Cosmic rays and air shower physics II

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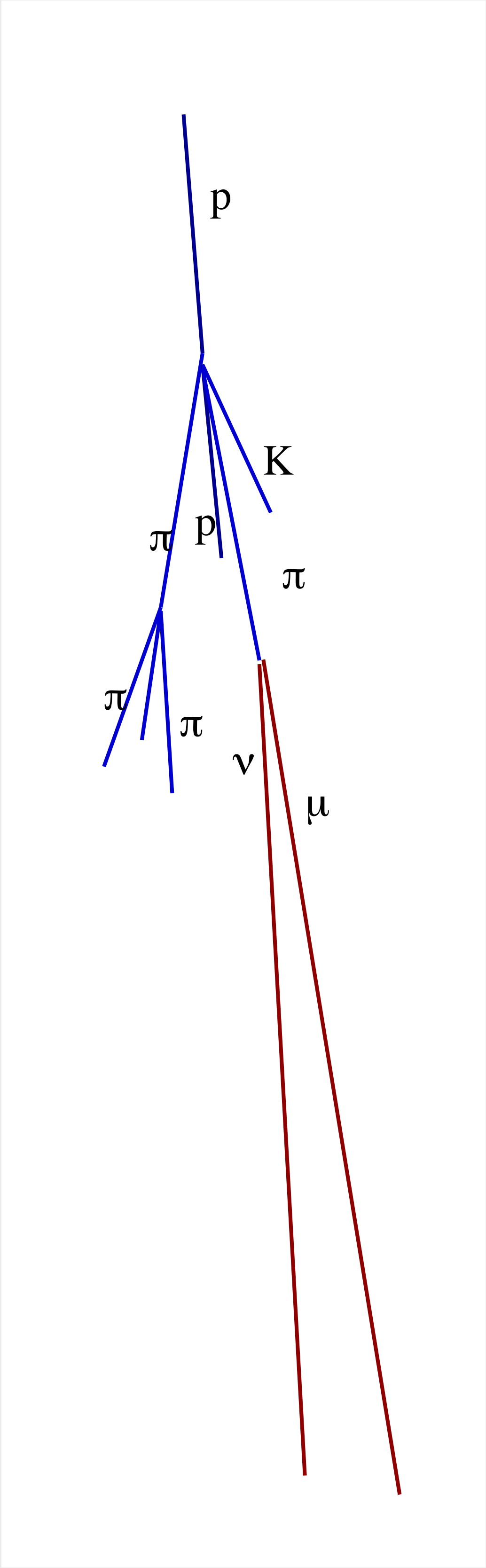
