Null Raychauduri: Canonical structure, Dressing time and Quantum geometry

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Based on 2309.03932, 2406.xxxx and WIP
with Luca Ciambelli and Rob Leigh

Carrollian Geometry

[Levy-Leblond '64, Ashtekar '78 -'24, Henneaux '81, Dautcourt '97, Duval-Gibbons-Horvarthy '14, …]

$\mathcal{N}$ 3d Null Manifold

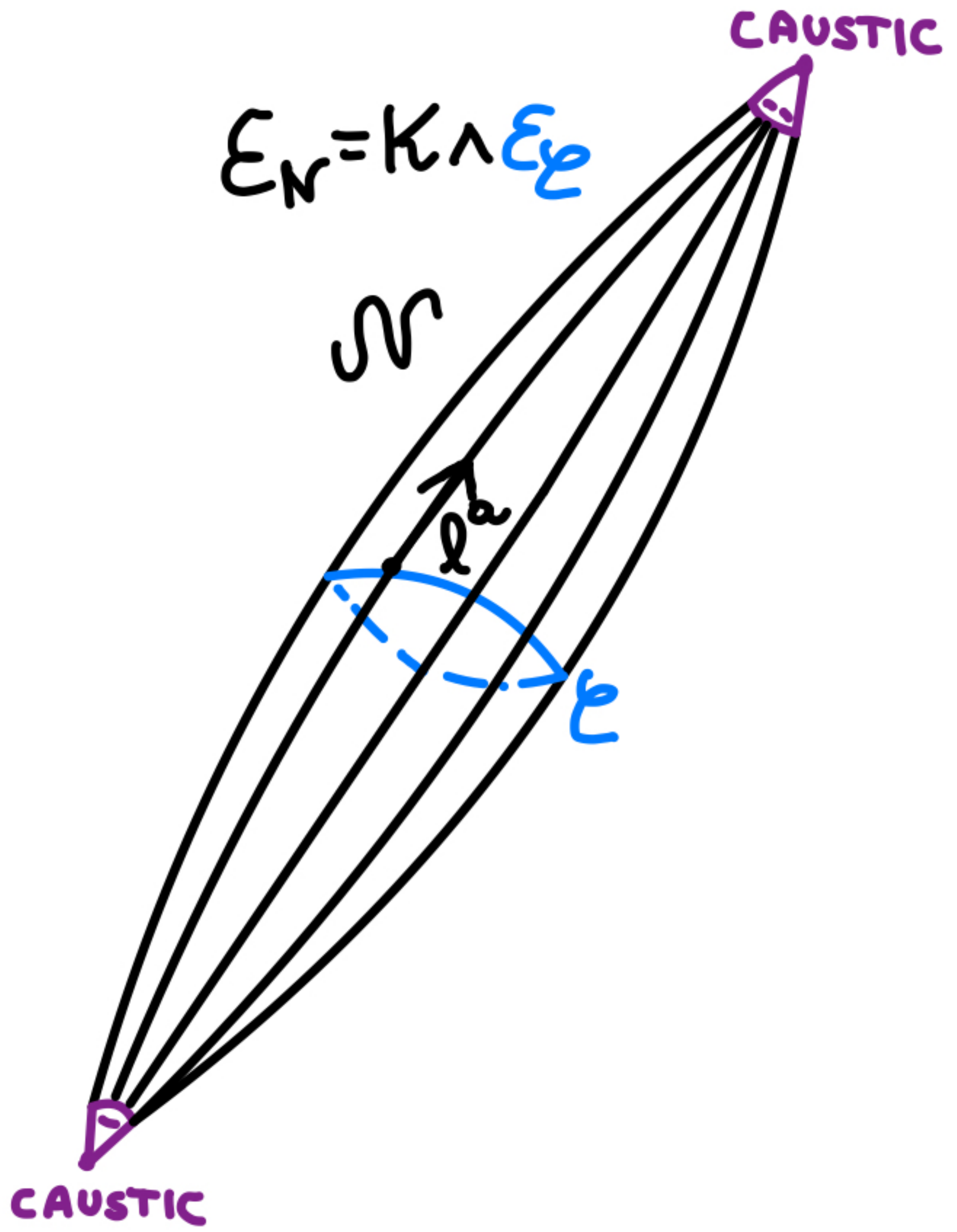
Carrollian Structure (ℓ^a, q_{ab}) with $\ell^a q_{ab} = 0$

Ruling k_a with $\ell^a k_a = 1$

Horizontal Projector $q_a{}^b = \delta_a{}^b - k_a \ell^b$

Traceless

Expansion Tensor $\theta_{ab} = \frac{1}{2}\mathcal{L}_\ell q_{ab}$ such that $\theta_a{}^b = \frac{\theta}{2} q_a{}^b + \sigma_a{}^b$



Geometry of Null Hypersurfaces

[Levy-Leblond '64, Ashtekar '78 -'24, Henneaux '81, Dautcourt '97, Duval-Gibbons-Horvarthy '14, …]

