



Beyond Brownian Diffusion

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> Physics diversity network (DIPHER24) Erlangen, 5-7 November 2024







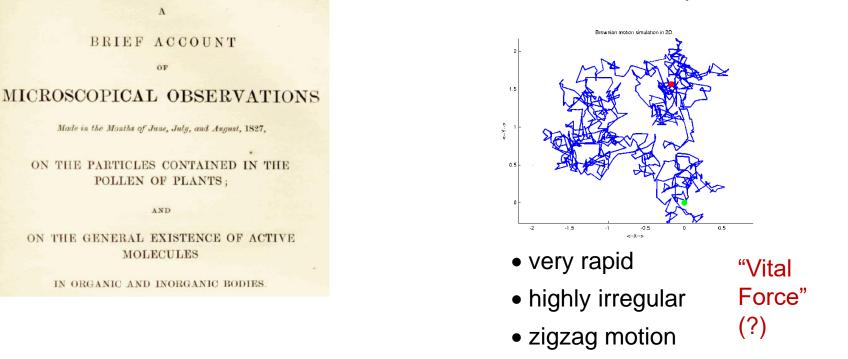
OUTLINE

- Introduction to diffusion and its anomalies
- Experimental evidence of Brownian yet not Gaussian Diffusion
- Diffusing Diffusivity models
- Effects of non-gaussianity on targeting problems

Individual experience with gender and diversity and at Padova University

The random motion of particles suspended in a medium is called Brownian Motion

What Robert Brown saw in his microscope ?



- living pollen
- dead pollen

• fine inorganic particles

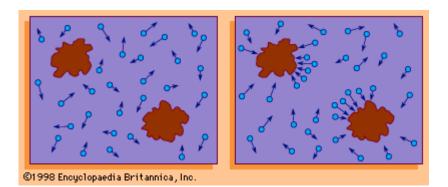


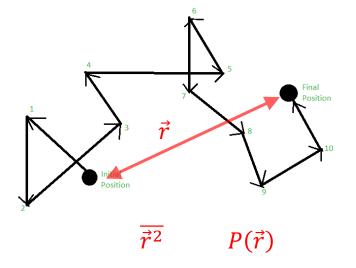
Display identical motion having a physical origin (but which ???)

This motion is not caused by currents (flows) in the liquid, nor by convection, nor by evaporation of the solvent

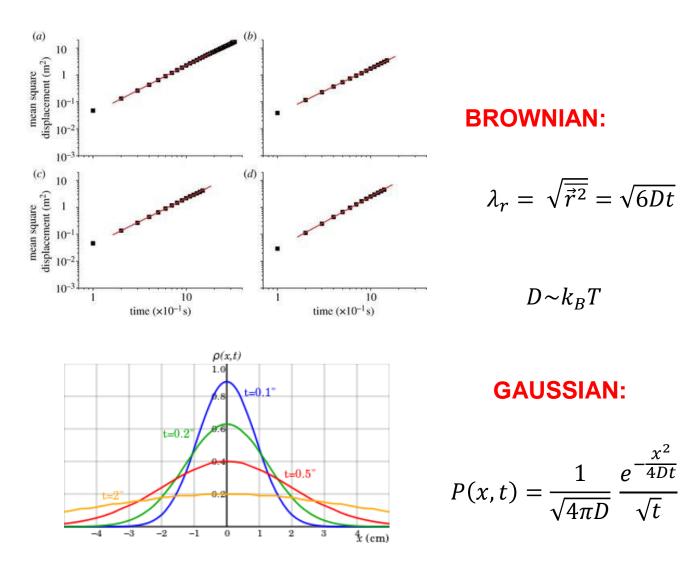
Explanation of Brownian motion – Einstein and Smoluchowksy

- Movement of Brownian particles are caused by collisions with molecules of the solvent
- These molecules move erratically in display of their thermal energy, of which the temperature is a certain measure
- Molecules of the solvent are too *little* to be observed directly
- Particlers of the suspension, even though they are tiny from a human point of view, are *true giants* in comparison withe molecules of the solvent , and can be observed directly





Normal diffusion is **BROWNIAN** and **GAUSSIAN**:

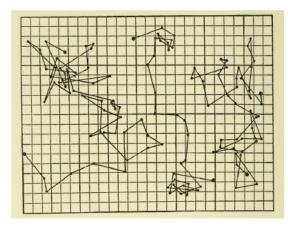


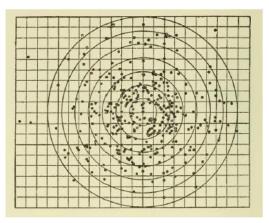




UBIQUITOUS phenomenon that play a SIGNIFICANT ROLE in virtually ALL MOLECULAR PROCESSES

J. B. Perrin (1909): experimental work to test and verify Einstein – Smoluchowski's theory of Brownian motion

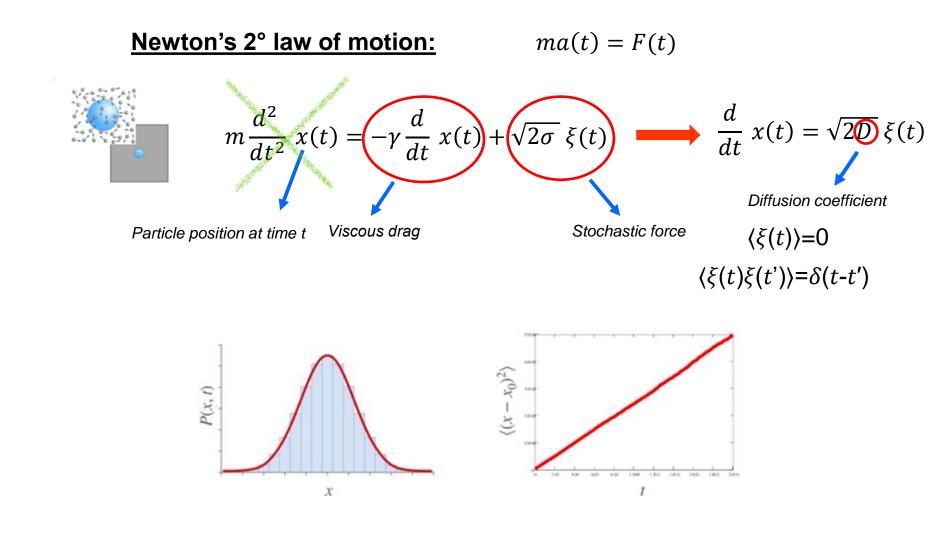




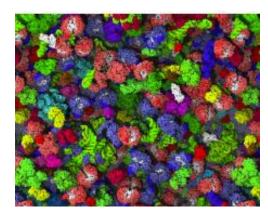
J. Perrin, Brownian Movement and Molecular Reality. London, Taylor and Francis (1910) http://web.mit.edu/swangroup/footer/perrin_bm.pdf

