Non-Perturbative Quantisation of Impulsive Radiative Data

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25-06-2024

Outline

1. Motivation: Planck luminosity
2. Quantisation of impulsive null initial data
3. Planck luminosity in quantum gravity
4. Outlook and conclusion

Motivation: Planck Luminosity

Observation: in $D = 4$ spacetime dimensions, the Planck power (luminosity) is independent of $\hbar$

$$\mathscr{L}_{\mathrm{P}} = \frac{m_{\mathrm{P}} c^2}{t_{\mathrm{P}}} = \frac{\hbar^{\frac{D-4}{D-2}} c^{\frac{2D+2}{D-2}}}{G^{\frac{2}{D-2}}}.$$

Only in $D = 4$, can we have a formula

$$\boxed{\mathscr{L}_{peak} = \mathscr{L}_{\mathrm{P}} \times f(\text{scale-independent observables}).}$$

Humanity has come close to observing such power

$$\mathscr{L}_{\mathrm{P}} = \frac{c^5}{G} \approx 3,63 \times 10^{52}\mathrm{W},$$

$$\mathscr{L}_{peak}\Big|_{\mathrm{GW170729}} \approx 4 \times 10^{49}\mathrm{W}.$$

First evidence for luminosity bound

Take the quadrupole formula (in $D=4$) for the emitted power

$$\mathscr{L}_{\mathrm{GW}} \sim \frac{G}{c^5}(\dddot{I})^2 \sim \frac{G}{c^5}(M\omega^3)^2 R^4$$

Virial theorem links average kinetic energy with potential energy

$$\bar{E}_{kin} = -\tfrac{1}{2}\bar{E}_{pot} \Rightarrow M\omega^2 R^2 \sim \frac{GM^2}{R}.$$

Emission can only happen before forming a black hole, $R \gtrsim 2GM/c^2$

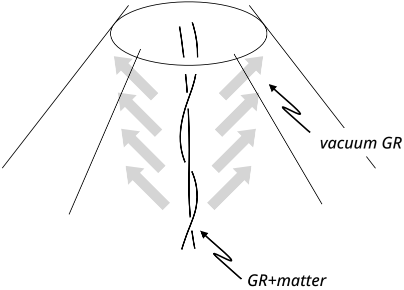
$$\boxed{\mathscr{L}_{\mathrm{GW}} \sim \frac{c^5}{G}\left(\frac{GM}{c^2 R}\right)^5 \lesssim \mathscr{L}_{\mathrm{P}}.}$$

Comments:

- If there is such a bound, it can only exist in $D=4$.
- Likely invisible in perturbative S-matrix approach, where we have expansion in $\varkappa = \sqrt{8\pi G/c^3}$.
- Rest of the talk: explore $\mathscr{L}_{\mathrm{P}}$ in non-perturbative quantum gravity.

Register radiation at null surface far away from the sources.

- Quantize null initial data.
- Constraint-free data: shear+corner data (area density).
- Truncation: piecewise constant shear (impulsive waves).



Step 1: Parametrisation of geometry

Signature $(0{+}{+})$ metric.

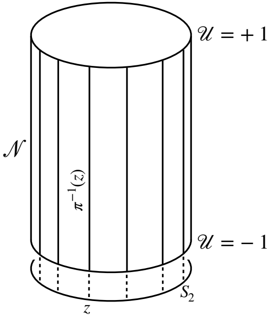
$$\varphi^*_{\mathcal{N}} g_{ab} = q_{ab} = \delta_{ij} e^i{}_a e^j{}_b, \qquad i,j = 1,2.$$

Parametrisation of the co-dyad:

$$\boxed{e^i = \Omega\, S^i{}_m e^m_{(o)}.}$$

- Conformal factor Ω parametrizes the overall scale.
- $SL(2,\mathbb{R})$-Holonomy $S^i{}_m$ determines the shape degrees of freedom.
- Fiducial background dyad $e^j_{(o)}$, e.g.