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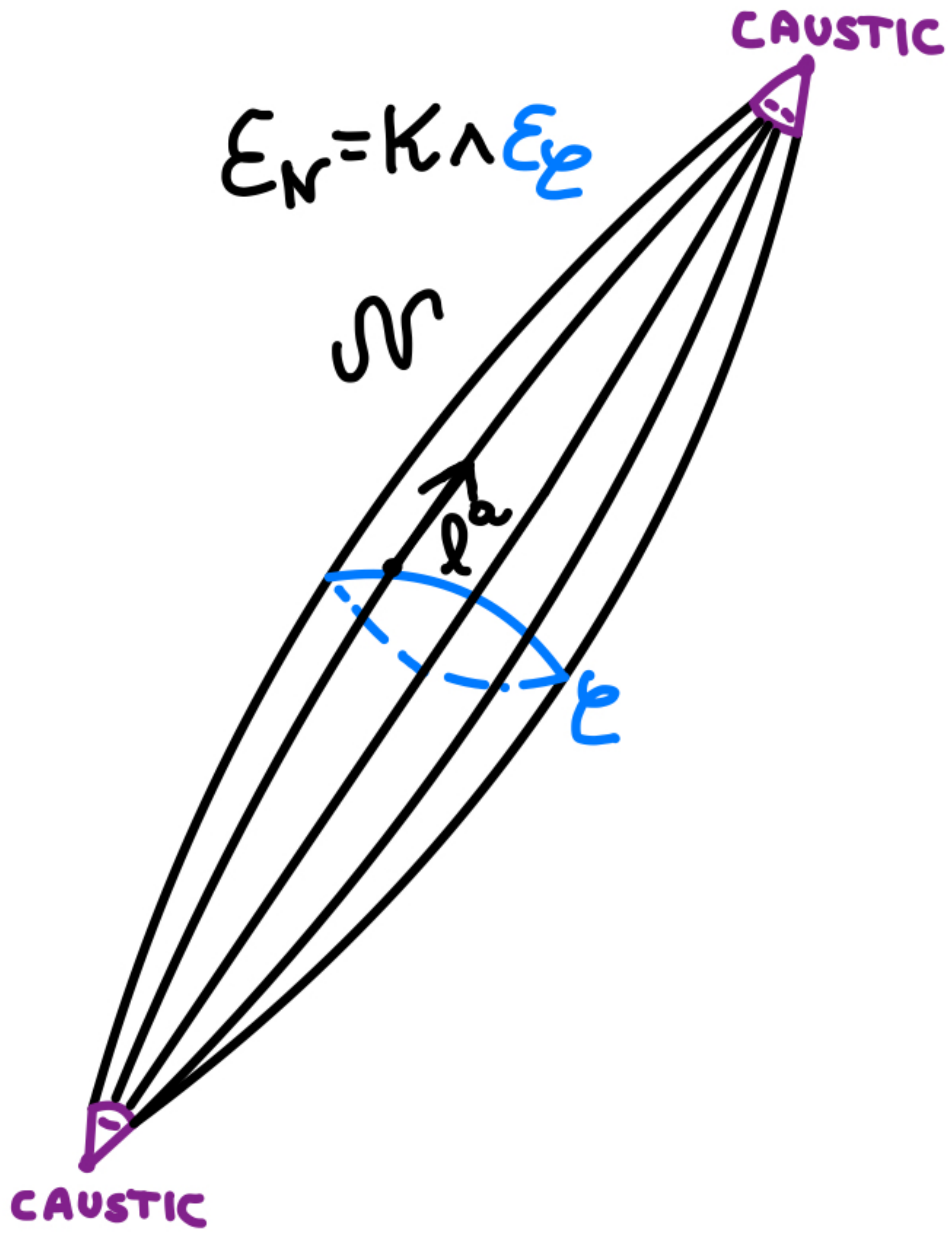
Expansion Tensor $\theta_{ab} = \frac{1}{2}\mathcal{L}_\ell q_{ab}$ such that $\theta_a{}^b = \frac{\theta}{2} q_a{}^b + \sigma_a{}^b$ (Traceless ↓ $\sigma_a{}^b$)

Carrollian Connection: 'preserve' the metric as

$$D_a q_{bc} = -(k_b \theta_{ac} + k_c \theta_{ab})$$



It defines a boost connection $D_a \varepsilon_\mathcal{N} = -\omega_a \varepsilon_\mathcal{N}$ With $\omega_a = \kappa k_a + \pi_a$

Dynamics on Null Hypersurfaces

[Ciambelli-Petropoulos'19
Donnay-Marteau '19,
Chandrasekaran-Flanagan-
Shehzad-Speranza '21]
[Freidel-Jai-akson '22]

Null Brown York Tensor $T_a{}^b := D_a \ell^b - \delta_a^b D_c \ell^c$

Einstein Gravity projected to $\mathcal{N}$: $D_b T_a{}^b = T^{\text{mat}}_{a\ell}$