Perrin wrote that his results "cannot leave any doubt of the rigorous exactitude of the formula proposed by Einstein," and his work later earned him his own Nobel Prize in Physics, in 1926.

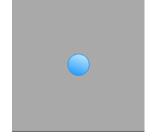
Langevin approach



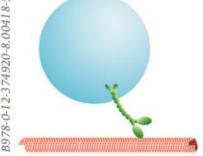
Beyond Gaussian Diffusion



Confined motion



978-0-12-374920-8.00418-5 doi:10.1016/



Free Motion

Active Motion



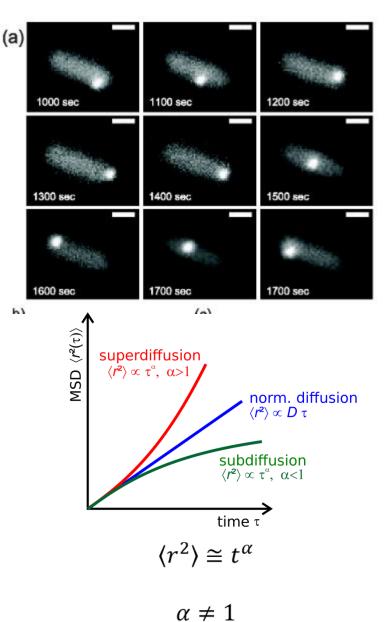




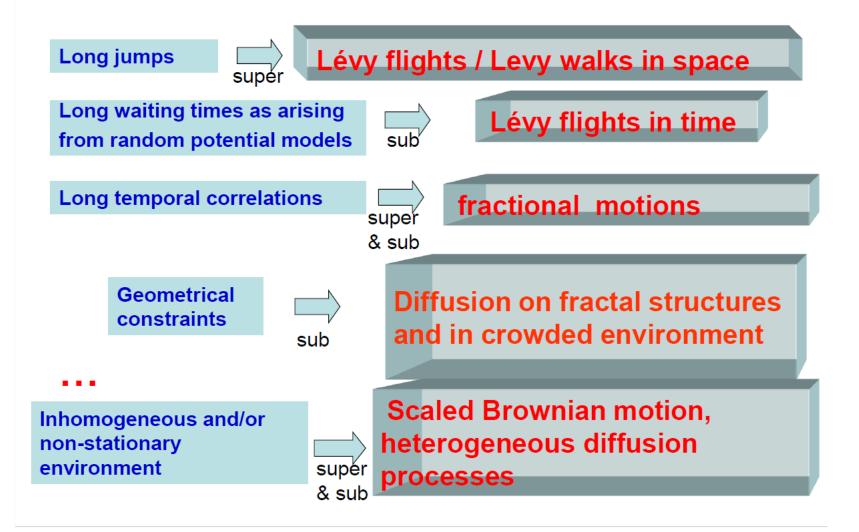
Single particle tracking experiments are rising new challenges (powerful immaging techniques such single molecule spectroscopy)

Table 2. Overview of <i>in vivo</i> experiments on crowded cellular fluids as a guide to the discussion in section 5.1.1.						
Cell type	Probe (size)	Experimental technique	Temporal and spatial scales investigated	Subdiffusion exponent α or diffusion constant D	Year	Ref.
S. pombe	Lipid granules (300 nm)	Laser tracking, video microscopy	50 μs-1 ms, 40 ms-3 min	0.70-0.74	2004	[231]
		Laser tracking	0.1-10 ms (PSD)	0.81 (interphase), 0.84 (cell division)	2009	[232]
		Video microscopy	10 ms-10 s	0.4 (time average)	2010	[233]
		Video microscopy,	>10 ms,	0.8,	2011	[234]
		laser tracking	0.1 ms-1 s	normal for $t \lesssim 3 \text{ ms}$, then $\beta = 1 - \alpha \approx 0.2$		
E. coli	mRNA (100 nm)	Video microscopy	1-30 s, 10 ⁻³ -1 Hz (PSD)	$0.70 \pm 0.07, 0.77 \pm 0.03$	2006	[235]
		Video microscopy	$1-10^3$ s	0.71 ± 0.10	2010	[236]
E. coli, C. crescentus	Chromosomal loci (GFP labelled)	Video microscopy	1–10 ³ s	0.39 ± 0.04 (ensemble and time averages coincide)	2010	[236]
Human ostero-sarcoma cells (nucleus)	Telomeres (GFP labelled)	Video microscopy	10 ms-1 h	Subdiffusion with α varying with lag time, $\alpha(t)$: $0.32 \rightarrow 0.51 \rightarrow 1$.	2009	[237]
SV-80 cells	PS beads $(3 \mu m)$	Video microscopy	40 ms-50 s	0.5–1 for $t > 10$ s, 3/2 for t < 3 s (motor proteins)	2002	[238]
Mammalian and plant cells	Rhodamine dye	FCS	$0.1ms < \tau_{1/2} < 1s$	0.6, or 2 components: $D_1 = D_{aq}/5$, $D_1/D_2 = 40$	1999	[239]
COS-7 and AT-1 cells	EGFP proteins	FCS	$\tau_{1/2}\approx 1ms$	0.7–1, or 2 components: $D_1 = D_{ag}/5, D_1/D_2 = 10$	2000	[240]
HeLa cells	FITC-dextran (1.8–14.4 nm)	FCS	$0.4ms < \tau_{1/2} < 16ms$	0.71–0.84 (depending on tracer size)	2004	[241]
HeLa and liver cells	Gold beads (5 nm)	FCS	10 μs –1 s, $\tau_{1/2} \approx 0.3ms$	0.53, with sucrose added: 0.66	2007	[242]
Mammalian cells	Gold beads (5 nm)	FCS	$0.1ms < \tau_{1/2} < 0.9ms$	0.52 (cytoplasm), 0.58 (nucleoplasm)	2007	[243]

