Understanding Shock Waves in Gravitational Collapse

June 27, 2024

- Models of gravitational collapse
- Shock waves: some background
- Classical dust collapse: simulations
- An effective quantum gravity model: simulations

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PRL 128 (2022) 12, 121301; e-Print:2109.08667

PRD 106 (2022) 2, 024014; e-Print: 2203.04238

... and to appear (with Mehmood Hassan)

well-studied:

- Null dust: Vaidya and generalizations
- Particle models: $x(t), p(t)$ — Oppenheimer-Snyder, thin shell, and variations
- Field theory models: Lemaitre-Tolman-Bondi dust, GR + scalar field, GR + perfect fluid, $\cdots$; matter fields $\phi(r,t)$:

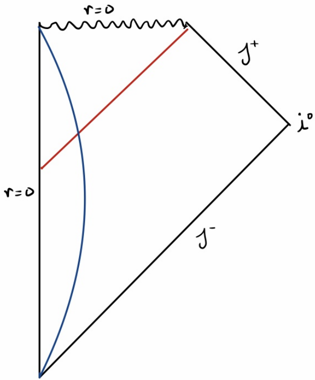
$$ds^2 = -f(r,t)dt^2 + g(r,t)dr^2 + r^2 d\Omega^2$$

and variations.

Choptuik (1993)

- complex behaviour of scalar field dynamics: scaling, discrete self-similarity
- weak data $\longrightarrow$ no black holes; strong data $\longrightarrow$ black holes
- critical behaviour at the onset of black hole formation: $M \approx (A - A_*)^\gamma$, A is a parameter in the initial data.
- infalling data does not become a shock wave

In QG, what happens to the scalar field if the singularity is avoided?



represents all classical collapse models

What happens in quantum gravity?

matter infall $\longrightarrow$ dynamical horizon formation $\longrightarrow$ matter bounce $\longrightarrow$ matter outflow $\longrightarrow$??

- Bounce?
- Remnant: metastable "star" with collapse and bounce pressures balanced?
- Repeated collapse and bounce?
- Black hole $\longrightarrow$ White hole transition?

A new possibility

Black holes end in a gravitational shock wave

—based on calculations in an effective QG model for dust collapse

Understanding shocks

- a shock is a propagating discontinuity in a physical field
- observed in many systems: fluids, plasmas, traffic modelling $\cdots$
- arise as "weak solutions" to PDEs: solutions of integrated version of PDE
- established numerical method: Godunov algorithm

The "harmonic oscillator" of shock waves:

$$u_t + uu_x = 0$$

Shock solution: $u(x,t) = f(x - st)$

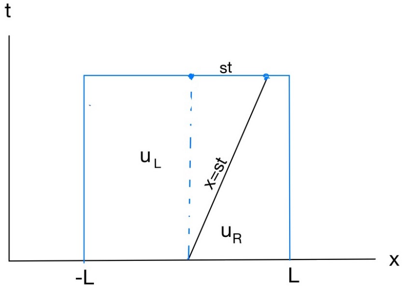
$$f(z) = \begin{cases} u_L, & z < 0 \\ u_R, & z > 0 \end{cases} \qquad u_L > u_R$$

shock speed s found by integrating the eqn:

$$\frac{d}{dt}\int_{-L}^{L} dx\ u(x,t)dx = -\int_{-L}^{L} dx\ uu'$$

$$\frac{d}{dt}\left[(L+st)u_L+(L-st)u_R\right]=\frac{1}{2}\left(u_L^2-u_R^2\right)$$

$$\implies s=\frac{\left(u_L^2-u_R^2\right)/2}{u_L-u_R}$$



- This is a solution of the integrated equation—weak solution

The same argument applies to the more general conservation equation

$$u_t + f(u)_x = 0.$$

It has shock solutions with speed

$$s = \frac{f(u_L) - f(u_R)}{u_L - u_R} = \frac{\text{jump in f}}{\text{jump in u}}$$