Projected to ℓ^a : $(\mathcal{L}_\ell + \theta)\theta = \mu\theta - \sigma_a{}^b \sigma_b{}^a - R_{\ell\ell}$ Null Raychaudhuri Equation

Projected to $q_c{}^a$: $(\mathcal{L}_\ell + \theta)\pi_a = D_b(\mu q_a{}^b - \sigma_a{}^b) + R_{a\ell}$ Damour Equation

$$\mu = \kappa + \frac{\theta}{2}$$ Surface Tension

Fluid-Gravity perspective, Membrane paradigm

[Price-Thorne '88]

Einstein Projected to Null Hypersurface

Fluid-Gravity perspective,

Membrane paradigm

[Raychaudhuri ’55, Sachs ’61, Landau ’75]

Null Raychaudhuri

[Damour ’79, Thorne-Price-Macdonald ’86, Penna ’17, Donnay-Marteau, Ciambelli-Leigh-Marteau-Petropoulos ’19, Freidel-Jai-akson ’22, LC-Freidel-Leigh ’23]

Fluid-Gravity perspective, Membrane paradigm

Intrinsic Null Conservation Law
With Matter Source

$$(\mathcal{L}_\ell + \theta)\theta = \mu\theta - \sigma_a{}^b\sigma_b{}^a - 8\pi G T^{\rm mat}_{\ell\ell} \qquad \Leftrightarrow \qquad \ell^a D_b T_a{}^b = T^{\rm mat}_{\ell\ell}$$

We impose $8\pi G = 1$

Symplectica

Starting from the gravitational action $\delta L_{\text{EH}} = \mathrm{d}\Theta_{\text{EH}} + G^{ab}\delta g_{ab}$

$$\Theta_{\text{EH}} = \tfrac{1}{2} D_a \left(\sqrt{g}\delta\ell^a\right) - \delta\left(\sqrt{g} D_a \ell^a\right) + \Theta_{\text{can}}$$

We extract after a canonical transformation a canonical Carrollian-fluid potential

Intrinsic to $\mathcal{N}$

$$\Theta^{\text{can}} = \int_{\mathcal{N}} \varepsilon_{\mathcal{N}} \left(\frac{1}{2}\tau^{ab}\delta q_{ab} - \tau_a \delta\ell^a\right)$$

$$T_a{}^b = \tau_a \ell^b + \tau_a{}^b$$

$$\tau_a = \pi_a - \theta k_a,$$
$$\tau_a{}^b = \sigma_a{}^b - \mu q_a{}^b$$

[Hayward '93, Ashtekar '00, Lewandowski '04, Lehner-Myers-Poisson-Sorkin '16, Parattu-Chakraborty-Padmanabhan '16, LF-Hopfmueller'16, DePaoli-Speziale '17, Adami-Grumiller-Sheikh-Jabbari-Taghiloo-Yavartanoo-Zwikel '21, Chandrasekaran-Speranza '21, Chandrasekaran-Flanagan-Shehzad-Speranza '21, LF-Jai-akson '22]

Symplectica

Intrinsic Canonical Symplectic Potential

$$T_a{}^b = \tau_a \ell^b + \tau_a{}^b \qquad \Theta^{\mathsf{can}} = \int_{\mathcal{N}} \varepsilon_{\mathcal{N}} \Big(\frac{1}{2} \tau^{ab} \delta q_{ab} - \tau_a \delta \ell^a \Big)$$

Dynamics is an expression of diffeomorphism symmetry

Noether Charges are corner charges on-shell

Diffeomorphism: $Q_\xi = I_{\hat{\xi}} \Theta^{\mathsf{can}} = - \int_{\mathcal{N}} \varepsilon_{\mathcal{N}} \, \xi^a D_b T_a{}^b + \int_{\mathcal{C}} \xi^a T_a{}^b \varepsilon_b$

Symplectica

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Dynamics is an expression of diffeomorphism symmetry

Noether Charges are corner charges on-shell

Diffeomorphism: $Q_\xi \hat{=} \int_{\mathcal{C}} \xi^a T_a{}^b \varepsilon_b$

Local Lorentz: $\delta k_a = -\lambda k_a + \zeta_a$ (internal boost: $-\lambda k_a$; shift: ζ_a) $\qquad Q_{(\lambda,\zeta)} = \int_C \lambda \Omega \epsilon_{\mathcal{C}}^{(0)}$

Corner symmetry group: BMSW $=\text{Diff}(S) \ltimes \mathbb{R}^S \ltimes \mathbb{R}^S$

Chandrasekaran, Flanagan, Prabhu '18
LF, Oliveri, Pranzetti, Speziale '21

Symplectica

Intrinsic Canonical Symplectic Potential

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Dynamics is an expression of diffeomorphism symmetry