Hofling F and Franosch T Anomalous transport in the crowded world of biological cells *Rep. Progr. Phys. 76 046602 (2013)*

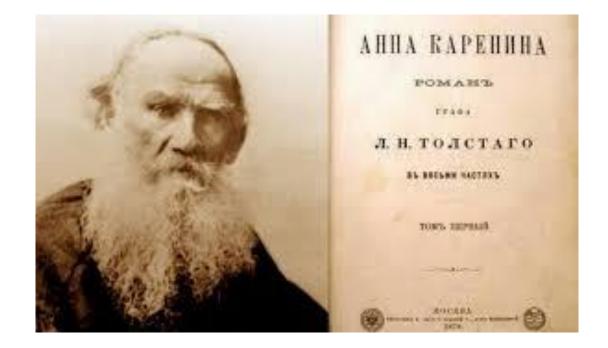


Different sources of anomaly



Normal diffusions are all alike; every anomalous diffusion is anomalous in its own way

L. Tolstoi, Anna Karenina (slightly modified)



Anomalous yet Brownian

Bo Wang^a, Stephen M. Anthony^b, Sung Chul Bae^a, and Steve Granick^{a,b,c,d,1} 15160–15164 | PNAS | September 8, 2009 | vol. 106 | no. 36

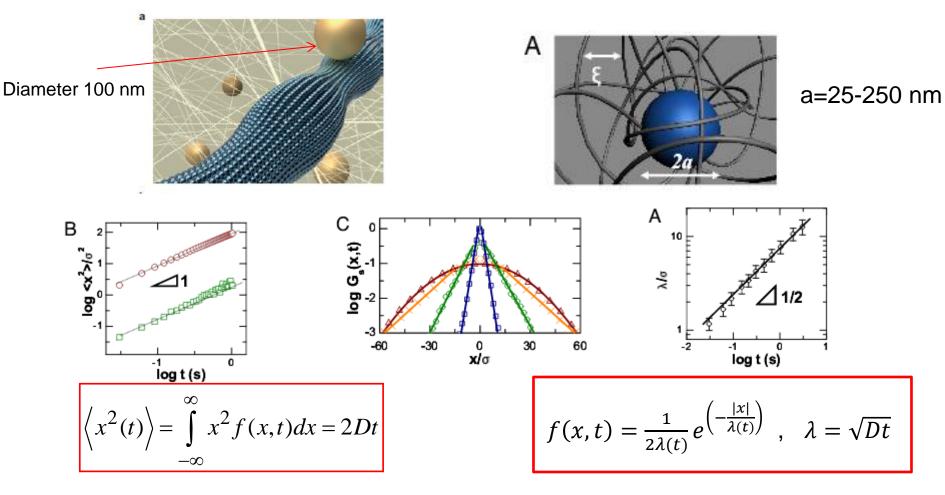
Diffusion of colloids on phospholipid tubes

When Brownian diffusion is not Gaussian

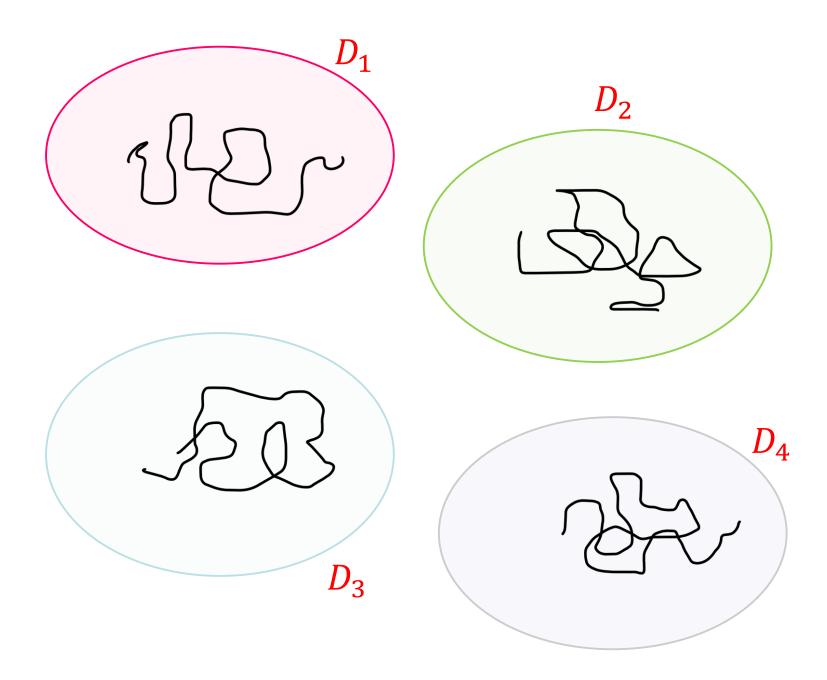
Bo Wang, James Kuo, Sung Chul Bae and Steve Granick

NATURE MATERIALS | VOL 11 | JUNE 2012 | www.nature.com/naturematerials

Colloidal beads in entangled actin suspensions



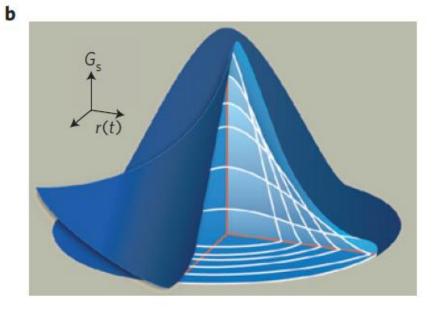
Brownian yet not Gaussian dynamics !!!



