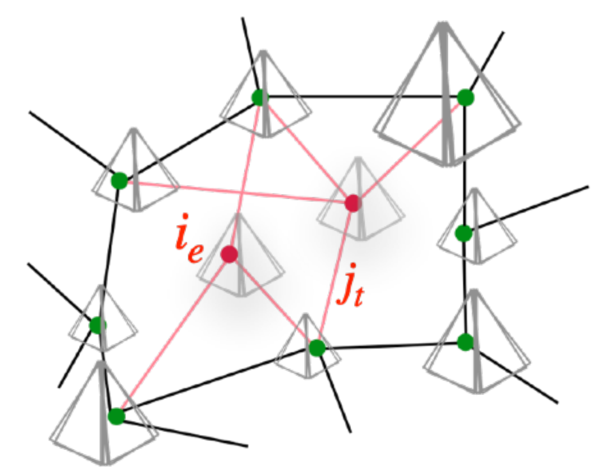


4-2 triangulation



5-1 triangulation

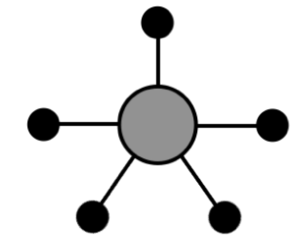
$$Z[A;B] = \sum_{\{i_e, j_t\}} f\left(\right)$$



Follow the connectivity/combinatorics of the triangulation

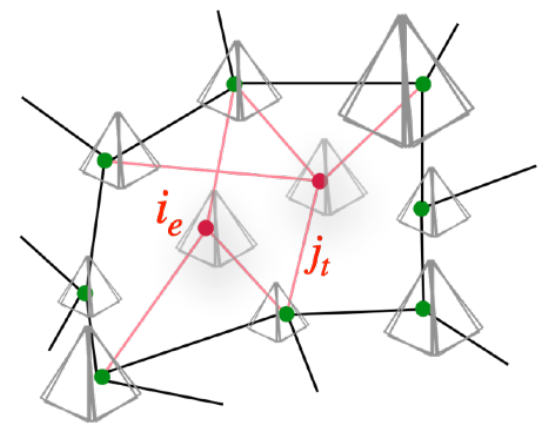
Partition function

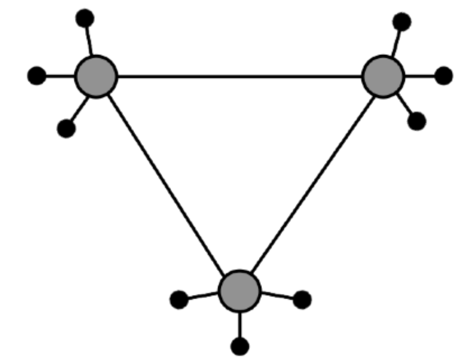
Spin foam models



Technical detail: How do the partition functions look like?

$$Z[A;B] = \sum_{\{i_e, j_t\}} f\Bigg(\ \Bigg)$$

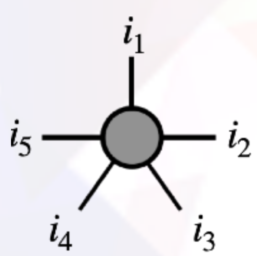




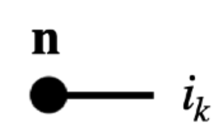
$$Z(\Delta; \{j, \mathbf{n}\}) = \sum_{\{i_e, j_f\}} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f, i_e) \prod_k C_{i_k}(j_f, \mathbf{n})$$

$$A_f(j_f) = 2j_f + 1 = d_j\,, \qquad A_e(i_e) = \langle i_e \,|\, i_e \rangle^{-1}$$

$$A_v(j_f, i_e) = \{15j(i_e, j_f)\} = \begin{Bmatrix} i_1 & j_{13} & i_3 & j_{35} & i_5 \\ j_{12} & j_{23} & j_{34} & j_{45} & j_{15} \\ j_{25} & i_2 & j_{24} & i_4 & j_{14} \end{Bmatrix}$$