Noether Charges are corner charges on-shell

Unimodular decomposition $q_{ab} = \Omega \bar{q}_{ab} \;\Rightarrow\; \varepsilon_{\mathcal{N}} = \Omega \varepsilon_{\mathcal{N}}^{(0)},\; \varepsilon_{\mathcal{C}} = \Omega \varepsilon_{\mathcal{C}}^{(0)}$

Dynamical data: Spin-0 $\{\Omega, \mu\}$ Spin-1 $\{\pi_a, \ell^a\}$ Spin-2 $\{\sigma^{ab}, \bar{q}_{ab}\}$

Symplectica

Intrinsic Canonical Symplectic Potential

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Raychaudhuri has only Spin-0 & Spin-2, we can decouple the Spin-1 Sector

by chosing the boost parameter such that $\delta \ell = 0$

Prime Phase Space

Since $\delta\ell = 0$ on the prime phase space, choose $\ell = \partial_v$

Diffeomorphisms: $\xi^a = f\ell^a + Y^b {q_b}^a$, Focus on time reparameterization $Y^a = 0$

Raychaudhuri constraint

$$C = \partial_v^2\Omega - \mu\partial_v\Omega + \Omega\left(\sigma^2 + 8\pi G T_{vv}^{\text{mat}}\right) = 0$$

Time Reparameterization Charge:

$$\mathcal{E}_f = \frac{1}{8\pi G}\int_{\mathcal{N}} \varepsilon_{\mathcal{N}}^{(0)} f C + \frac{1}{8\pi G}\int_{\mathcal{C}} \varepsilon_{\mathcal{C}}^{(0)} \left(\Omega\partial_v f - f\partial_v\Omega\right)$$

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Kinematical Poisson Brackets

Spin-1 Decouples on the Prime Phase Space

$$\Omega^{\text{can}} = \frac{1}{8\pi G}\int_{\mathcal{N}} \varepsilon_{\mathcal{N}}^{(0)} \left(\delta\left(\tfrac{1}{2}\Omega\sigma^{ab}\right) \wedge \delta q_{ab} - \delta\mu \wedge \delta\Omega \right)$$

Spin2 : q_{ab} and $\Omega\sigma^{ab}$ are conjugate variable : Radiation modes

Spin0 : Ω and μ are conjugate variable : Newtonian modes

Kinematical Poisson Brackets

Spin-1 Decouples on the Prime Phase Space

$$\Omega^{\text{can}} = \frac{1}{8\pi G}\int_{\mathcal{N}} \varepsilon_{\mathcal{N}}^{(0)} \left(\delta\left(\tfrac{1}{2}\Omega\sigma^{ab}\right)\wedge\delta q_{ab} - \delta\mu\wedge\delta\Omega\right)$$

To Compute the Poisson Bracket

Reisenberger '07 '12 '17 '18,
LF,Ciambelli, Leigh '23

Beltrami Differentials: $\overline{q}_{ab}\mathrm{d}x^a\mathrm{d}x^b = 2\frac{|\mathrm{d}z+\zeta\mathrm{d}\bar{z}|^2}{\beta}$ with $\beta = 1-\zeta\overline{\zeta}$

Define the Propagator: $\overline{\mathcal{P}}_{12} = \dfrac{e^{-\int_{v_1}^{v_2}(\zeta\partial_v\bar{\zeta}-\bar{\zeta}\partial_v\zeta)\mathrm{d}v}}{\sqrt{\Omega_1\Omega_2}}\theta(v_1-v_2)\delta^{(2)}(z_1-z_2)$

$$\mathcal{P}_{12} = \left(D_v + \tfrac{1}{2}\theta\right)^{-1}_{12}$$

Kinematical Poisson Brackets

[Sachs '62, Gambini-Restuccia '78, Penrose '80, Torre '86, Goldberg-Robinson-Soteriou '96, Ashtekar '00, Lewandowski '04, Reisenberger '07 '12 '17 '18, Wieland '17 '19 '21,'24, Ciambelli-LF-Leigh '23

$$\{\Omega_1, \mu_2\} = 8\pi G\, \delta^{(3)}(x_1 - x_2)$$
$$\{\zeta_1, \bar{\zeta}_2\} = 4\pi G\, \beta_1 \mathcal{P}_{12} \beta_2$$