ENSEMBLE BEHAVIOUR IN TERMS OF DISTRIBUTIONS OF DIFFUSIVITIES OF INDIVIDUAL TRACER PARTICLES (SUPERSTATISTICAL APPROACH)

$$p(D) = \frac{1}{D_*} exp\left(-\frac{D}{D_*}\right)$$

$$G(r,t) = \frac{1}{\sqrt[d]{4\pi Dt}} e^{-r^2/4Dt}$$



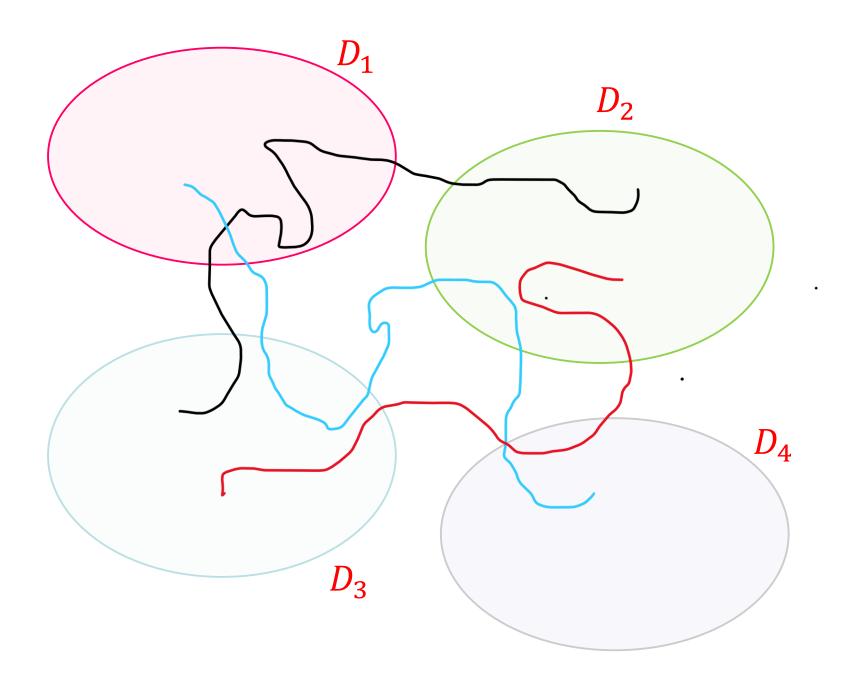
$$f(r,t) = \int_0^\infty G(r,t)p(D)dD$$

$$f(x,t) = \frac{1}{2\sqrt{D_*t}} exp\left(-\frac{|x|}{\sqrt{D_*t}}\right)$$

0

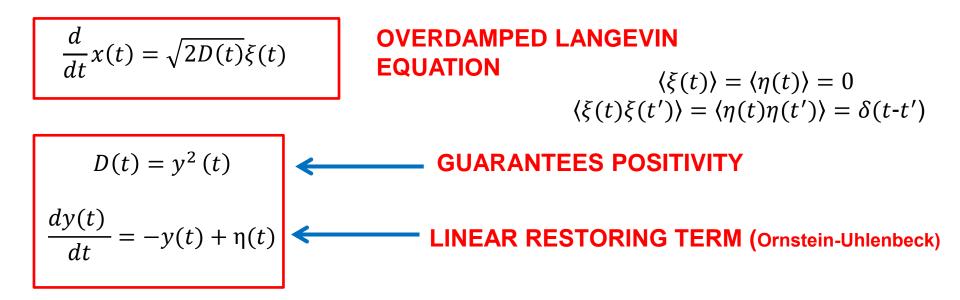
$$\langle x^2(t) \rangle = \frac{\sqrt{\pi}}{4} D_* t$$
 K

No crossover to the gaussian behaviour!



Minimal Langevin Model for Diffusing Diffusivity

NON INTERACTING PARTICLES DIFFUSE IN ONE DIMENSION: EACH ONE WITH ITS OWN INSTANTANEOUS DIFFUSION COEFFICIENT THAT VARIES WITH TIME



It guarantees stationary dynamics with finite correlation time Distribution of probability is exponential

Minimal Langevin model for diffusing diffusivities

$$\frac{d}{dt}x(t) = \sqrt{2D(t)}\xi(t)$$
$$\frac{d\tau}{dt} = D(t)$$

Subordination approach (Bochner, 1949) (financial mathematics)



A subordinator associates a random time increment with the number of steps of the subordinated process

$$\tau(t) = \int_{0}^{t} dt' D(t') = \int_{0}^{t} dt' y^{2}(t')$$

Integral formula for subordination (Feller, vol.II)

$$\frac{\partial}{\partial t} p(x,t) = D(t) \frac{\partial^2}{\partial x^2} p(x,t)$$

$$\frac{dx(\tau)}{d\tau} = \sqrt{2}\xi(\tau)^{2}$$

$$p(x,t) = \int_{0}^{\infty} d\tau G(x,\tau)T(\tau,t)$$

$$G(x,\tau) = \frac{1}{\sqrt{4\pi\tau}} \exp\left(-\frac{x^2}{4\tau}\right)$$

$$T(\tau, t)$$
 is the PDF of the process $\tau(t)$

$$\hat{p}(k,t) = \int_{0}^{\infty} d\tau \,\hat{G}(k,\tau) T(\tau,t) = \int_{0}^{\infty} d\tau \, e^{-k^{2}\tau} T(\tau,t) = \tilde{T}(k^{2},t)$$

The Fourier transform of p(x,t) is the Laplace transform of the density function $T(\tau,t)$

 $\tau(t)$ is the integrated square of the Ornstein-Uhlenbeck process

THOUGH PROBLEM....BUT



integrated square of the Ornstein-Uhlenbeck process

Cerca con Google

Mi sento fortunato

ON THE DISTRIBUTION OF THE INTEGRATED SQUARE OF THE ORNSTEIN-UHLENBECK PROCESS*

THAD DANKEL, JR.†

Abstract. Using functional integral methods, the Laplace transform of the square of the Ornstein-Uhlenbeck process X(t) integrated over $0 \le t \le T$ is calculated. Via the complex inversion integral, the asymptotic behavior as $T \to \infty$ of the density and distribution functions, as well as these functions conditioned on the event X(T) = 0, are studied. It is found that the approximation by an inverse Gaussian distribution, introduced earlier by Grenander, Pollak, and Slepian, is asymptotically correct (to within a constant factor) in the conditional case, but not in the unconditional case.

