Spherical Collapse and Black Hole Evaporation

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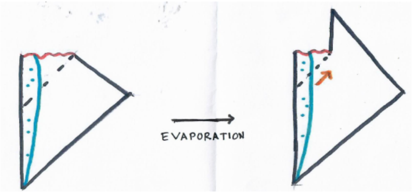
Introductory Remarks:

- Hawking showed that BHs radiate at $kT_H \approx \frac{m_P}{M}\ m_P c^2$.
- Two time scales are relevant to the radiation process:

1. $T_{\mathrm{evap}} \sim \frac{M}{\dot{M}}, \quad \dot{M} \sim \sigma\ T_H^4 R_S^2$
2. $T_{\mathrm{settling\ down}}$: When BH radiates and loses mass, it takes some time to settle down. This can be estimated as the time one part of the BH takes to communicate with another:
   $T_{\mathrm{settling\ down}} \sim \frac{R_S}{c}$

- $\frac{T_{\mathrm{settling\ down}}}{T_{\mathrm{evap}}} \sim \frac{\mathbf{m_P^2}}{\mathbf{M^2}} << 1$
  'Practically instantaneous settling down' implies we can model process through QFT on 1 parameter family of fixed BH geometries of decreasing mass. Valid till mass of BH is approx m_P at which point QG effects become important.
- Final State $= m_P +$ Thermal Radiation $=$ Mixed State.
  Initial State: Matter in a pure quantum state
  INFO LOSS.



Note: Hawking's calculation is of QFT on a fixed sptime geometry whereas in reality the bh radiation back-reacts on the geometry. Backreaction effects are estimated through physical arguments resting on quasistaticity but not computed.

Question: Does an actual *computation* of back reaction alter Hawking's proposed picture of info loss in anyway?
To answer this, we need a classically solvable system of collapse with computable back reaction.

To this end, I consider a system of spherically symmetric scalar field collapse with following key features:

- Geometry is that of Spherically symm GR.
  Ensures $T_H \sim 1/M$.
- Axis of symmetry is part of sptime.
  Ensures a single set of asymptotic regions rather than 'Kruskal' type situation.
- Matter coupling depends on the Areal Radius in a specific way.
  Ensures classical solvability (Vaidya).
  Ensures computability of matter stress energy exp value, and hence, formulation of semiclassical Einstein equations.
  Allows informed speculation on the fate of information.

Plan of Talk:

1. Kinematic Setup in Spherical Symmetry
2. Classical Dynamics: Einstein eqns, Vaidya solution
3. Semiclassical Dynamics: Semiclassical Einstein equations, Semiclassical Solution
4. Asymptotic Analysis of Semiclassical Einstein Equations
5. Informed Speculation: Arena for true degrees of freedom, Fate of Information

1. Kinematics: Geometry in Spherical Symmetry

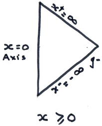
$ds^2 = {}^{(2)}g_{\mu\nu}dx^\mu dx^\nu + R^2(d\Omega)^2, \;\; \mu,\nu = 1,2$

Choose conformal coordinates $x^\pm$ along radial outgoing/ingoing light rays: ${}^{(2)}g_{\mu\nu}dx^\mu dx^\nu = -e^{2\rho}dx^+dx^-$

- $4\pi R^2$ is area of spherical light front at fixed x^+, x^-, $R = R(x^+, x^-)$. Outgoing/Ingoing expansions are proportional to $\partial_+ R, \partial_- R$.
- Geometry is such that *Axis of Symmetry* is part of our sptime. Restrict attention to sptimes where Axis of Symmetry is *timelike* curve.
- 'Straighten' out axis by choice of conformal coordinates: Axis curve can be written as $x^+ = a(x^-)$. By choosing $a(x^-)$ as our new x^-, axis becomes straight line $x^+ = x^-$. Setting $x^\pm = t \pm x$, Axis is at $x = 0$.
- In summary: Region of interest is the $x \geq 0$ part of the $x - t$ plane.

  Note: By definition $R = 0$ at axis and geometry near axis is *non-singular* (as opposed to $R = 0$ BH singularity). In solns of interest geometry at axis turns out to be flat.

Initial conditions on Geometry:



Require metric to be asymp flat at $\mathcal{I}^-$ and require $\mathcal{I}^-$ to be located at $x^- = -\infty$.