–Rankine-Hugoniot condition

Similar equations arises for classical and effective dust collapse

$$u_t + uu_x = 0$$

Solution using method of characteristics: let $u = u(x(s), t(s))$. Then

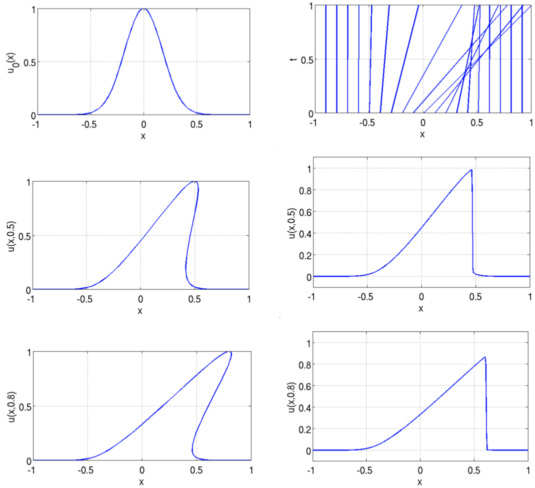
$$\frac{du}{ds} = u_t \frac{dt}{ds} + u_x \frac{dx}{ds} = 0$$

provided the characteristic equations hold:

$$\frac{dt}{ds} = 1, \quad \frac{dx}{ds} = u.$$

$\longrightarrow$ the implicit solution for initial data $u(x, 0) = f(x)$ is

$$u(x, t) = f(x - u(x, t)t)$$



L column: solution by method of characteristics (wave breaking)
Top right: crossing of characteristics
R column: shock wave solution (numerical weak solution) (...movie)

Lessons from Burgers' eqn

- physically reasonable initial data can evolve to "solutions" that are not functions: characteristic crossing occurs.
- seek weak solutions: may give shock waves.

dust time + areal gauge $\longrightarrow$

$$ds^2 = -dt^2 + \left(dr + \frac{B(r,t)}{r}\, dt\right)^2 + r^2 d\Omega^2$$

eom : $\dot{B} + J'(B, r) = 0,$ energy density : $\rho = \frac{J'}{8\pi r^2}$

Classical: $J_c = \frac{B^2}{2r}$

Effective LQG: $J_{eff} = \frac{r^3}{2} \sin^2\left(\frac{B}{r^2}\right).$

—from an effective Hamiltonian formulation with holonomy corrections.

These are conservation equations like Burgers'
— except for radial dependence in current

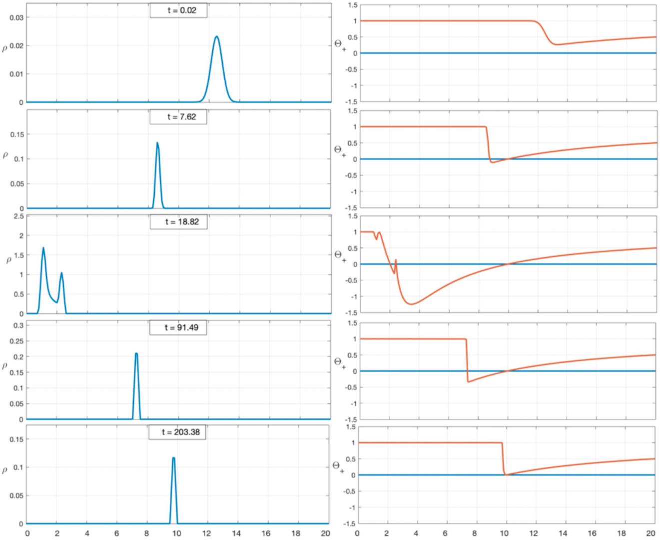
"The same equations have the same solutions"

— Feynman Lectures II.12

classical movie ...