Key words. Ornstein-Uhlenbeck process, functional integrals, asymptotic distributions, Laplace transforms, inverse Gaussian distribution

AMS(MOS) subject classifications. 60G10, 60G15, 60J25, 41A60, 28C20

$$\tilde{T}(s,t) = \frac{e^{t/2}}{\left[\frac{1}{2}\left(\sqrt{1+2s} + \frac{1}{\sqrt{1+2s}}\right)\sinh\left(t\sqrt{1+2s}\right) + \cosh\left(t\sqrt{1+2s}\right)\right]^{1/2}}$$

Times Cited: 1 (from Web of Science Core Collection)

Usage Count ~

NOW 16!

Exact solution of the diffusing diffusivity model

Sho

• Short times,
$$t \ll 1$$

$$p(x,t) = \frac{1}{\pi\sqrt{t}} \int_{0}^{\infty} dk \frac{\cos(kx)}{(k^{2} + \frac{1}{t})^{1/2}} = \frac{1}{\pi\sqrt{t}} K_{0} \left(\frac{|x|}{\sqrt{t}}\right)$$

$$(x,t) = \frac{1}{\sqrt{4\pi} \langle D \rangle_{st} t} \exp\left(-\frac{x^{2}}{4 \langle D \rangle_{st} t}\right)$$

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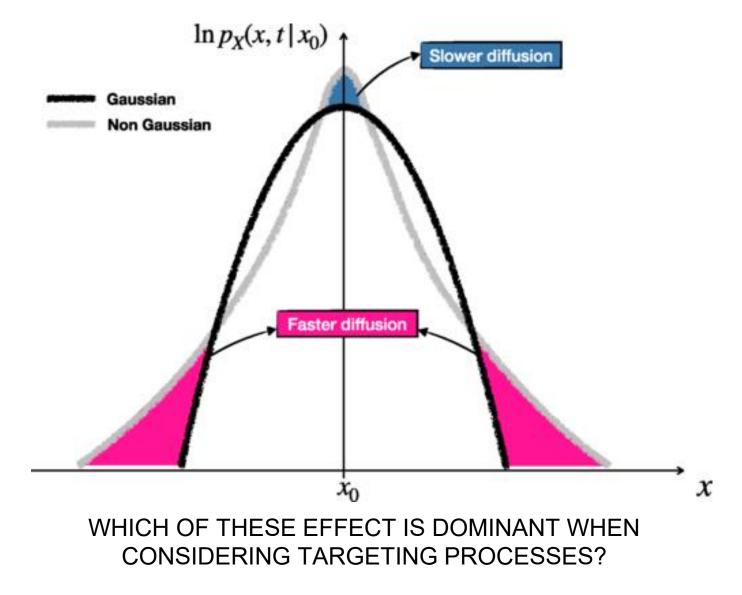
$$(x,t) = \frac{1}{\sqrt{4\pi} \langle D \rangle_{st} t} \exp\left(-\frac{x^{2}}{4 \langle D \rangle_{st} t}\right)$$

0.1

t

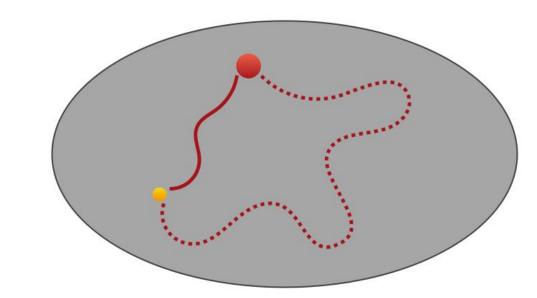
From a practical viewpoint:

Why all this matters ? As the non-Gaussian character mostly appears in the tails of the distribution, and thus with low probability...



TARGETING PROCESSES

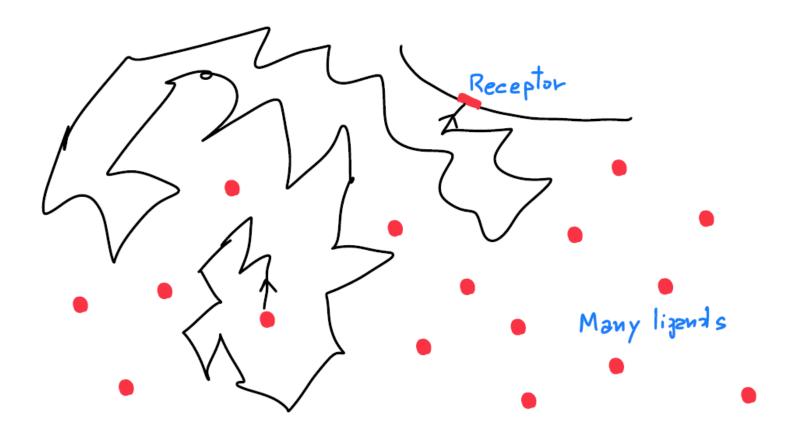
- Fluorescence quenching
- Integrate and fire neurons
- Execution of buy/sell orders when a stock price firt reach a threshold
- Biology:
- Transcriptor factors
- Drug binding
- Chemical signalling