  $(e^1_{(o)}, e^2_{(o)}) = (\mathrm{d}\vartheta, \sin\vartheta \mathrm{d}\varphi)$



Teleological clock

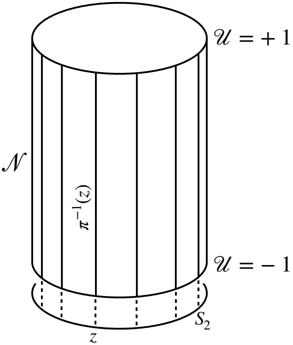
We consider a null strip $\mathscr{N}$ with two corners. No unique clock along $\mathscr{N}$. Convenient choice

Boundary condition at $\partial\mathscr{N} = \mathscr{C}_+ \cup \mathscr{C}_-$,

$$\mathscr{U}(\partial\mathscr{N}, z, \bar{z}) = \pm 1,$$

Affinity proportional to expansion

$$\partial^b_{\mathscr{U}} \nabla_b \partial^a_{\mathscr{U}} = -\frac{1}{2}(\Omega^{-2}\frac{\mathrm{d}}{\mathrm{d}\mathscr{U}}\Omega^2)\partial^a_{\mathscr{U}}$$



Comments:

- $\delta\mathscr{U} \neq 0$, but $\delta\mathscr{U}\big|_{\partial\mathscr{N}} = 0$.
- The clock knows that it has to land at the upper boundary with value $\mathscr{U} = +1$. The clock is teleological (*telos* means goal).

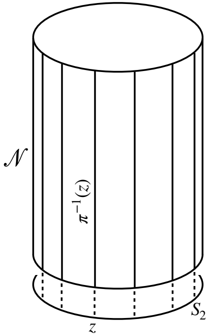
Upon fixing the direction of the null rays, we are left with two constraints.

Raychaudhuri equation: $G_{ab}\ell^a\ell^b = 0$

$$\frac{\mathrm{d}^2}{\mathrm{d}\mathscr{U}^2}\Omega^2 = -2\sigma\bar{\sigma}\Omega^2.$$

Transport equation for $SL(2,\mathbb{R})$ parallel propagator

$$\frac{\mathrm{d}}{\mathrm{d}\mathscr{U}}S = \Big(\varphi J + \big(\sigma\bar{X} + \mathrm{cc.}\big)\Big)S.$$



Note:

- $U(1)$ connection φ on $\mathscr{N}$.
- σ is the shear (free radiative data) on $\mathscr{N}$.
- The matrices J, X and $\bar{X}$ are $SL(2,\mathbb{R})$ generators:

  $J^2 = -\mathbb{1}, \quad [J,X] = -2\mathrm{i}\,X, \quad [X,\bar{X}] = \mathrm{i}\,J.$

Step 2: Symplectic structure (from action)

In $D = 4$, there are two Lorentz scalars that we can build from the curvature tensor:

$$R[\omega, e] = R^{\alpha\beta}{}_{ab}[\omega] e_\alpha{}^a e_\beta{}^b,$$
$$R^*[\omega, e] = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} R_{\alpha\beta ab}[\omega] e_\mu{}^a e_\nu{}^b \approx 0.$$

Note: If the torsionless equation is satisfied, R^* vanishes.

In the first-order formalism, there are *two coupling constants* at linear order in the curvature,

$$S = \frac{1}{16\pi G} \int_{\mathscr{M}} d^4 v \left[R - \frac{1}{\gamma} R^* \right] + \text{boundary terms.}$$

G is Newton's constant, γ is the Barbero–Immirzi parameter.

The Barbero–Immirzi parameter is akin to the θ-angle in QCD.

Phase space and symplectic structure

Symplectic structure $\Theta = p\,\mathbb{d}q$ determines Heisenberg relations $[\hat{q},\hat{p}] = \mathrm{i}\hbar$.
Starting from the action, we obtain the symplectic structure on $\mathscr{N}$.

$$\begin{aligned}\Theta_{\mathscr{N}} = &-\frac{1}{16\pi\gamma G}\int_{\partial\mathscr{N}} d^2v_o\,\Omega^2\ \mathrm{Tr}\left(J\mathbb{d}SS^{-1}\right)+\\ &-\frac{1}{8\pi G}\int_{\mathscr{N}} \mathrm{d}\mathscr{U}\wedge d^2v_o\,\Omega^2\ \mathrm{Tr}\left((\sigma_I\bar{X}+\bar{\sigma}_I X)\mathbb{D}S_I S_I^{-1}\right)+\\ &-\frac{1}{8\pi G}\int_{\mathscr{N}} \mathrm{d}\mathscr{U}\wedge d^2v_o\,\mathbb{d}\mathscr{U}\left(\frac{\mathrm{d}^2}{\mathrm{d}\mathscr{U}^2}\Omega^2+2\sigma_I\bar{\sigma}_I\Omega^2\right).\end{aligned}$$

Key Observations:

- Local $SL(2,\mathbb{R})$ symplectic structure on $\mathscr{N}$.
- First line: initial data for the Raychaudhuri equation.
- Second line: radiative data.
- Third line: Clock variable is conjugate to the constraint.