This is achieved by requiring that as $x^- \to -\infty$:

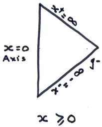
$R = \frac{x^+ - x^-}{2} + O(1/x^-) \;\; \epsilon^{2\rho} = 1 + O(1/(x^-)^2)$

Vaidya solution satisfies these with mass info in $O(1/x^-)$ part of R and in $O(1/(x^-)^2)$ part of $e^{2\rho}$.

This together with location of axis at $x = 0$ fixes all freedom in choice of conformal coordinates.

- In summary: Region of interest is the $x \geq 0$ part of the $x - t$ plane. $\mathcal{I}^-$ is located at $x^- = -\infty$. Axis is located at $x = 0$.

Behavior of matter at the axis and at $\mathcal{I}^-$



At the axis: A point on the x^+, x^- plane away from the axis represents a 2d sphere of radius $R(x^+, x^-)$. A point on the axis is just a 0d point. Matter field f is spherically symmetric $f = f(x^+, x^-)$ and differentiable in 4d at the axis. It turns out that this implies that at the axis $\partial_+ f = \partial_- f$

At $\mathcal{I}^-$: We require the matter field to be of compact support on $\mathcal{I}^-$ so that $f(x^+, -\infty) = g(x^+)$, g comp support in x^+.
We also require $g(x^+)$ to satisfy a certain condition of prompt collapse appropriate to the Vaidya solution. Roughly, this requires a discontinuous jump in the derivative of g at its initial support.

2. Dynamics: Action

- Geometry: $S_{geometry} = \frac{1}{8\pi G} \int d^4x \sqrt{-^{(4)}g}\,^{(4)}R$
  In order to facilitate comparision with 2d gravity conventions in literature hereon we set $R^2 = R^2_{old}/\kappa^2$. (In 2d gravity, $R^2 =: e^{-2\phi}$ where ϕ is called the dilaton.) κ is an arbitrarily chosen (but fixed) constant with dimensions of length.

  Integrating over angles and removing total deriv terms:
  $S_{geometry} = \frac{1}{2G\kappa^2} \int d^2x \sqrt{-^{(2)}g} R^2 [^{(2)}R + 2(\frac{\nabla R}{R})^2 + 2\kappa^2]$
- Matter coupling: The matter coupling is chosen to depend on the Areal Radius R so that the matter action is:
  $S_{matter} = -\frac{1}{8\pi} \int d^4x \sqrt{-^{(4)}g}\,^{(4)}g^{ab} \frac{1}{\kappa^2 R^2} (\nabla_a f \nabla_b f)$
  Integrating over angles, we obtain:
  $S_{matter} = -\frac{1}{2} \int d^2x \sqrt{-^{(2)}g} (\nabla f)^2$.
  The coupling reduces to conformal coupling to the 2d metric.

  The total action is then $S_{geometry} + S_{matter}$. In what follows it is convenient to set $G = c = \kappa = 1$. Thus $\hbar$ is not set to unity.

EoM: Einstein's Equations

4d Einstein equations in spherical symmetry in (x^+, x^-, Ω) coordinates

$G_{\Omega\Omega} = 0 \equiv \partial_+\partial_-\rho + \frac{1}{R}\partial_+\partial_- R = 0$ ($T_{\Omega\Omega} = 0$)

$R^2\ G_{+-} = 2R\partial_+\partial_- R + 2\partial_+\partial_- R + \frac{1}{2}e^{2\rho} = 0$ (Traceless T_{ab})

$R^2\ G_{\pm\pm} = R^2[-\frac{2}{R}(\partial_\pm^2 R - 2\partial_\pm\rho\partial_\pm R)] = (\partial_\pm f)^2$

Since $G_{ab} = 8\pi G T_{ab}$, we have that the only non-trivial stress energy components are $T_{\pm\pm} = \frac{1}{4\pi R^2}(\frac{1}{2}(\partial_\pm f)^2)$.

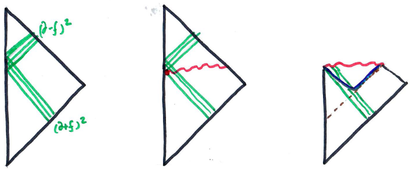
This is exactly the form of a pair of (ingoing, outgoing) streams of null dust. The Vaidya solution only has an ingoing stream...