Searcher that moves according to a specific diffusive model

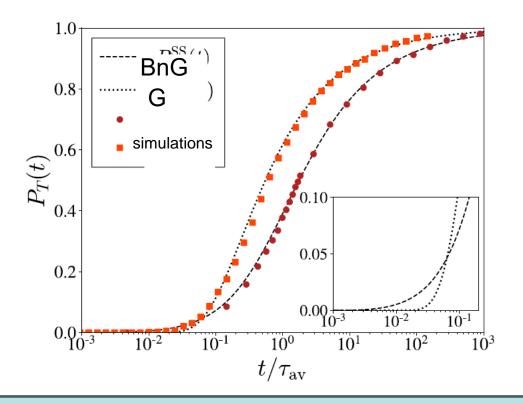
Perfectly absorbing target

MEAN FIRST PASSAGE TIME



In manu cases activation is triggered when many searches reach the target (robustness), e.g. QUORUM SENSING

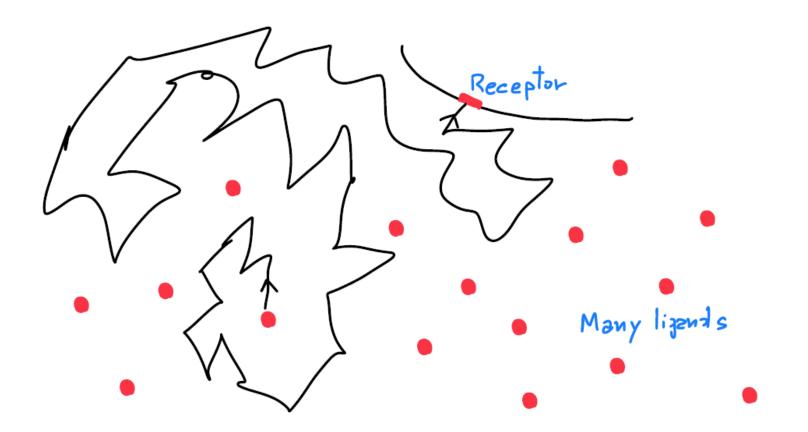
Text book on stochastic processes: $P_T(x_0, t, D) = 1 - \int P_X(x, t | x_0, D)$



$$\tau_T^{BnG}(x_0) > \tau_T^G(x_0, D_{a\nu})$$

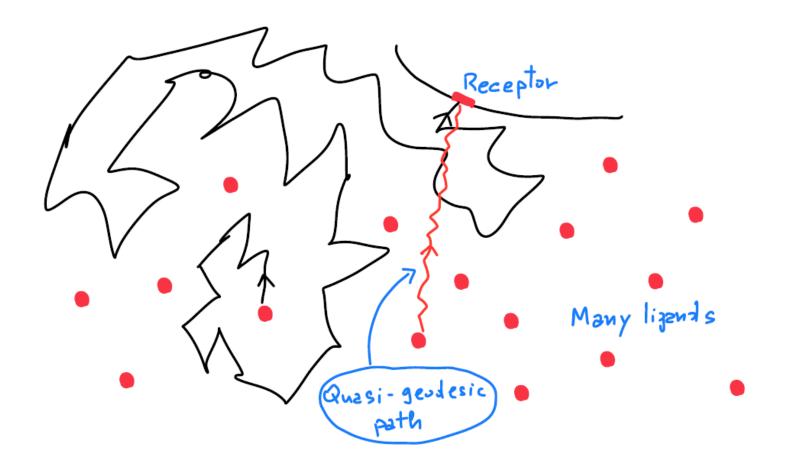
The characteristic time to target of a single searcher for subordinated non-Gaussian diffusion processes is larger than in ordinary Gaussian diffusion!

EXTREME FIRST PASSAGE TIME



In other cases activation is triggered when the first searcher reach the target (a paradigmatic example is human reproduction, in which a single sperm cell out of M \sim 10^8 finds and fertilizes the egg.)

Extreme FPT is governed by rare trajectories which are the few among the many to follow a quasi-geodesic path to the target



The tail effect non-Gaussianity adds to these rare events the possibility for the searcher to diffuse faster

EXTREME FIRST PASSAGE TIME

$M \gg 1$ INDIPENDENT SEARCHERS WITH ARRIVAL TIME τ_i

$$T_M = \min_i [\{\tau_1, \tau_2, \dots, \tau_M\}]$$

Heterogeneity for diffusing particles with $p_D(D_n)$ $(n = 1, 2, ...) \Rightarrow D_{av}$

$$P(T_M > t) = \prod_{i=1}^{M} P(\tau_i > t) = \prod_n \left[S_{D_n}(t) \right]^{M_n}$$

 M_n number of searcher with diffusion coefficient D_n

 $S_{D_n}(t)$ Survival probability not easy to compute

M>1 suggests that the computation of $P(T_M > t)$ can be approximated by the short time behaviour when $S_{D_n}(t) \approx 1$

EXTREME FIRST PASSAGE TIME

$$\tau_{\rm av} \equiv \frac{\ell^2}{2D_{\rm av}},$$

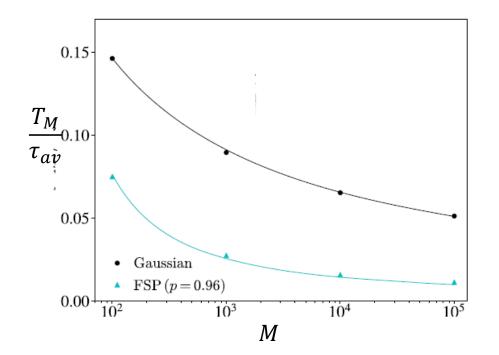
Average time to cover a distance ℓ

Gaussian case, only one D

$$\frac{T_M}{\tau_{av}} \approx \frac{1}{2 \ln M} \qquad M \gg 1$$

Non Gaussian case, distribution of D

$$\frac{T_M}{\tau_{av}} \approx \frac{1}{(\ln M)^2} \qquad M \gg 1$$

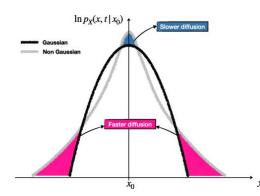


CONCLUSIONS

- We considered processes in which the origin of non-Gaussianity is ascribed to heterogeneity of the ensembles of diffusers and/or of the environment
- Such heterogeneity implies both an excess of probability in the central part and in the tails of the displacements distribution, when compared with the Gaussian one in diverse searchers strongly enhances the fast targetting of the first few instances
- In particular we have shown that a higher probability for few, faster diffusers, pointing out that a redundant information stored (tail effect) influences extreme searches