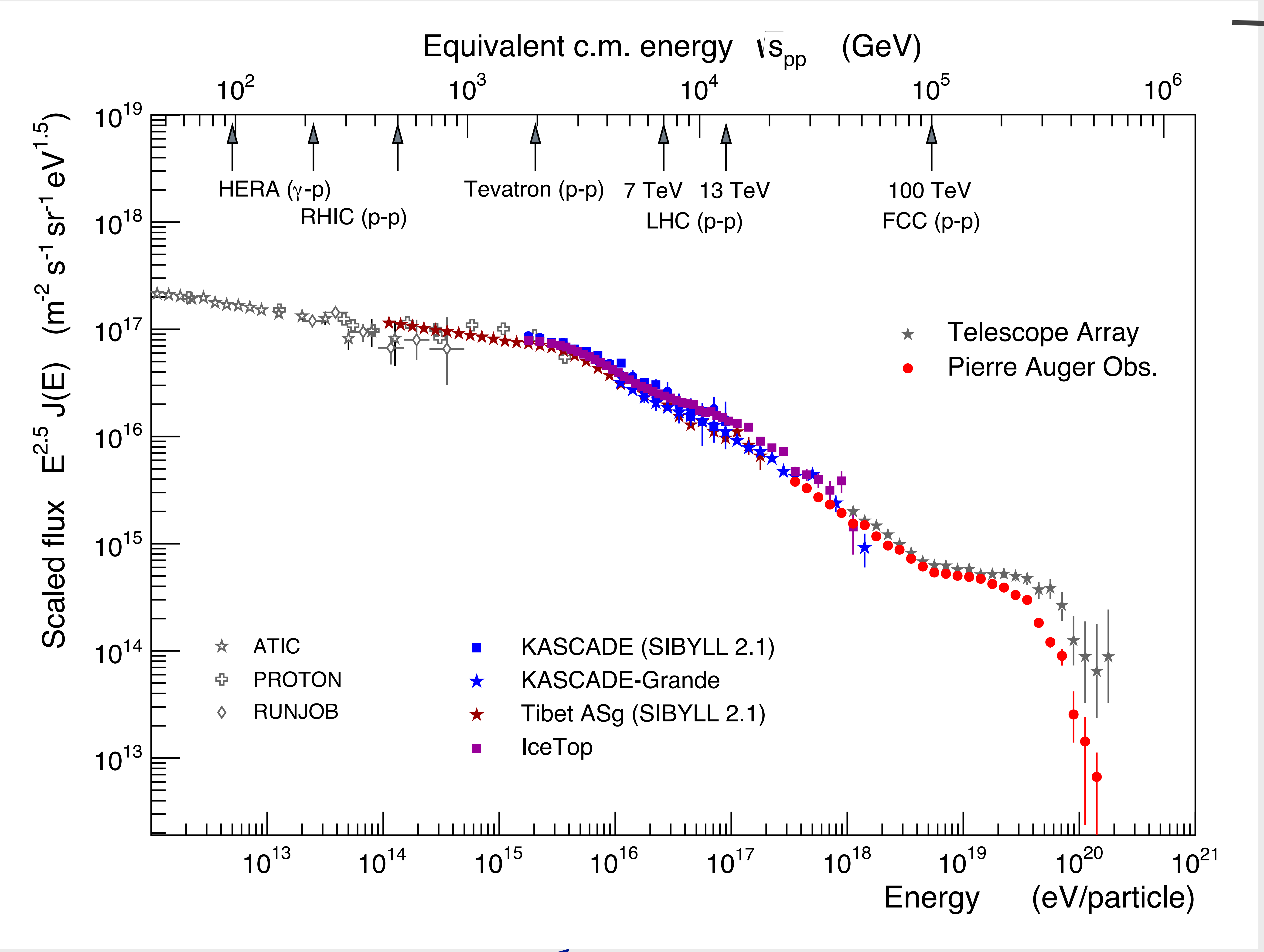
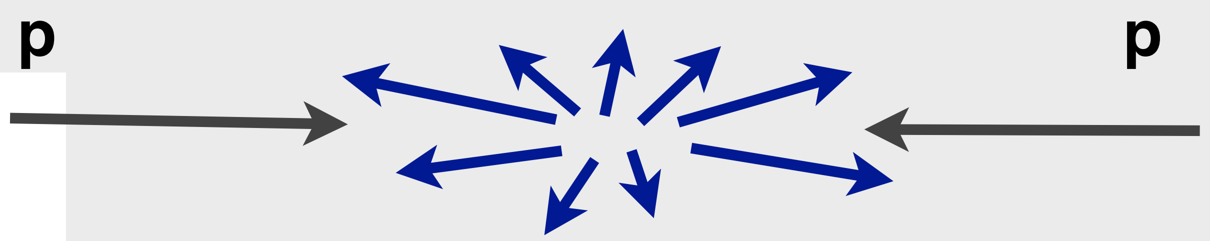
(photograph by S. Saffi)

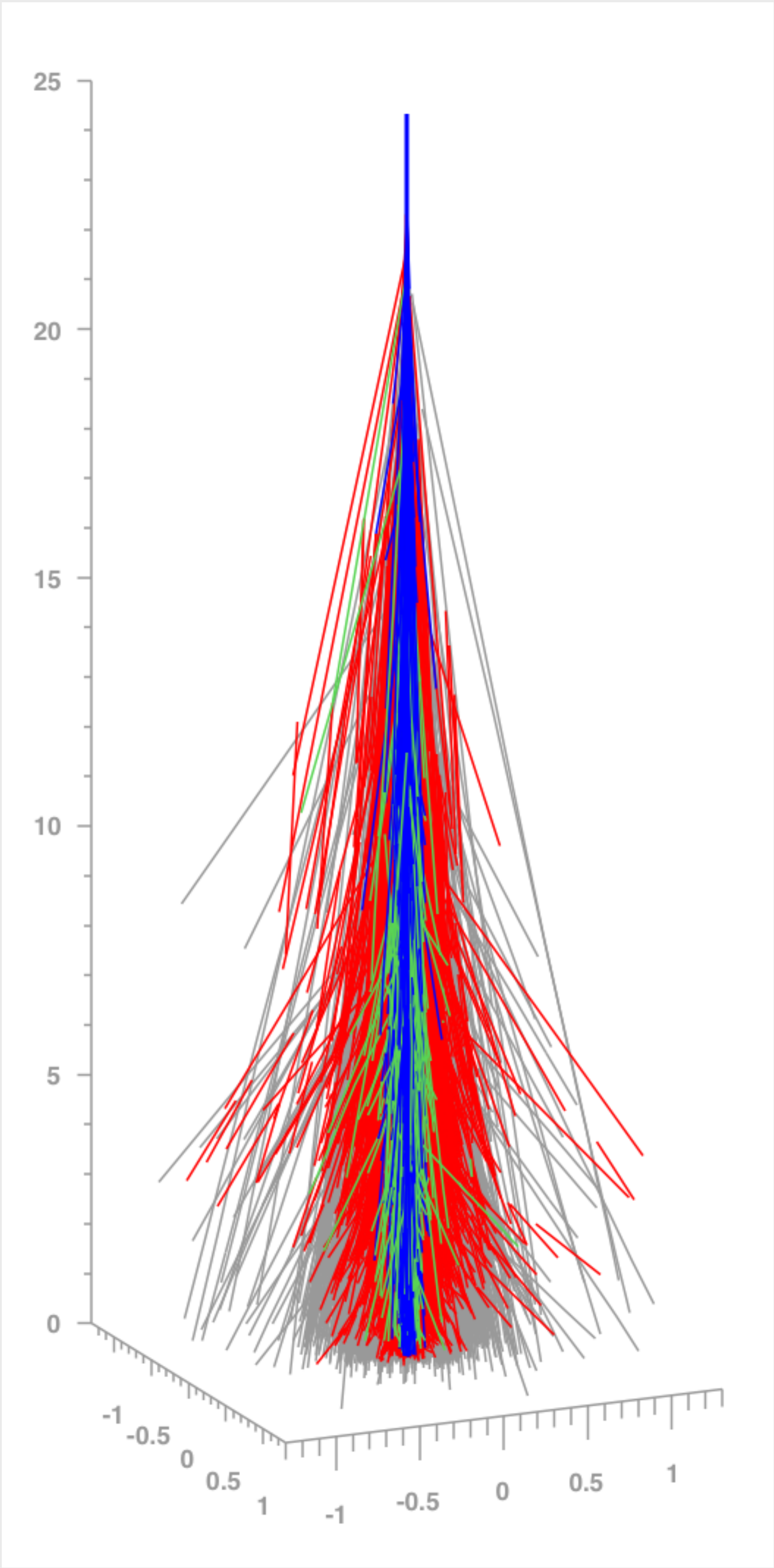