$$C_{i_k}(j_f, \mathbf{n}) =$$

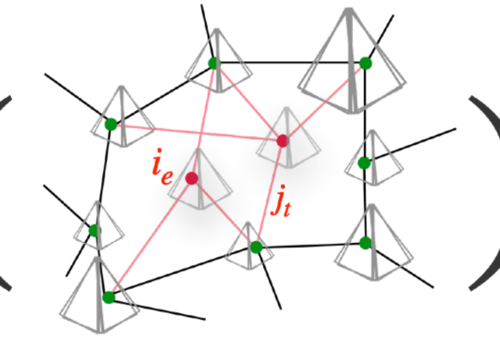


coherent vector

Challenges

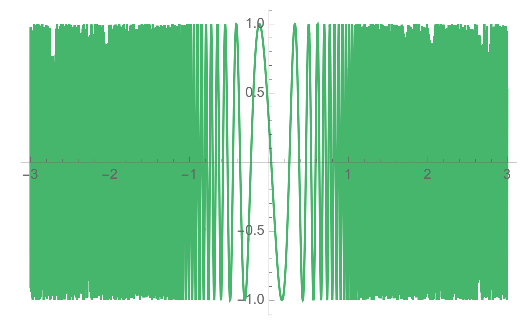
Computations

$$Z[A;B] = \sum_{\{i_e, j_t\}} f\Bigg(\quad\Bigg)$$



Computational challenges

- ✦ Sum over many degrees of freedom
- ✦ Highly oscillatory functions
- ✦ High dimensional integrals



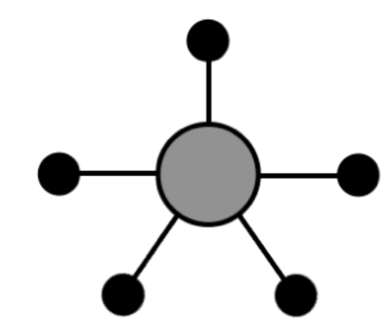
Limited analytical results

- Asymptotic formula for ‘simplest’ possible graph

[Barett et. al .....]

$$Z_v \sim \mathcal{N} \cos\left(S_{\text{Regge}} + \kappa\right)$$

- connection to discrete gravity (Regge action)

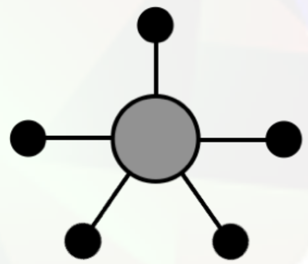


[Speziale, Dona, Sarno, Gozzini, Fanizza, Frisoni, Han, Liu, Qu, Huang, Bahr, Steinhaus, Simao, SKA...]

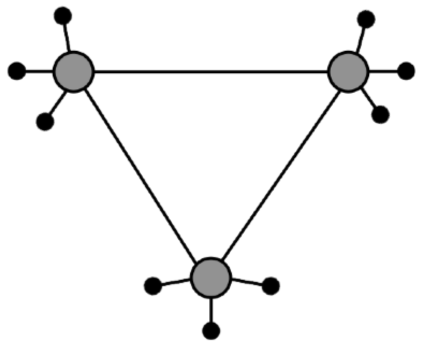
Challenges

Numerical computations

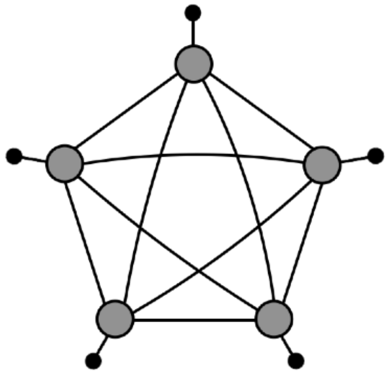
Accelerate computations



4-simplex



3-3 triangulation



5-1 triangulation

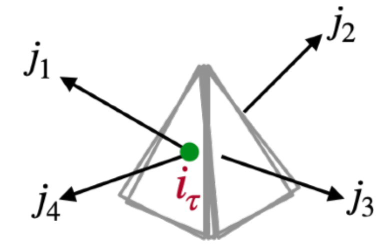
Libraries: **`sl2cfoam`**, **`sl2cfoam-next`**

- perform sums using **tensor contractions**
- make use of **HPC** resources

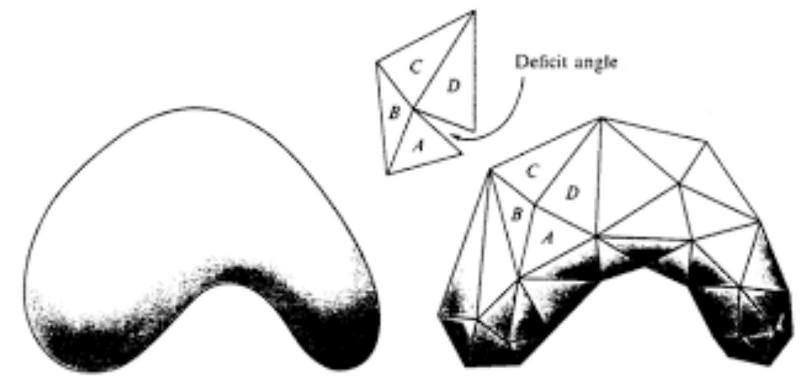
Effective spin foam models

[SKA, Dittrich, Haggard]