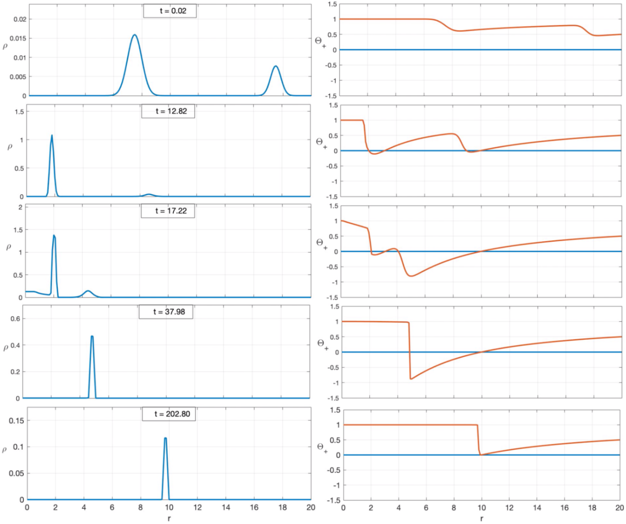
Effective collapse: Gaussian initial density

density | outward null expansion



Effective collapse: double Gaussian initial density

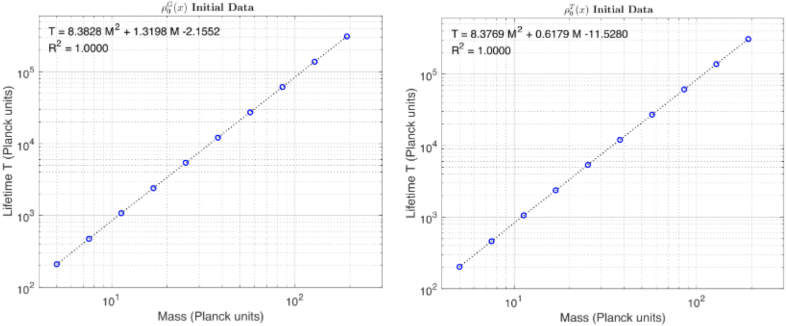
density | outward null expansion



In simulations

- check outward null expansion function Θ_+ for roots.
- record time at which first root occurs t_i
- record time when no roots remain t_f
- plot ADM mass M vs $T = t_f - t_i$

Black hole lifetime



$$T_{BH} \approx \frac{8\pi}{3} M^2$$

– code available online

The effective equation can be solved for a thin shell by taking

$$\rho = \frac{M}{4\pi r^2}\delta(r - L(t))$$

and finding the shock velocity $dL(t)/dt$ using the Rankine-Hugoniot condition.

This gives an estimate of black hole lifetime:

$$\begin{aligned} T &= t_{\text{in}} + t_{out} \\ &= \int_{R_S}^{L_{min}} \left(\frac{dL}{dt}\right)_{in}^{-1} dL + \int_{L_{min}}^{R_S} \left(\frac{dL}{dt}\right)_{out}^{-1} dL \\ &\approx \frac{2R_S}{3} + \frac{2\pi R_S^2}{3l_P} \end{aligned} \tag{1}$$

$L_{min} \approx (l_p^2 R_S)^{1/3}$.

New Penrose diagram

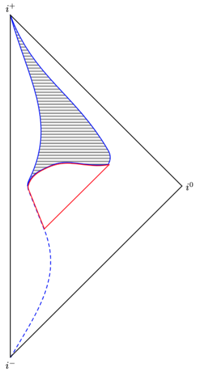


FIG. 4. Conformal diagram for dust collapse and bounce. Red lines show the outer and inner apparent horizons—the outer horizon becomes and remains null once all in-falling matter has passed through it; the dashed blue line shows a typical ingoing dust trajectory; the solid blue lines show the outgoing shock wave trajectory according to the interior and exterior metrics respectively—these metrics differ due to the shock discontinuity; the shaded portion of the diagram is excised and the two solid blue lines are identified. To an outside observer the outer horizon disappears as the shock wave emerges.

Summary

- Shock waves can form in classical dust collapse
- Shock waves *always* form in effective dust collapse
- Dust black holes end in an emerging shock wave
- Black hole lifetime $\approx M^2$: end before Hawking time M^3

Future

A lot to think about: other matter types, other effective equations, gauges, observables, $\cdots$

The dust case is easier relative to what we are seeing for the scalar field