Cosmic ray flux and interaction energies



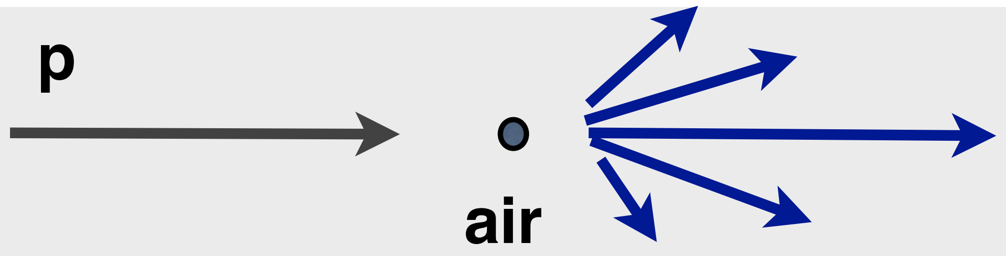




Center-of-mass energy



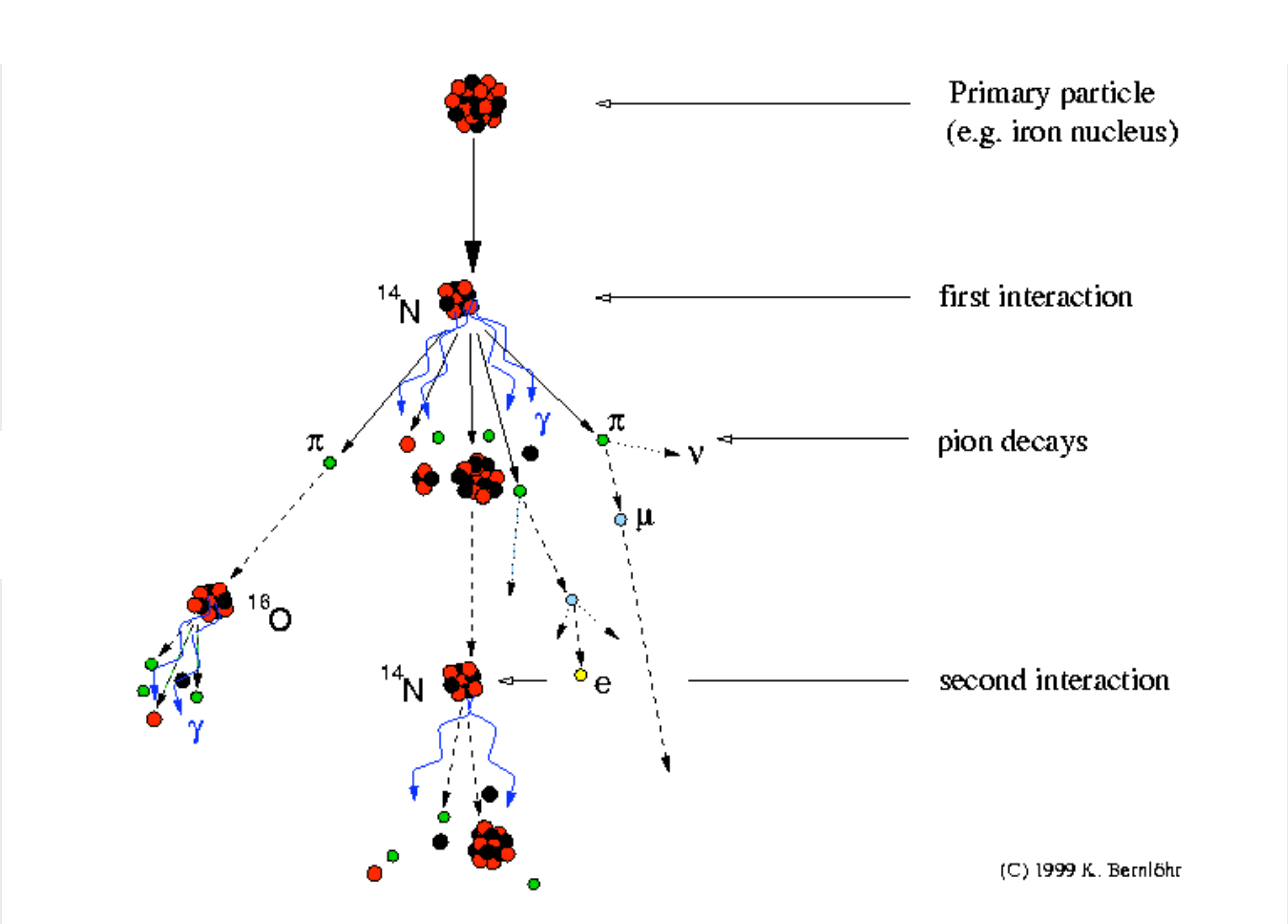