Idea:

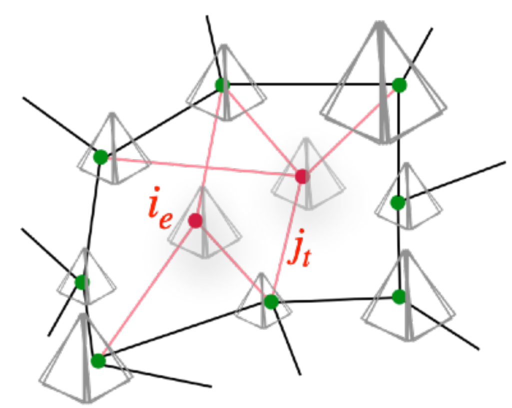


- Keep the principles of discrete quantum geometries
  - discrete area spectrum, non-commutativity



- Use simple amplitudes
  - Regge calculus [Regge 61]

$$Z[A;B] = \sum_{\{i_e, j_t\}} f_{\text{Regge}} \Bigg(\quad \Bigg)$$



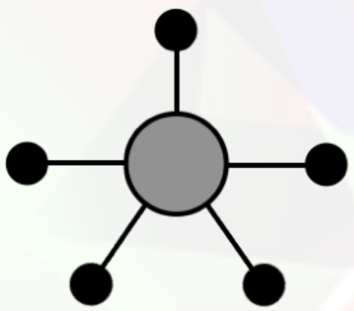
Advantages:

Fast computations, control and test features with complex examples

Improving numerics

A tensor-network algorithm

[Steinhaus, SKA]
[2406.19676 [gr-qc]]



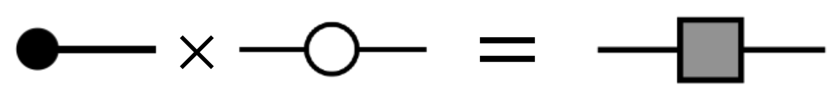
$\simeq \mathcal{O}(d_j^6)$

Simple algorithm

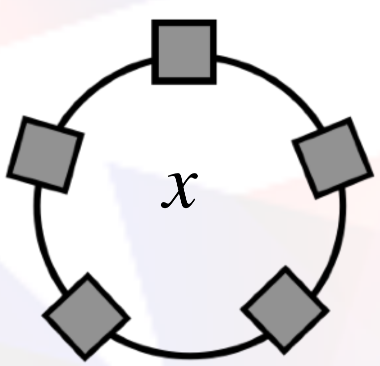
julia

`TNAlgo-su2bf`

- reorganize sums and products of functions to contract low-valent tensor
  - reduced to matrix contractions