Starting from the γ-action, we obtain the symplectic structure on $\mathcal{N}$.

$$\begin{aligned}\Theta_{\mathcal{N}} = &-\frac{1}{16\pi\gamma G}\int_{\partial\mathcal{N}} d^2 v_o\,\Omega^2\ \mathrm{Tr}\left(J\mathbb{d}SS^{-1}\right)+\\ &-\frac{1}{8\pi G}\int_{\mathcal{N}} \mathrm{d}\mathcal{U}\wedge d^2v_o\,\Omega^2\ \mathrm{Tr}\left((\sigma_I\bar{X}+\bar{\sigma}_I X)\mathbb{D}S_I S_I^{-1}\right)+\\ &-\frac{1}{8\pi G}\int_{\mathcal{N}} \mathrm{d}\mathcal{U}\wedge d^2 v_o\,\mathbb{d}\mathcal{U}\left(\frac{\mathrm{d}^2}{\mathrm{d}\mathcal{U}^2}\Omega^2+2\sigma_I\bar{\sigma}_I\Omega^2\right).\end{aligned}$$

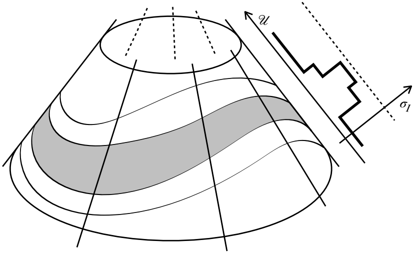
Technical remarks:

- Interaction picture: $S(\mathcal{U})=\exp(\int^{\mathcal{U}}\mathrm{d}\mathcal{U}'\varphi(\mathcal{U}')J)S_I=\mathrm{e}^{\Delta J}S_I$.
- Dressed field space derivative: $\mathbb{D}=\mathbb{d}-\mathbb{d}\mathcal{U}\frac{\mathrm{d}}{\mathrm{d}\mathcal{U}}$ [Carrozza, Hoehn, Riello, Gomes, Freidel, Ciambelli, Kabel, Brukner, ...]
- Barbero–Immirzi parameter only appears in corner term.
- Looks innocent but highly non-linear constraint between shear σ_I and conformal factor $\Omega=\Omega[\sigma_I,\Omega_\pm]$.

Truncation to impulsive null initial data

Split the shear profile (free radiative data) into sequence of pulses.

- Each pulse represents a quasi-local graviton.
- Strategy: Quantise each pulse, then glue many pulses together.
- Full non-linear and non-perturbative quantization of impulsive data.



Solving the constraints

We consider a single pulse

$$\frac{\mathrm{d}}{\mathrm{d}\mathscr{U}}\sigma_I = 0, \qquad \sigma_I \equiv \sigma_I(\vartheta, \varphi).$$

Conformal factor

$$\Omega^2(\mathscr{U}, \vartheta, \varphi) = \frac{E_+(\vartheta,\varphi) + E_-(\vartheta,\varphi)}{2} \frac{\cos\left(\sqrt{2\sigma_I\bar{\sigma}_I}\,\mathscr{U}\right)}{\cos\left(\sqrt{2\sigma\bar{\sigma}}\right)} + \\ + \frac{E_+(\vartheta,\varphi) - E_-(\vartheta,\varphi)}{2} \frac{\sin\left(\sqrt{2\sigma_I\bar{\sigma}_I}\,\mathscr{U}\right)}{\sin\left(\sqrt{2\sigma_I\bar{\sigma}_I}\right)},$$

for initial conditions

$$\Omega^2(\mathscr{U} = \pm 1, \vartheta, \varphi) = E_\pm(\vartheta, \varphi).$$

$SL(2,\mathbb{R})$ holonomy

$$H = \mathrm{ch}\left(\sqrt{\sigma_I\bar{\sigma}_I}(\mathscr{U}+1)\right)\mathbb{1} + \frac{1}{\sqrt{\sigma_I\bar{\sigma}_I}}\left(\bar{\sigma}_I X + \sigma_I \bar{X}\right)\mathrm{sh}\left(\sqrt{\sigma_I\bar{\sigma}_I}(\mathscr{U}+1)\right),$$

$$S = \mathrm{e}^{\Delta J} S_I = \underbrace{\mathrm{e}^{\Delta J}}_{U(1)} H S_-, \qquad S(\mathscr{U} = \pm 1) = S_\pm.$$

Double role of shear: euclidean angle in Ω, boost angle in H. Recalll $A = \Gamma + \gamma K$, Immirzi parameter mixes euclidean and boost angles.