Matter EoM and the Vaidya Solution

Matter eqns: f doesnt see conformal factor, satisfies 2d wave equation on $x - t$ plane, $f = f_{(+)}(x^+) + f_{(-)}(x^-)$.

The axis conditions imply:

$\partial_+ f_{(+)}(x^+) = \partial_- f_{(-)}(x^-)$ at axis i.e. at $x^+ = x^-$.



Axis reflection of stress energy

Vaidya

So till the compactly supported infalling matter hits the axis there is no outgoing matter stream. But in the (prompt collapse) Vaidya solution, as soon as the matter hits the axis a spacelike singularity forms.

It can be checked in all technical detail that the Vaidya metric, expressed in null coordinates, is a classical solution in which axis is located at $x = 0$ and which is asymp flat as $x^- \to -\infty$.

3. Semiclassical Theory: Matter field quantization

Field Operator:Conformally coupled massless scalar field does not 'see' metric conformal factor, propagates freely on the fiducuial flat $x - t$ spacetime. Can expand $\hat{f}$ in terms of left, right moving modes. Axis boundary conditions imply these are not independent:

$$\hat{f}(x^+, x^-) = f_{(+)}(x^+) + f_{(-)}(x^-)$$

$$= \int_0^\infty dk \frac{1}{\sqrt{4\pi k}}(\hat{a}(k)e^{-ikx^+} + h.c) + \int_0^\infty dk \frac{1}{\sqrt{4\pi k}}(\hat{a}(k)e^{-ikx^-} + h.c)$$

(Mode functions are $\frac{\cos kxe^{-ikt}}{\sqrt{\pi k}}$)

Hilbert Space: Fock space wrto $\hat{a}(k), \hat{a}^\dagger(k)$. Note that since the 4-metric is asym flat at $\mathcal{I}^-$, (x, t) coordinates are freely falling there so that 'fiducial' and physical vacuum coincide at early times.

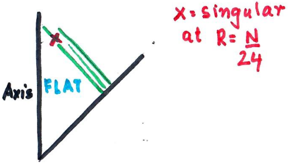
- Above quantization can be used either for test field on Vaidya sptime, or for defining semiclassical gravity $G_{ab} =< T_{ab} >$ or even perhaps for fundamental quantum gravity-matter system...
- For the quantum test field, an argument similar to Hawking's establishes late time thermal radiation at the Hawking Temperature.

Semiclassical Einstein Equations:

Since the scalar field is conformally coupled to the 2d metric, its quantum stress energy expectation value can be calculated using the results of Davies and Fulling. Their results continue to hold in the presence of the boundary at the axis because the axis is a straight line in the flat sptime coordinates.

Recall that due to asym flatness, the fiducial flat sptime coordinates are freely falling at $\mathcal{I}^-$. If the field is in a coherent state modelled on the classical data at $\mathcal{I}^-$, $< \hat{T}_{ab} >$ has in addition to its 'classical' contribution, a vacuum fluctuation part proportional to $\hbar$. In order to include quantum fluctuations in the matter but neglect those in the geometry, we enhance the vacuum contribution by a factor of N by conformally coupling an additional $N-1$ number of matter fields each in their vacuum state with $N\hbar$ held fixed as $\hbar \to 0$.

- The semiclassical Einstein eqns $G_{ab} = 8\pi G < \hat{T}_{ab} >$ take the form:
  $G_{\Omega\Omega} = 0 \equiv \partial_+\partial_-\rho + \frac{1}{R}\partial_+\partial_- R = 0$
  $R^2\ G_{+-} = 2R\partial_+\partial_- R + 2\partial_+ R\partial_- R + \frac{1}{2}e^{2\rho} = -\frac{N\hbar}{12\pi}\partial_+\partial_-\rho$
  $R^2\ G_{\pm\pm} = R^2[-\frac{2}{R}(\partial_\pm^2 R - 2\partial_\pm\rho\partial_\pm R)] = (\partial_\pm f)^2 - \frac{N\hbar}{12\pi}((\partial_\pm\rho)^2 - \partial_\pm^2\rho)$
- Note that when $\rho = 0$, quantum part vanishes. Thus, when f vanishes, classical flat spacetime (with $e^{2\rho} = 1$) remains a soln. Note also that we can eliminate $\partial_+\partial_-\rho$ between the first two eqns: $\frac{1}{R}\partial_+\partial_- R = -\frac{\partial_+ R\partial_- R + \frac{1}{4}e^{2\rho}}{R^2 - \frac{N\hbar}{24\pi}}$. Denominator vanishes at $R^2 = \frac{N\hbar}{24\pi}$. Analysis of the pde's implies that for generic matter data on $\mathcal{I}^-$, there is a curvature singularity at $R^2 = N\hbar/24\pi$.