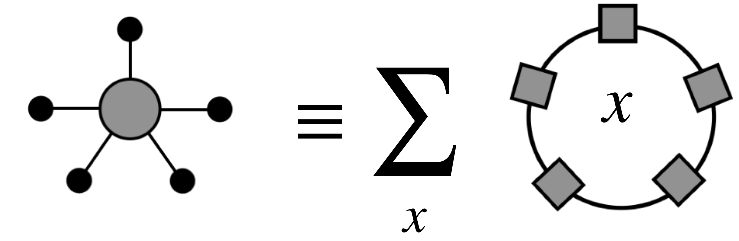
$\sum_x$



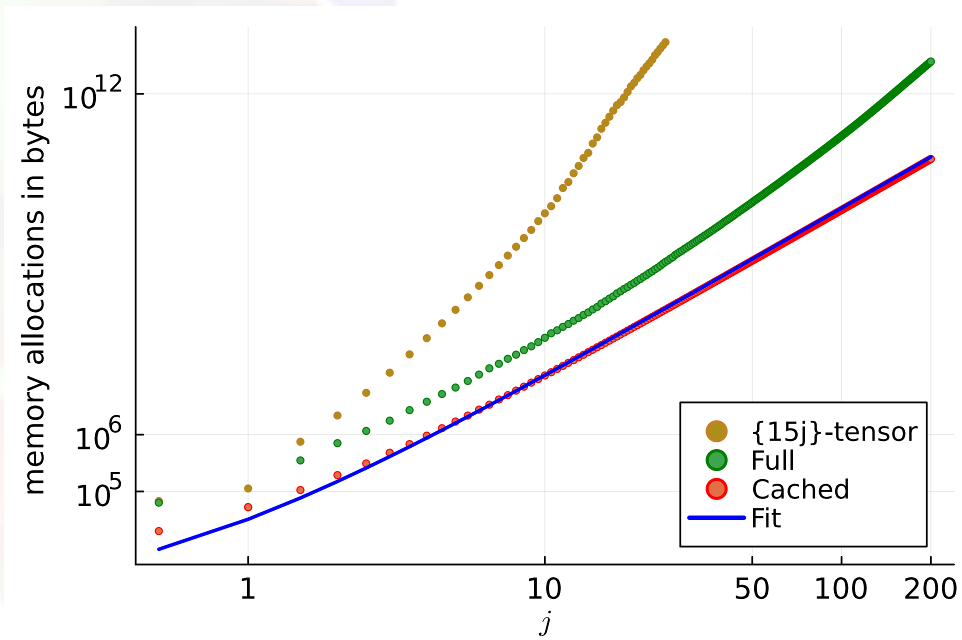
$\simeq \mathcal{O}(d_j^4)$

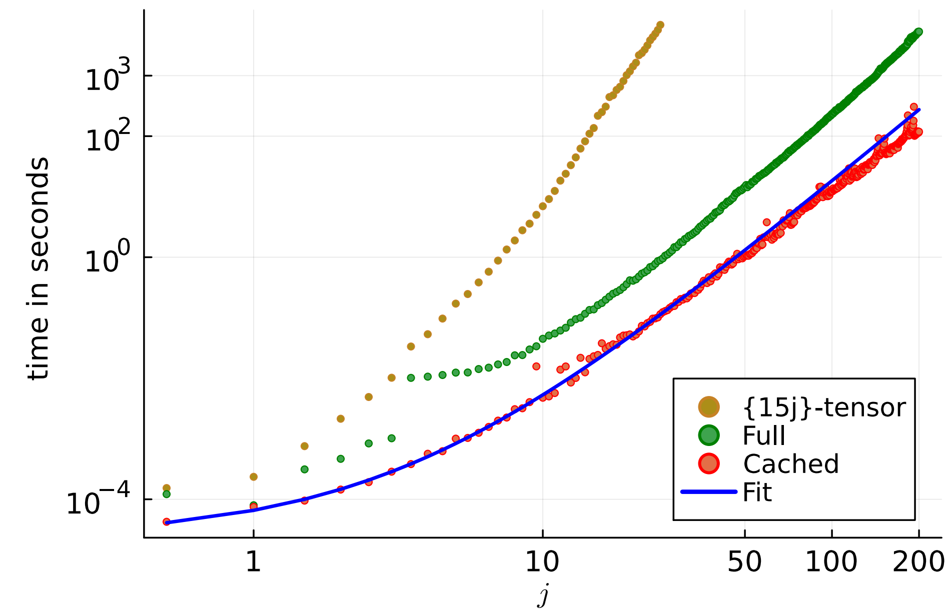
Tensor network algorithm

Benchmarks



Equilateral vertex : parametrized by a boundary spin j





Full $\simeq 10^{-8}\mathcal{O}(d_j^4)$ Cached $\simeq 10^{3}\mathcal{O}(d_j^3)$

Time (secs)

Spins	15j-tensor	TN-Algo.	Cached
$j = 10$	6.95	0.045	0.005
$j = 25$	5692.78	0.86	0.10
$j = 100$	—	234.03	12.32