Pull-back of symplectic potential $\Theta_{\mathscr{N}}$ to configurations of a single pulse.
Finite-dimensional mechanical system on each null ray.

$$\begin{aligned}
\big\{a(\boldsymbol{z}),\bar{a}(\boldsymbol{z}')\big\} &= \mathrm{i}\,\delta^{(2)}(\boldsymbol{z}|\boldsymbol{z}'),\\
\big\{b(\boldsymbol{z}),\bar{b}(\boldsymbol{z}')\big\} &= \mathrm{i}\,\delta^{(2)}(\boldsymbol{z}|\boldsymbol{z}'),\\
\big\{c(\boldsymbol{z}),\bar{c}(\boldsymbol{z}')\big\} &= 2\,\mathrm{i}\,\delta^{(2)}(\boldsymbol{z}|\boldsymbol{z}')\,L(\boldsymbol{z}),\\
\big\{L(\boldsymbol{z}),c(\boldsymbol{z}')\big\} &= -\mathrm{i}\,\delta^{(2)}(\boldsymbol{z}|\boldsymbol{z}')\,c(\boldsymbol{z}),\\
\big\{L(\boldsymbol{z}),\bar{c}(\boldsymbol{z}')\big\} &= +\mathrm{i}\,\delta^{(2)}(\boldsymbol{z}|\boldsymbol{z}')\,\bar{c}(\boldsymbol{z}),
\end{aligned}$$

with $\boldsymbol{z}=(\vartheta,\varphi)$ and

$$\begin{aligned}
\big\{c(\boldsymbol{z}),U(\boldsymbol{z}')\big\} &= XU(\boldsymbol{z})\,\delta^{(2)}(\boldsymbol{z}|\boldsymbol{z}'),\\
\big\{\bar{c}(\boldsymbol{z}),U(\boldsymbol{z}')\big\} &= \bar{X}U(\boldsymbol{z})\,\delta^{(2)}(\boldsymbol{z}|\boldsymbol{z}'),\\
\big\{L(\boldsymbol{z}),U(\boldsymbol{z}')\big\} &= -\frac{1}{2}JU(\boldsymbol{z})\,\delta^{(2)}(\boldsymbol{z}|\boldsymbol{z}').
\end{aligned}$$

Highly non-linear definition of canonical variables.

$SL(2,\mathbb{R})$ holonomy

$$U = \mathrm{e}^{\gamma \ln\left(\tan\left(\sqrt{2\sigma\bar{\sigma}}\right)/\sqrt{2\sigma\bar{\sigma}}\right) J} S_- .$$

Heisenberg charges

$$a = \frac{\sqrt{E_+}}{\sqrt{8\pi\gamma G}} \,\mathrm{ch}\left(2\sqrt{\sigma\bar{\sigma}}\right) \mathrm{e}^{-\mathrm{i}\left[\Delta_+ + 2\gamma \ln\left(\cos\left(\sqrt{2\sigma\bar{\sigma}}\right)\right)\right]},$$

$$b = \frac{\sqrt{E_+}}{\sqrt{8\pi\gamma G}} \,\mathrm{sh}\left(2\sqrt{\sigma\bar{\sigma}}\right) \mathrm{e}^{\mathrm{i}\left[\Delta_+ + \phi + 2\gamma \ln\left(\frac{\sin\left(\sqrt{2\sigma\bar{\sigma}}\right)}{\sqrt{2\sigma\bar{\sigma}}}\right)\right]}.$$

Area operators and shear operators

Area operators are mere number operators.

$$E_- + E_+ = 16\pi\gamma G \left(L + a\bar{a}\right),$$
$$E_- - E_+ = 16\pi\gamma G \left(L + b\bar{b}\right).$$

Shear operator: quotient of the two oscillators

$$\mathrm{th}\big(2\sqrt{\sigma\bar{\sigma}}\big) = \sqrt{\frac{\bar{b}b}{\bar{a}a}}.$$

There are operator ordering ambiguities, but they are mild.

One residual constraint

$$\boxed{c\,\bar{a}\,\bar{b} = -\gamma\,(L + \bar{a}a)\,\sqrt{2\,\bar{a}a\,\bar{b}b}\,\tan\left(\frac{1}{2\sqrt{2}}\ln\left(\frac{\sqrt{\bar{a}a} + \sqrt{\bar{b}b}}{\sqrt{\bar{a}a} - \sqrt{\bar{b}b}}\right)\right) - \mathrm{i}\,\bar{a}a\,\bar{b}b.}$$

Key Observations:

- Left hand side: ladder operators.
- Right hand side: number operators.
- $SL(2,\mathbb{R})$ Casimir commutes with the constraint.
- This is a simple recurrence relation.