$$\overline{\mathcal{P}}_{12} \sim \theta(v_1 - v_2)\delta^{(2)}(z_1 - z_2)$$

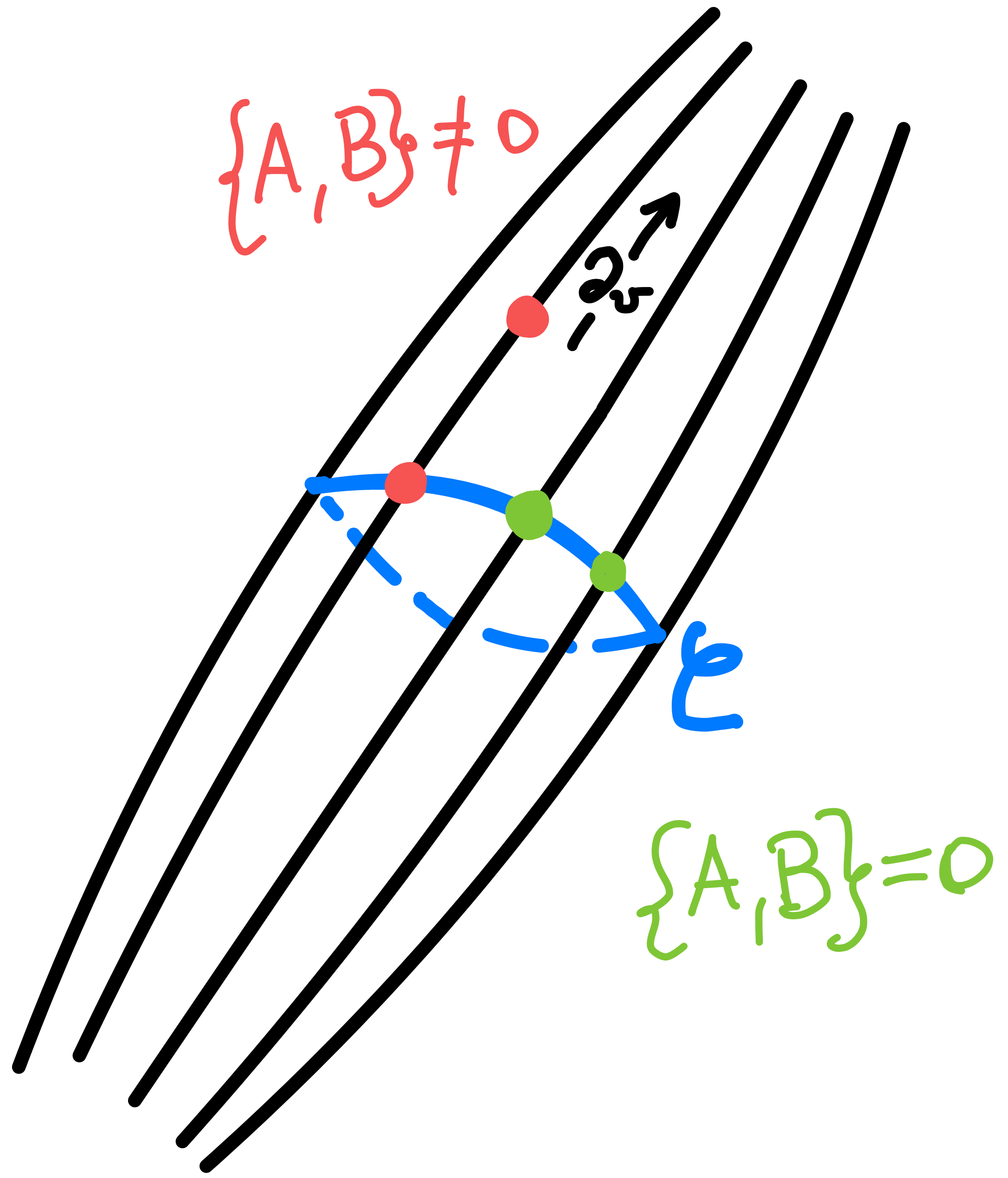
Mixing spin-0 spin-2

$$\{\mu_1, \zeta_2\} = -4\pi G\, \frac{\partial_{v_1}\zeta_1}{\beta_1}\, \overline{\mathcal{P}}_{12}\beta_2$$

$$\{\mu_1, \mu_2\} = 4\pi G \left(\frac{\partial_{v_1}\zeta_1}{\beta_1} \overline{\mathcal{P}}_{12} \frac{\partial_{v_2}\bar{\zeta}_2}{\beta_2} + c.c \right).$$

Ultralocal Structure on the Cut

Non-Commutativity on Null Lines



Ultralocality of Carrollian Physics
BMSW symmetry

Dressing Time

The surface tension μ, transforms anomalously under time reparametrisation

$$\delta_f \mu = f\partial_v \mu + \mu \partial_v f + \partial_v^2 f$$

We can therefore trade it for a dynamical time variable

The dressing time is a diffeo $V : v \to V(v,\sigma)$ which maps $\mu \to 0$

$$\mu = \frac{\partial_v^2 V}{\partial_v V}$$

Dressing Time

The surface tension μ, transforms anomalously under time reparametrisation $\mu = \kappa + \frac{1}{2}\theta$

$$\delta_f \mu = f\partial_v \mu + \mu \partial_v f + \partial_v^2 f$$

We can therefore trade it for a dynamical time variable

The dressing time is a diffeo $V : v \to V(v, \sigma)$ which maps $\mu \to 0$

$$\mu = \frac{\partial_v^2 V}{\partial_v V}$$

It allows the construction of gauge invariant observables

$$\tilde{\Omega} := \Omega \circ V^{-1}, \ \tilde{q}_{ab} := q_{ab} \circ V^{-1}, \ldots$$