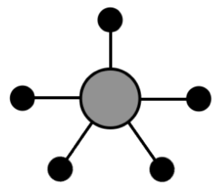
© APPLE M2 PRO-16GB RAM

Tensor network algorithm

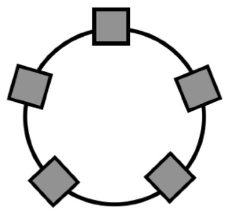
First application

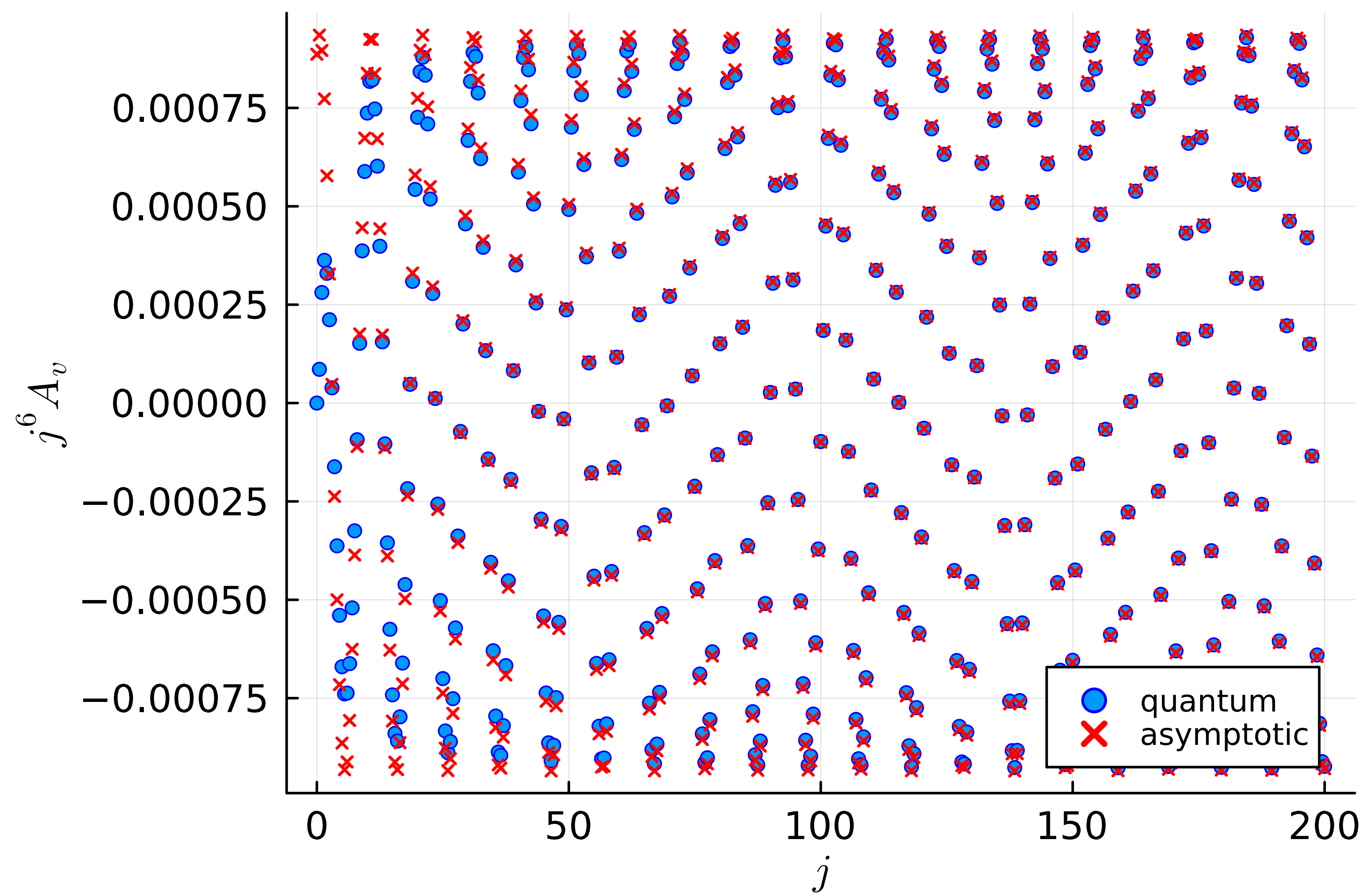
Vertex amplitudes

Equilateral vertex



- boundary data $\{j, \mathbf{n}\}$



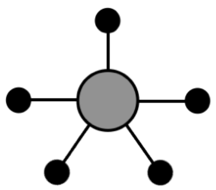


- Can scale up computations !