Critical shear and physical states

Physical states can be classified by the value of the $SL(2,\mathbb{R})$ Casimir.

$$L^2 - c\bar{c} = \frac{1}{8\pi G}\left[\frac{1}{4\gamma^2}(E_- - E_+)^2 + \right. \\ \left. - \frac{1}{\gamma^2}\,\mathrm{sh}^2\big(2\sqrt{\sigma\bar{\sigma}}\big)E_+E_- - \frac{1}{2}(E_+ + E_-)^2\tan^2\big(\sqrt{2\sigma\bar{\sigma}}\big)\right].$$

Discrete series unitary representations: $L^2 > c\bar{c}$, the $U(1)$ generator $|L|$ is bounded from below, recurrence relations terminate.

Continuous series unitary representations: $L^2 < c\bar{c}$, the $U(1)$ generator $|L|$ is unbounded, recurrence relations do not terminate. Physical states are quantum superpositions that contain caustics.

For small shear, the critical value is

$$|\sigma_{crit.}|^2 = \frac{1}{4}\frac{(E_- - E_+)^2}{\gamma^2(E_+ + E_-)^2 + 4E_+E_-} + \mathcal{O}(|\sigma_{crit}|^3).$$

Use Bondi $1/r$-expansion to evaluate $|\sigma_{crit.}|$ near future null infinity.

Note: $E_\pm = \mathcal{O}(r^2)$, but $E_- - E_+ = 2r(\Delta u)(\vartheta,\varphi) + \mathcal{O}(r^0)$.

- Bondi time u vs. teleological time $\mathcal{U}$:

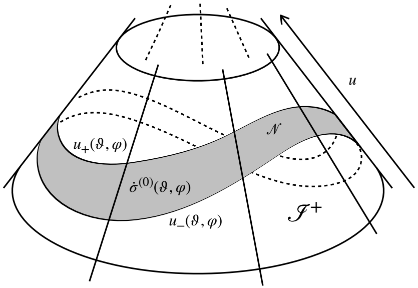
$$\partial^a_{\mathcal{U}} = \frac{(\Delta u)(\vartheta,\varphi)}{2}\partial^a_u + \mathcal{O}(r^{-1}).$$

- Asymptotic shear:

$$\sigma_{crit.} = \frac{(\Delta u)(\vartheta,\varphi)}{2}\frac{\dot\sigma^{(0)}_{crit.}}{r} + \mathcal{O}(r^{-2}).$$

- Critical news:

$$\boxed{\dot\sigma^{(0)}_{crit.} = \frac{1}{\sqrt{\gamma^2+1}}.}$$



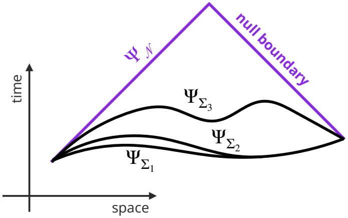
Critical luminosity

$$\mathscr{L}_{crit.} = \frac{c^5}{4\pi G} \oint_{S_2} d^2\Omega \, |\dot{\sigma}^{(0)}_{crit.}|^2 = \frac{\mathscr{L}_{\mathrm{P}}}{\gamma^2 + 1}.$$

Outlook and Summary

Quantum gravity in causal regions

- Initial data: three-metric h_{ab} and extrinsic curvature $\tilde{\pi}^{ab} \sim K_{ab} \sim \dot{h}_{ab}$.
- Constraints $\mathscr{H}[h, \tilde{\pi}] = 0$ and $\mathscr{H}_a[h, \tilde{\pi}] = 0$ generate gauge redundancies on phase space.
- Gauge redundancies: states on Σ_1, Σ_2, …are gauge equivalent.



- Basic idea: Characterize the entire gauge equivalence class $[\Psi_{\Sigma_i}]$ by pushing the time-evolution to its extreme.
- The boundary of the future Cauchy development of Σ_i is a null (light-like) boundary. Quantize gravity at light-like boundary. Problem simplifies. Less constraints.

[Ashtekar, Speziale, Reisenberger, Freidel, Donnelly, Ciambelli, Leigh, Geiller, Pranzetti, …, Donney, Grumiller, Fiorucci, Ruzziconi, … , Hoehn, Carrozza, … , Barnich, Prabhu, Chandrasekaran, Flanagan, Compère, …]

Key results:

1. Non-perturbative quantisation of impulsive null initial data.
2. Quantum geometry includes radiative data and corner data.
3. Planck power separates discrete and continuous $SL(2, \mathbb{R})$ representations.
4. Above the Planck power, states contain caustics. Contradiction with implicit assumption of smooth $\mathscr{I}^+$ above the Planck power.
5. Window into phenomenology—perhaps.