Dressing time differs from affine time when $\theta \neq 0$ $\tilde{\mu} = 0 \to \tilde{\kappa} = -\frac{1}{2}\tilde{\theta}$

The Dressing Time

The canonical structure can then be written in terms of the gauge invariant observables

The dressing time is conjugated to the constraint in the bulk

[a Specific Dynamical Frame: LF-Donnelly'16, Rovelli '19, Wieland '21, Carrozza-Hoehn '21, Goeller-Hoehn-Kirklin '22]

It provides a specific quantum clock (QRF)

At the corner, the boost time is conjugated to the area form

[Carlip-Teitelboim '93, Wald '93]

The area form is shown to be equal to the modular Hamiltonian

$$K = -\frac{1}{2\pi} \ln \rho$$

$$\rho = \mathrm{tr}_D(|0\rangle\langle 0|)$$

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At the quantum level, allowing finite resolution smoothen modular divergences

$$\Delta V \neq 0 \to \Delta K < \infty$$

[Useful in the Crossed Product: Connes '73, Connes-Rovelli '94, Witten '21, Jensen-Sorce-Speranza '23, Klinger-Leigh '23]

It allows the definition of a dynamical entropy that satisfies a Generalized Second Law

$$\boxed{\tilde{S}_T = (1 - (V - T)\partial_V)\Omega}$$

Charge associated to Boosts $f_T \partial_V = (V - T)\partial_V$

is monotonic even with Expansion $\partial_V S_T \geq 0$

[Hollands-Wald-Zhang '24, Visser-Yan '24]

Constraint algebra

At the quantum level: Promote Raychaudhuri to a quantum stress operator balance law

$$\hat{T}^0 = \partial_v^2\hat{\Omega} - :\mu\partial_v\hat{\Omega}: \qquad \hat{T}^2 =: \hat{\Omega}\hat{\sigma}_a{}^b\hat{\sigma}_b{}^a: \qquad \hat{T}^M =: \Omega\, T_{vv}^{\text{mat}}:$$

$$\boldsymbol{C} = \hat{T}^0 + \hat{T}^2 + \hat{T}^{\text{mat}}$$

Carrollian Ultralocality $\rightarrow$ Each null ray carry a chiral CFT

[Schroer' 10, Wall '11]

$\rightarrow$ 4d Quantum gravity reduces to a collection of 1 dimensional CFTs!

The algebra develop a central charge which can be evaluated perturbatively and non-perturbatively

Quantization

At the quantum level: Promote Raychaudhuri to a quantum stress operator balance law

$$\boldsymbol{C}_f = \int_{\mathcal{N}} f \boldsymbol{C}(u, z, \bar{z})$$

generates time reparametrisation

$$e^{i\boldsymbol{C}_f} \phi e^{-i\boldsymbol{C}_f} = \phi \circ F_f \qquad F_f(v) = e^{f\partial_v}(v).$$

It satisfies the algebra

$$[\boldsymbol{C}_f, \boldsymbol{C}_g] = \boldsymbol{C}_{\{f,g\}} + cN \int_{\mathcal{N}} (\partial_v^3 f g - \partial_v^3 g f)$$

$c =$ central charge per null generator

$N =$ number point points on the cut $\mathcal{C}$

$\{f, g\} = f\partial_v g - g\partial_v f$ de Witt bracket

Quantization and Time

It satisfies the algebra

$$[\boldsymbol{C}_f, \boldsymbol{C}_g] = \boldsymbol{C}_{\{f,g\}} + cN \int_{\mathcal{N}} (\partial_v^3 f g - \partial_v^3 g f)$$

Physical observables commute with $\boldsymbol{C}_f$

States depend on time $|f\rangle := e^{i\boldsymbol{C}_f}|0\rangle \neq |0\rangle$

Expectation value of Physical observables are time independent $\langle f|O|f\rangle = \langle 0|O|0\rangle$

Quantization depends on time through the normal order $\quad :A::B:=:A\star B:$

Weak Gravity Expansion

Perturbative expansion can be performed in Stationary Background $\partial_v \Omega_0 = 0$

Perturbative Decoupling of Spin-$0\&2\&\mathrm{mat}$: $\{\bar{\mu}_1, \zeta_2\} = 0 \;\Rightarrow\; [T^0, T^2] = 0$