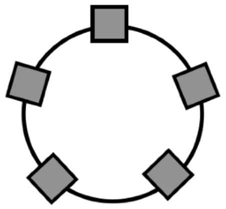
Tensor network algorithm

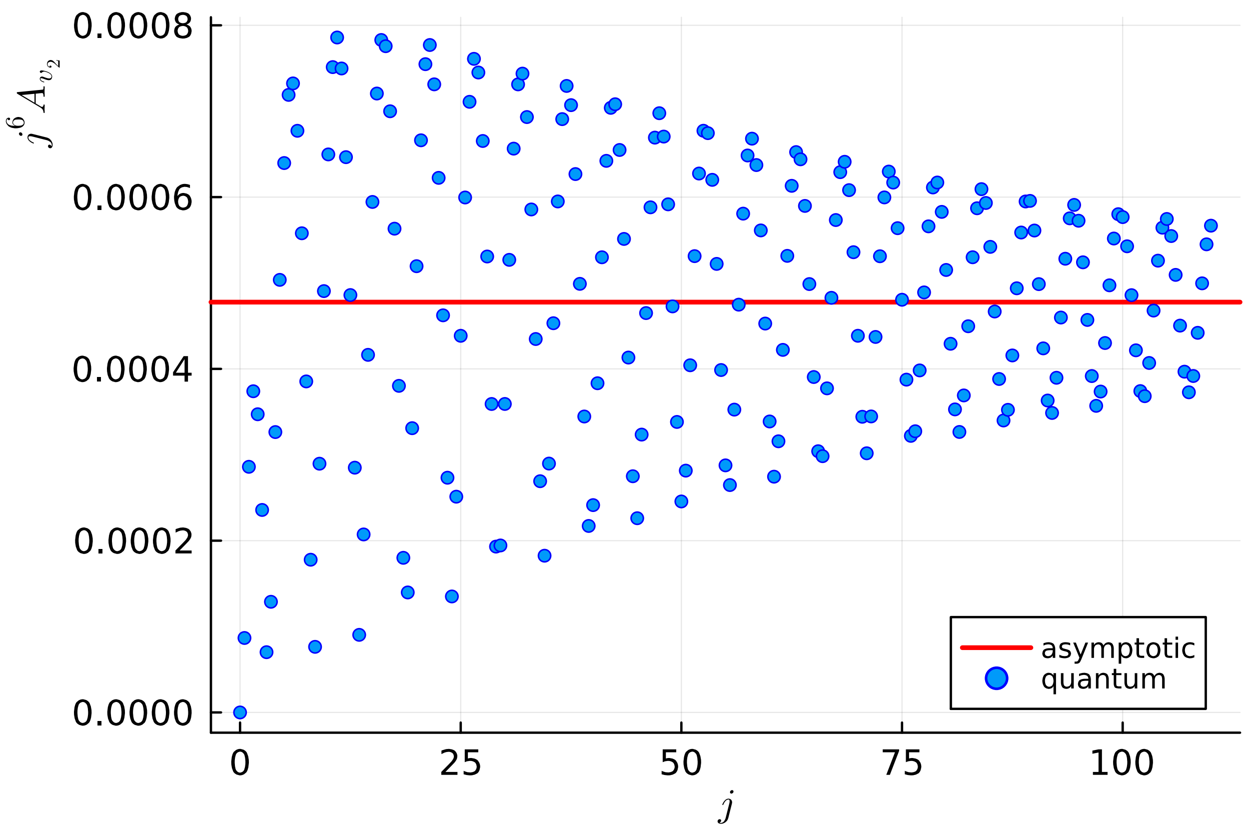
First application

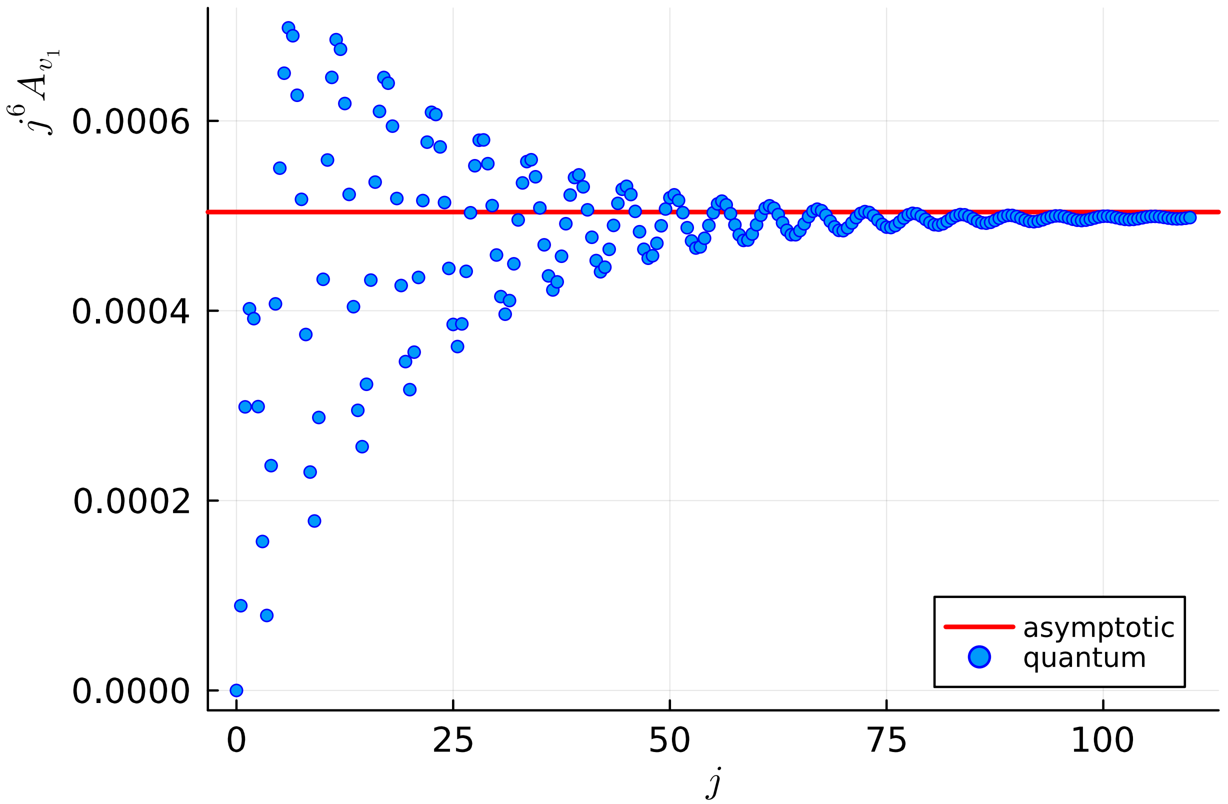
Coherent vertex amplitudes



- vector geometry boundary data $\{j, \mathbf{n}\}$