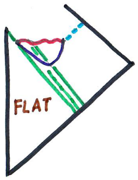


Semiclassical Soln

- One quasilocal characterization of BH is existence of outer marginally trapped surface (OMTS)
- $\partial_+ R = 0$=constant at OMTS. Its normal is: $(n_+ = \partial_+^2 R, n_- = \partial_- \partial_+ R)$.
  These components respond to stress energy via the Einstein eqns. Can show as expected that when $\langle \hat{T}_{++} \rangle$ is positive/zero/negative, OMTS is splike-expanding/null-nonexpanding/timelike- shrinking.
- One possible scenario which seems to be consistent with:
  - the semiclassical eqns,
  - the existence of singularity at $R^2 = \frac{N\hbar}{24\pi}$,
  - the early flat phase,
  - a post collapse evaporating phase,
  - agreement with the classical sptime when $\hbar = 0$

  is as follows..

An AH is born near the singularity along the initial classical matter infall line. At sing both sets of light rays converge so $\partial_- R, \partial_+ R < 0$
Since $R = \sqrt{\frac{N\hbar}{24}}$ is constant along the singularity, its normal $=(\partial_+ R, \partial_- R)$. Normal is timelike so singularity is splike.
$\langle \hat{T}_{++} \rangle$ is expected to dominated by the classical piece till Hawking evaporation starts at which point on $\langle \hat{T}_{++} \rangle$ is negative and AH transmutes to timelike Outer Marginally Trapped Tube. Timelike OMTT and splike sing either intersect and a Cauchy horizon develops (or both asymptote to a null 'thunderbolt)



Above (no thunderbolt) picture is supported by old simulations by Lowe, Parentani-Piran.

4. Asymptotic Analysis:

We make well motivated assumption that metric is asymp flat as $x^+ \to \infty$. Then Sph.symmetry + outgoing Hawking radiation + asymp. flatness suggest that the metric takes the 'outgoing' Vaidya form at $\mathcal{I}^+$ in outgoing Eddington-Finkelstein coordinates $\bar{u}, R$. $\bar{u}$ is an outgoing null coordinate which need not coincide with x^- so we have $\bar{u} = \bar{u}(x^-)$.

- Under these assumptions we obtain:

  $-\frac{1}{2}R^2 G_{\bar{u}\bar{u}} = \frac{dM_B}{d\bar{u}}$ ($M_B =$ Bondi mass along $\mathcal{I}^+$) so that:

  $\frac{dM_B}{d\bar{u}} = -4\pi R^2 \langle T_{\bar{u}\bar{u}} \rangle = -\frac{1}{2}(\partial_{\bar{u}} f)^2 +$ *quantum contribution*.

- The quantum contribution is given by the standard Schwarz derivative term and we get:

  $\frac{dM_B}{d\bar{u}} = -\frac{1}{2}(\partial_{\bar{u}} f)^2 + \frac{N\hbar}{48\pi}(\frac{1}{\bar{u}'})^2[\frac{3}{2}(\frac{\bar{u}''}{\bar{u}'})^2 - \frac{\bar{u}'''}{\bar{u}'}]$

- Classical flux is explicitly +ve but not so for the red term. But red term can be re-written as:

  $\frac{N\hbar}{48\pi}(\frac{1}{\bar{u}'})^2[\frac{3}{2}(\frac{\bar{u}''}{\bar{u}'})^2 - \frac{\bar{u}'''}{\bar{u}'}] = \frac{N\hbar}{48\pi}[-\frac{1}{2}\frac{(\bar{u}'')^2}{(\bar{u}')^4} - \frac{d}{d\bar{u}}(\frac{\bar{u}''}{(\bar{u}')^2})]$ so that:

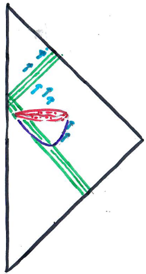
  $\frac{d}{d\bar{u}}[M_B + \frac{N\hbar}{48\pi}\frac{\bar{u}''}{(\bar{u}')^2}] = -\frac{1}{2}(\partial_{\bar{u}} f)^2 - \frac{N\hbar}{96\pi}\frac{(\bar{u}'')^2}{(\bar{u}')^4}$

Thus we may interpret $[M_B + \frac{N\hbar}{48\pi}(\frac{\bar{u}''}{(\bar{u}')^2}]$ as a back reaction corrected Bondi Mass which responds to the positive definite flux $\frac{1}{2}(\partial_{\bar{u}} f)^2 + \frac{N\hbar}{96\pi}\frac{(\bar{u}'')^2}{(\bar{u}')^4}$.

- Let us assume that the black hole stops radiating when it exhausts its corrected Bondi Mass. The flux must then vanish. The olive term vanishes once we go beyond its classical support as shown in the figure on the next slide. The red term must also vanish. Its vanishing implies that $\bar{u} = \alpha x^- + \beta$ for some constants α, β. This means that the physical $\mathcal{I}^+$ is as long as the fiducial $\mathcal{I}^+$. Suggests that there is a quantum extension of the classical Vaidya I^+.

6. Speculation: Arena for true degrees of freedom

- Recall that the quantum scalar field sees no singularities and lives on the fiducial spacetime. Classically if the scalar field is set to vanish, the only solution is flat sptime. This suggests that the scalar field can be thought of as coordinatizing the true degrees of freedom of the system.
- Speculation: If the true degrees of freedom are the scalar field ones, the fundamental Hilbert space for the entire system is the scalar field Fock space and the natural arena is the fiducial spacetime



Closing Remarks

- The scenario is supportive of the AB paradigm in which correlations with Hawking radiation emerge in the vast region beyond the singularity.
- Directly relevant tools to study purification of Hawking radiation have been developed by Agullo, Calizaya Cabrera, Elizaga Navascues. Seems to indicate that purification and info in matter profile are distinct "purification recovers the 'vacuum' part of the initial state"
- Earlier work on CGHS by Ori seems applicable so as to continue the geometry beyond the semiclassical singularity. Comparision with BH-WH geometry (Han, Rovelli, Soltani)
- From earlier work on CGHS by AA, FP, FR, potential for very interesting numerical discoveries along 'last ray' (universality).
- Good model to attempt systematic non-pert canonical quantization....

Details of my work in e-Print: 2406.09176 [gr-qc]

Additional Notes

- The old and corrected flux can be calculated at late times for classical Vaidya. They agree (!) and equal $\frac{N\hbar}{24\pi}(\frac{1}{64M^2})$.
  But new Bondi mass $= M_{old} + \frac{N\hbar}{48\pi}(\frac{1}{4M})$
- Parameterize the trajectory followed by the DH by its 'shape' $x^-(x^+)$.
  $\frac{d\partial_+ R}{dx^+} = \partial_+^2 R + \frac{dx^-}{dx^+}\partial_-\partial_+ R = 0$
  $\frac{dx^-}{dx^+} = -\frac{\partial_+^2 R}{\partial_-\partial_+ R} = -16\pi\langle T_{++}\rangle e^{-2\rho}(R^2 - \frac{N\hbar}{24\pi})$.
  $\frac{dR}{dx^+} = \partial_+ R + \frac{dx^-}{dx^+}\partial_- R$
  For large mass $M >> N$ black holes, can argue that the shape can be well approximated by calculating with classical Vaidya at $R = 2M$.
  Obtain $\frac{dR}{dx^+} = -\frac{N\hbar}{12\pi}\frac{1}{64M^2}$. If we set $R = 2M$ get:
  $\frac{dM}{dx^+} = -\frac{N\hbar}{24\pi}(\frac{1}{64M^2})$!!!
- Simulations: Lowe shows no thunderbolt. P+P show that flux falls off from its $\sim 1/m^2$ behavior at late times.