Each stress tensor $\hat{T}^0, \hat{T}^2, \hat{T}^{\mathrm{mat}}$ satisfy a chiral Virasoro algebra

$$\boldsymbol{C}(x_1)\boldsymbol{C}(x_2) \sim \left(\frac{cN}{2(v_{12}+i\epsilon)^4} + \frac{2\boldsymbol{C}(x_2)}{(v_{12}+i\epsilon)^2} + \frac{\partial_{v_2}\boldsymbol{C}(x_2)}{(v_{12}+i\epsilon)}\right)\delta^{(2)}(z_1 - z_2)$$

$$c = c_{\text{Spin-0}} + c_{\text{Spin-2}} + c_{\mathrm{mat}} = 2 + 2 + c_{\mathrm{mat}}$$

$$N = \delta^{(2)}(0) = \infty$$

Universal UV divergences in energy fluctuations

Molecular Quantization

[Connects to: Rovelli-Smolin ’94, Ashtekar-Lewandowski ’96, Rovelli ’96, Wieland ’21-23 and many others]

$$\hat{\Omega}(v,z) = \sum_{i=1}^{N} \hat{\Omega}_i(v)\delta^{(2)}(z-z_i) \longrightarrow \qquad \boldsymbol{T}(x) = \sum_i \Omega_i \boldsymbol{T}_i(v_1)\delta^{(2)}(z-z_1)$$

Leads to finite central charge for the Spin-0 sector

$\longrightarrow$ Continuous in the null generators

A property of the corner symmetry representation

[LC-Freidel-Leigh to appear ’24]

$$\boldsymbol{C}_i(x_1)\boldsymbol{C}_j(x_2) :\ = \delta_{ij}\left(\frac{c}{2(v_{12}-i\epsilon)^4} + \frac{2\boldsymbol{C}_j(v_2)}{(v_{12}-i\epsilon)^2} + \frac{\partial_{v_2}\boldsymbol{C}_j(v_2)}{v_{12}-i\epsilon}\right)$$

Molecular Quantization: Mesoscopic Quantum Gravity Scale

Fuzzy sphere regularization ?

Molecular Quantization, LQG and Holography

The area operator plays the role of a fluid density with constituents called "embadon" $\epsilon\mu\beta\alpha\delta o\nu$: LQG which are codimension 2 area elements

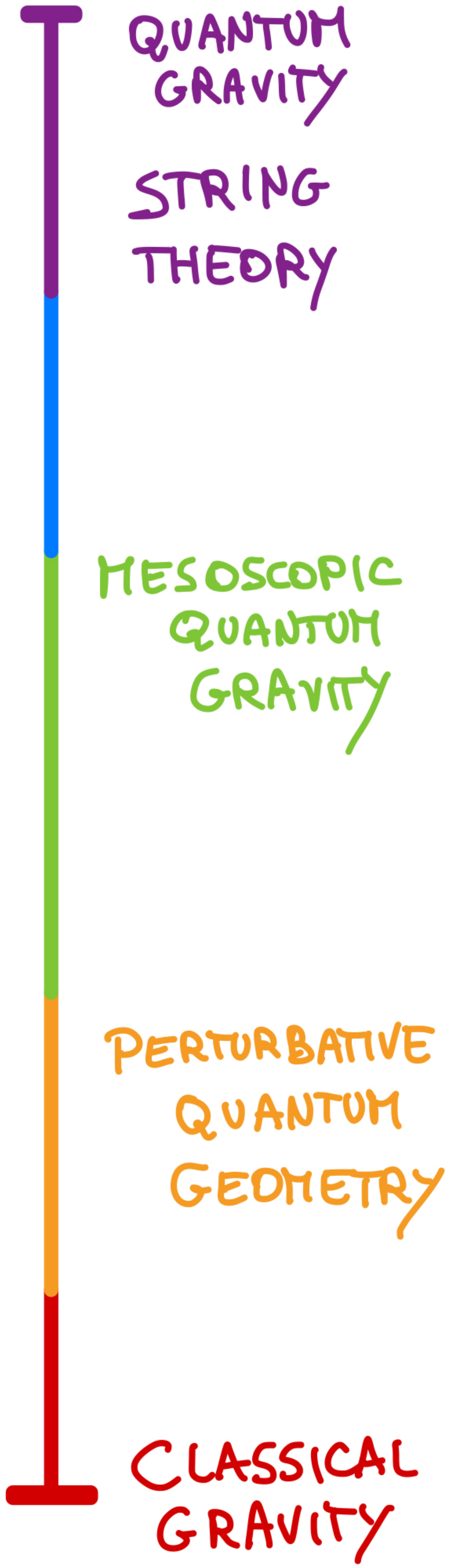
$$\hat{\Omega}(v,z) = \sum_{i=1}^{N} \hat{\Omega}_i(v)\delta^{(2)}(z - z_i)$$

The area form is equal to the modular Hamiltonian : Hol

→ These are the foundational elements of LQG and Holography

Key differences with LQG:

- The symmetry group is BMSW$=$Diff$(S) \ltimes \mathbb{R}^S \ltimes \mathbb{R}^S$ not SU(2)
- The spatial diffeomorphims generate entanglement and have a canonical generator
- Boost diffeomorphism is non trivially represented on every cut
- Null Time reparametrisation can be quantized and is centrally extended
- Each null line carries a Fock vaccum
- Usual Fock vaccum is recover as a continuum limit of embadons.