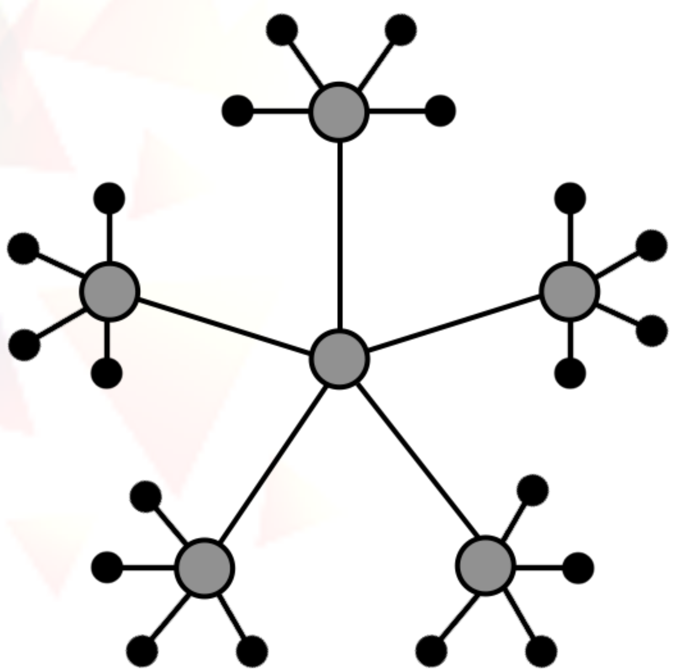
Provide new insights into parametrization of quantum ‘twisted’ geometries

More challenges

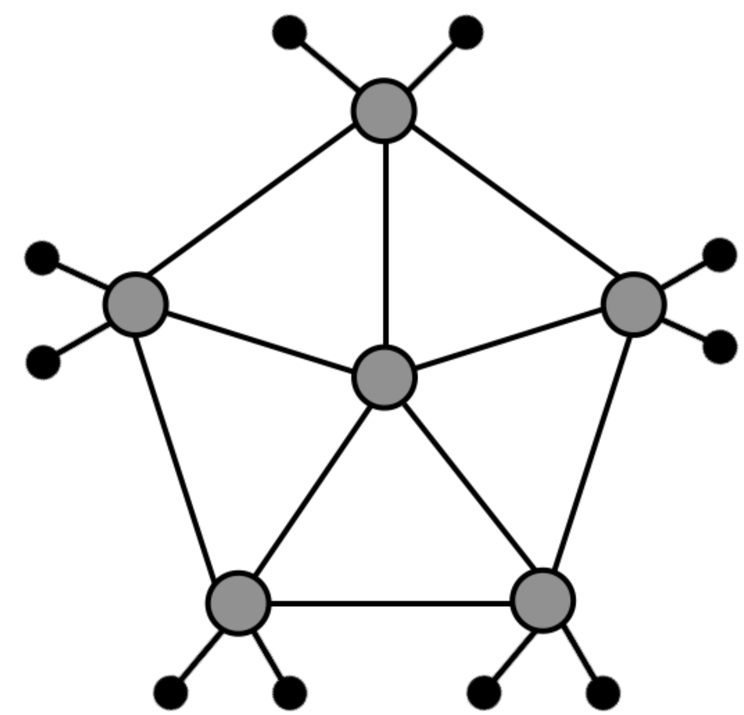
Beyond boundary vertices

Tensor network algorithm

Can we always reduce to low-valent tensors or matrix contractions ?