Non-Gaussianity is both disvantageous and advantageous:

- It is disvantageous when dealing with systems in which a large percentage of searchers need to reach the target.
- It is advantegeous when only a few searchers, among many, are required to reach the target



Being heterogeneous is disadvantageous: Brownian non-Gaussian searches

Vittoria Sposini[®],^{1,*} Sankaran Nampoothiri,^{2,†} Aleksei Chechkin[®],^{3,4,5,‡} Enzo Orlandini[®],^{6,§} Flavio Seno[®],^{6,∥} and Fulvio Baldovin[®],⁹

PHYSICAL REVIEW LETTERS 132, 117101 (2024)

Being Heterogeneous Is Advantageous: Extreme Brownian Non-Gaussian Searches

Vittoria Sposini[®],^{1,*} Sankaran Nampoothiri,^{2,†} Aleksei Chechkin[®],^{3,4,5,‡} Enzo Orlandini[®],^{6,§} Flavio Seno[®],^{6,∥} and Fulvio Baldovin[®],^{6,¶}



OXFORD (1096)

Scientific Bias: much easier to pubblish with the right preprint cover

The most snobbish and exclusive community in the world are Oxford Catholics





UNIVERSITY (1222)











ALUMNI



Janus Pannonius



Galileo Galilei 1592-1610



William Harvey



Vesalius



Falloppio



Torquato Tasso



Elena Lucrezia Cornaro

25 June 1678



Tullio Levi Civita



GENDER UMBALANCE

FIRST FEMALE FULL PROFESSOR



MASSIMILA BALDO CEOLIN (1963)



GENDER UMBALANCE

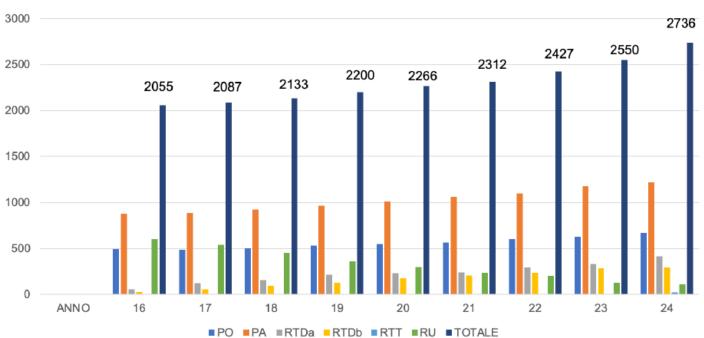
FIRST FEMALE RECTOR



DANIELA MAPELLI (2019)



L'ateneo in cifre DOCENTI



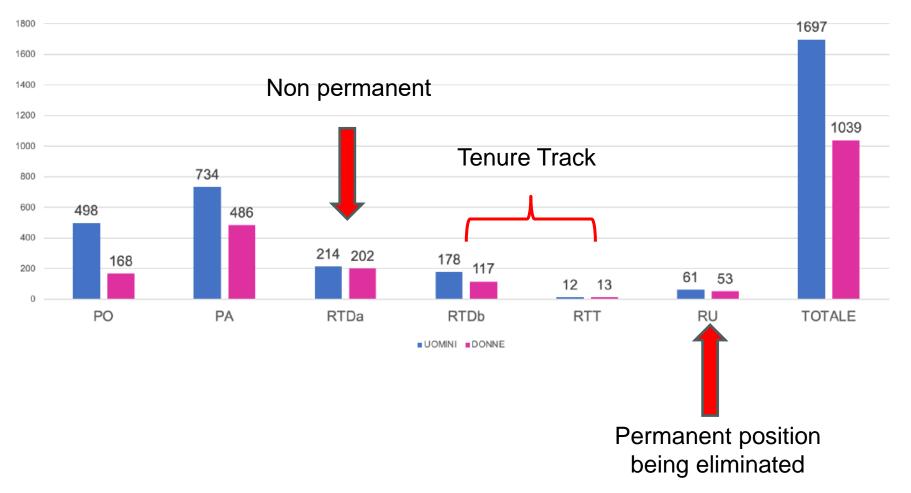
DOCENTI

Whoever has will be given more, and they will have an abundance. Whoever does not have, even what they have will be taken from them