Mesoscopic Quantum Gravity

The central charge vanishes classically

There are ∞-many null generators/points on the cut

The central charge diverges in perturbative QFT

Postulate: Quantum Gravity has finite central charge

At Mesoscopic Quantum Gravity Scale

Find discrete=molecular representation of BMSW

Quantum Area $\hat{\Omega}$ has reps with discrete support

Quantum Geometry provides a UV regulator

Ultralocality: it can be done preserving symmetries

Conclusion

Raychaudhuri Constraint as a Carrollian Conservation Law

Raychaudhuri Constraint as a Chiral CFT balance equation

Dressing Time and Boost Monotonicity

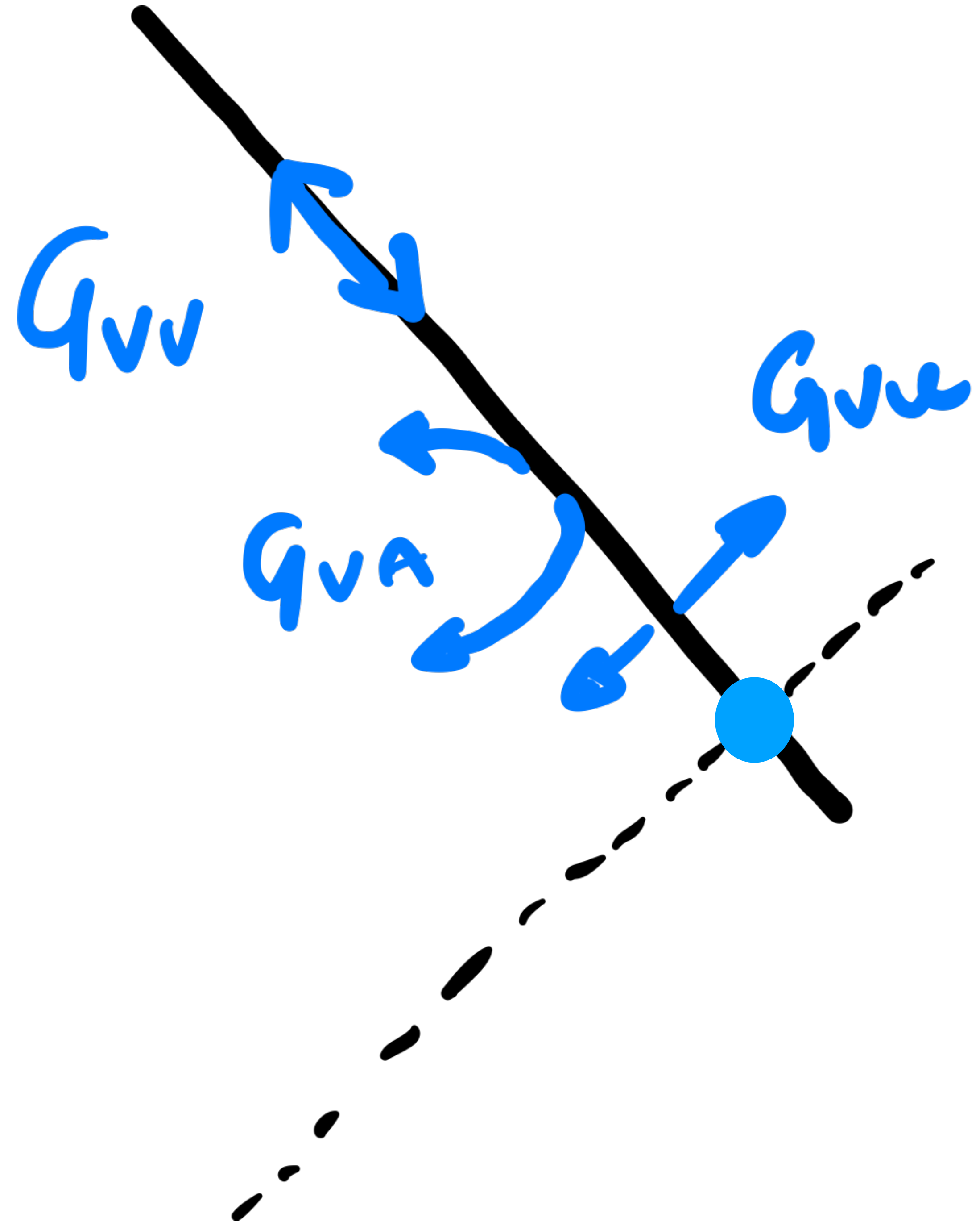
The area element is the modular Hamiltonian

Time exists due to the central charge

Molecular Quantization leads to UV finiteness

Connection with asymptotic infinity

$\partial_u \Omega - 2/r \to M_B$ [Kapec,Raclariu,Strominger '16]



Future: Damour and Spin-1

Intersecting light cones and study of

The Verlinde-Zurek Fluctuation equation