can avoid high-valent tensors



High-valent tensors unavoidable

Summary & Outlook

Good progress toward improving numerical computations of 'quantum space-time'

Tensor network algorithm julia

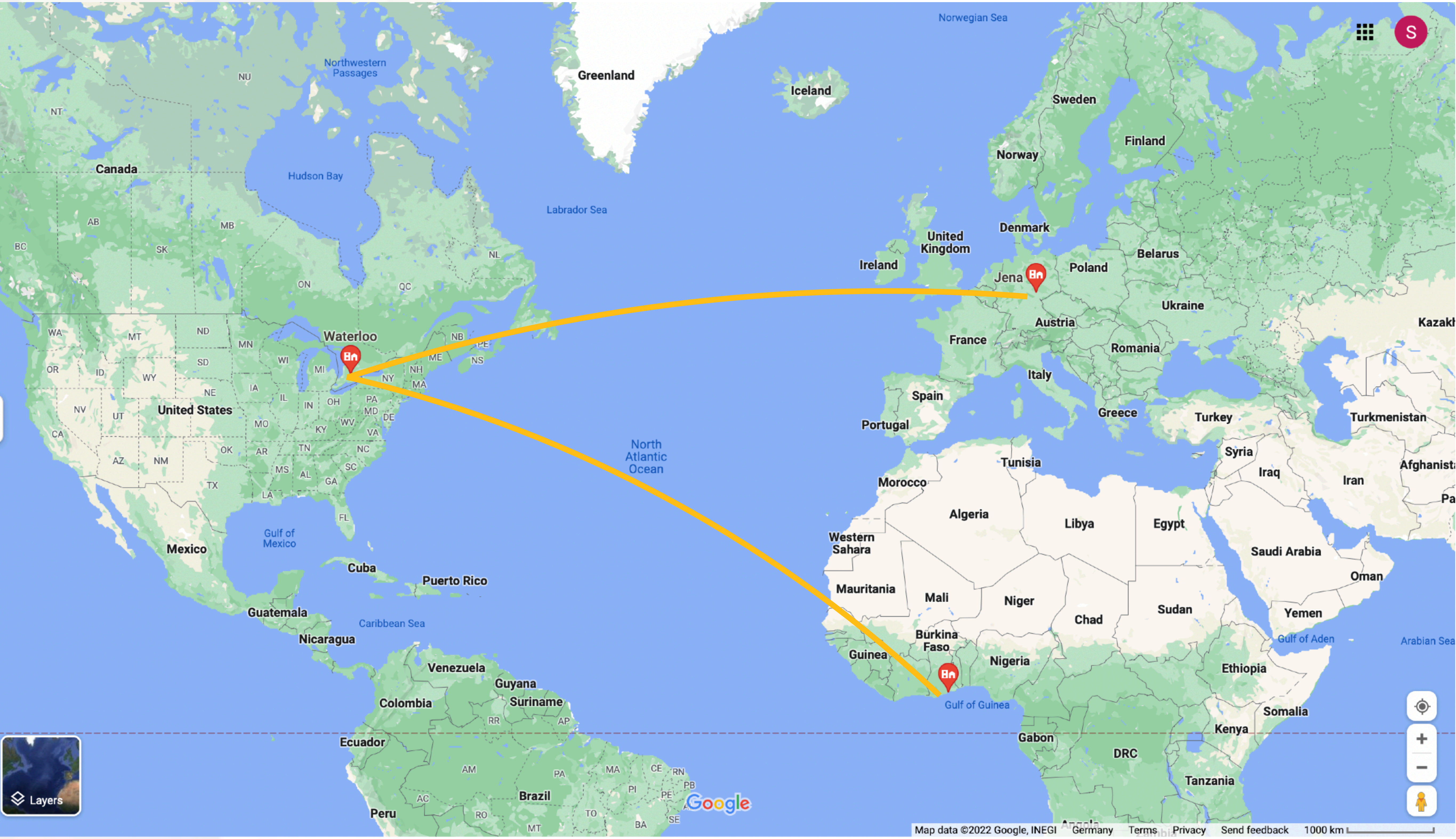
- Improve scalability, increase accessibility of spin foam quantum gravity

Lesson: tensor-networks tools are powerful for computations using low-valent tensors

Outlook:

- More room for optimization
- Provide guidance to improve analytical results

My journey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Mauritania
Mali
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Chad
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Guinea
Nigeria
Ethiopia
Gulf of Guinea
Somalia
Kenya
Gabon
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Hudson Bay
Labrador Sea
North Atlantic Ocean
United States
Mexico
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My Diversity Experiences

‘The good and the bad'

THANK YOU !

The IVY League