Matthew 13, 12











PA MACRO 1 MACRO 2 MACRO 3 ■PAm ■PAf

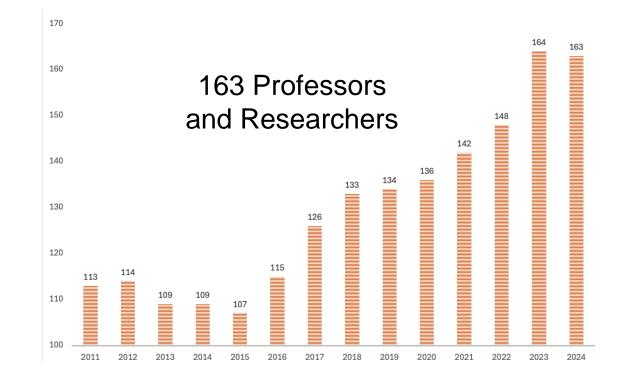
PE LS SH



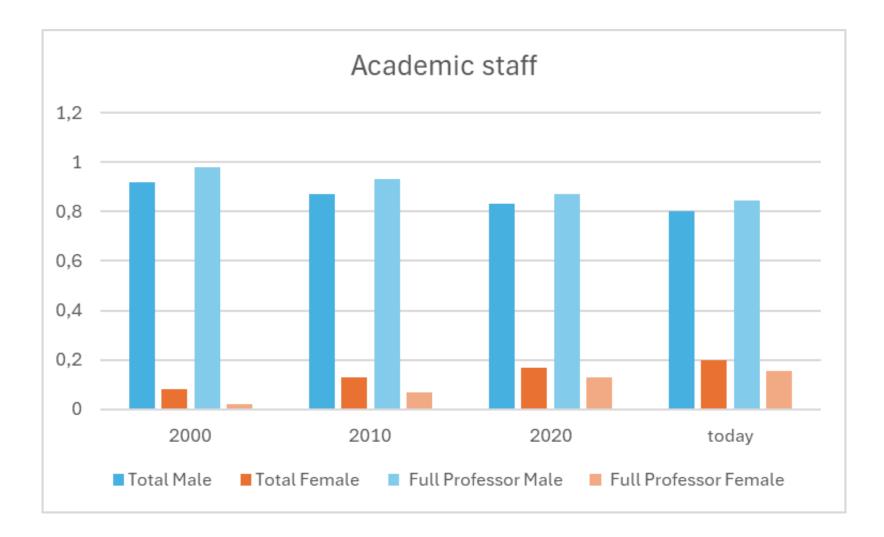


Department of Physics and Astronomy





78 Administrative and technical staff99 Postdocs150 Ph.D. students









Activities

Video project "International Day of Women and Girls in Science"



Le Sorelle del Cielo

di e con Grazia Dentoni e Simonetta Marcello



The sisters of heaven. Theatrical journey of narration, gesture and sound La propa vascosta Scienziate nella Fisica e nella Storia

Diversity that makes science better With Francesca Vidotto



The play offers a view on 20th century physics through the eyes of four female scientists who were its protagonists, including Prof. Milla Baldo Ceolin.

ō

Immer mehr Details zum Mord an Giulia: Mit Klebeband zum Schweigen gebracht

Nur 50 Meter von ihrem Zuhause entfernt könnte Filippo Turetta zum ersten Mal mit dem Messer auf Giulia Cecchettin eingestochen haben: Die Carabinieri rekonstruieren anhand von Spuren den Angriff, der sich am 11. November gegen 23.30 Uhr im Industriegebiet von Fossò bei Venedig ereignet hat.

A⁺ A⁻







The Department of Physics and Astronomy has established the Inclusion and Diversity Commission, dedicated to fostering a climate of welcome, respect and appreciation of differences within our community, technical and administrative staff, the student population and teachers and researchers. The objectives are Research and monitoring, Training and Awareness.

1. Questionnaire: Identify current obstacles (gender, mobbing), conducting and making available an analysis of harassment/bullying to possibly identify areas of intervention to improve inclusion.

2. **Short sessions** during department council meetings to address issues of inclusion and diversity, improving awareness and understanding of the challenges and opportunities for a more equitable academic environment.





QUESTIONNAIRE COMPLETION PERCENTAGE

DFA/INAF/INFN PARTECIPANTI %*

STUDENTS	362	15
PHD	57	41
POST DOC	30	30
PROFESSORS	91	79
RTDA/RTDB	36	78
TECHNICAL AND ADMINISTRATIVE STAFF	78	100



The online course "Equity and inclusion" is an initiative promoted by the Gender Equality Plan and is part of the equal training, culture and science interventions and, at the same time, intends to promote awareness of the different forms of harassment, violence and discrimination and knowledge of the services available in the University for those who suffer them.



November 25, 2024



UNO, NESSUNA, CENTOMILA

scheda approfondita [LINK]

LETTURA DUE ATTORI

TEMI AFFRONTATI

- La violenza come problema universale
- L'importanza di affrontare il tema della violenza
- L'importanza della sensibilizzazione continua
- Il ruolo delle donne nella lotta contro la violenza

SINOSSI

La lettura affronta il tema universale della violenza, esplorandone le diverse forme, alcune delle quali possono sembrare sottili o comiche, mentre altre generano rabbia e indignazione.

Tuttavia, l'opera promuove la **necessità di parlare apertamente della violenza**, sottolineando che il silenzio non è la soluzione e che solo attraverso il **dialogo aperto** possiamo sperare di provocare un cambiamento.

Nel suo potente epilogo, viene offerto un messaggio di speranza che evidenzia il ruolo fondamentale delle donne nella lotta contro la violenza e ci ricorda che questa discussione cruciale non deve essere limitata a occasioni specifiche, ma deve diventare una parte costante del nostro tessuto sociale.

Dai testi di Lella Costa, Franca Rame, Serena Dandini Con Evarossella Biolo e Marco Artusi Play: Uno, Nessuna, Centomila

Flash mob: TOYS? by Anna Piratti





MUSEO GIOVANNI POLENI





Science from the Islamic World to Today's Europe

A DFA and Giovanni Poleni Museum project

Science from the Islamic World to Today's Europe Cross-Fertilisation between Past and Future

Proposed by the Department of Physics and Astronomy with the collaboration of various partners, the project is funded by the University of Padua. It aims at presenting to the public various aspects of the development of science from the Islamic contributions up to cutting-edge physics and astronomy, shedding light on the exchanges and cross-fertilisation between various countries and cultures.



Inclusion and the active participation of the local community are two of the main features of the project, which is actually based on the co-creation of knowledge by the public. In this sense, mixed working groups will be formed, composed of members of the local Islamic communities and other foreign communities, and PhD students from the University of Padua; these groups will work in synergy, in a continuous exchange of ideas and knowledge, using the rich collection of scientific instruments of the Giovanni Poleni Museum. For the PhD students, the participation in the project is part of a Soft Skills PhD course. The working groups will have several training meetings, from October 2022 to March 2023. Each group will then develop a new science communication project, which will be presented to the public in Padua in April-May 2023.















United Nations ICTP - East African Institute Educational, Scientific and for Fundamental Research Cultural Organization under the auspices of UNESCO

Stellenbosch









July 5-6 (School on complex systems/Organized Tour) July 7-10 (Workshop), 2025

Kigali, College of Sciences and Technology, University of Rwanda

Satellite meeting of





THANK YOU FOR YOUR ATTENTION...