Laboratory energy



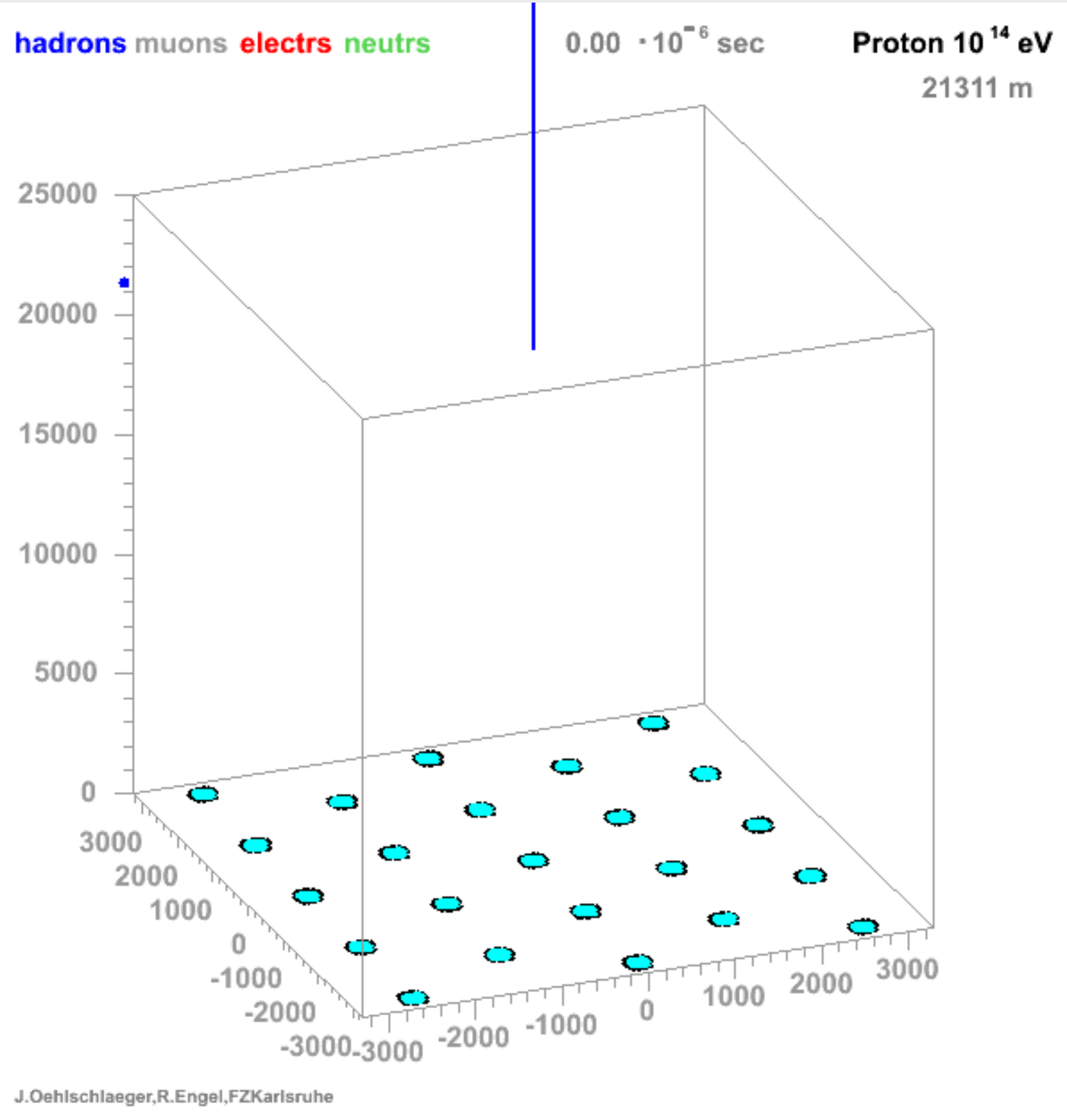
Outline of lectures

- Cosmic rays below the knee – direct measurements
- Physics of extensive air showers
- Discussion and exercises *(topics to be decided)*
- Cosmic rays of very high energy – indirect measurements

1. Simulations



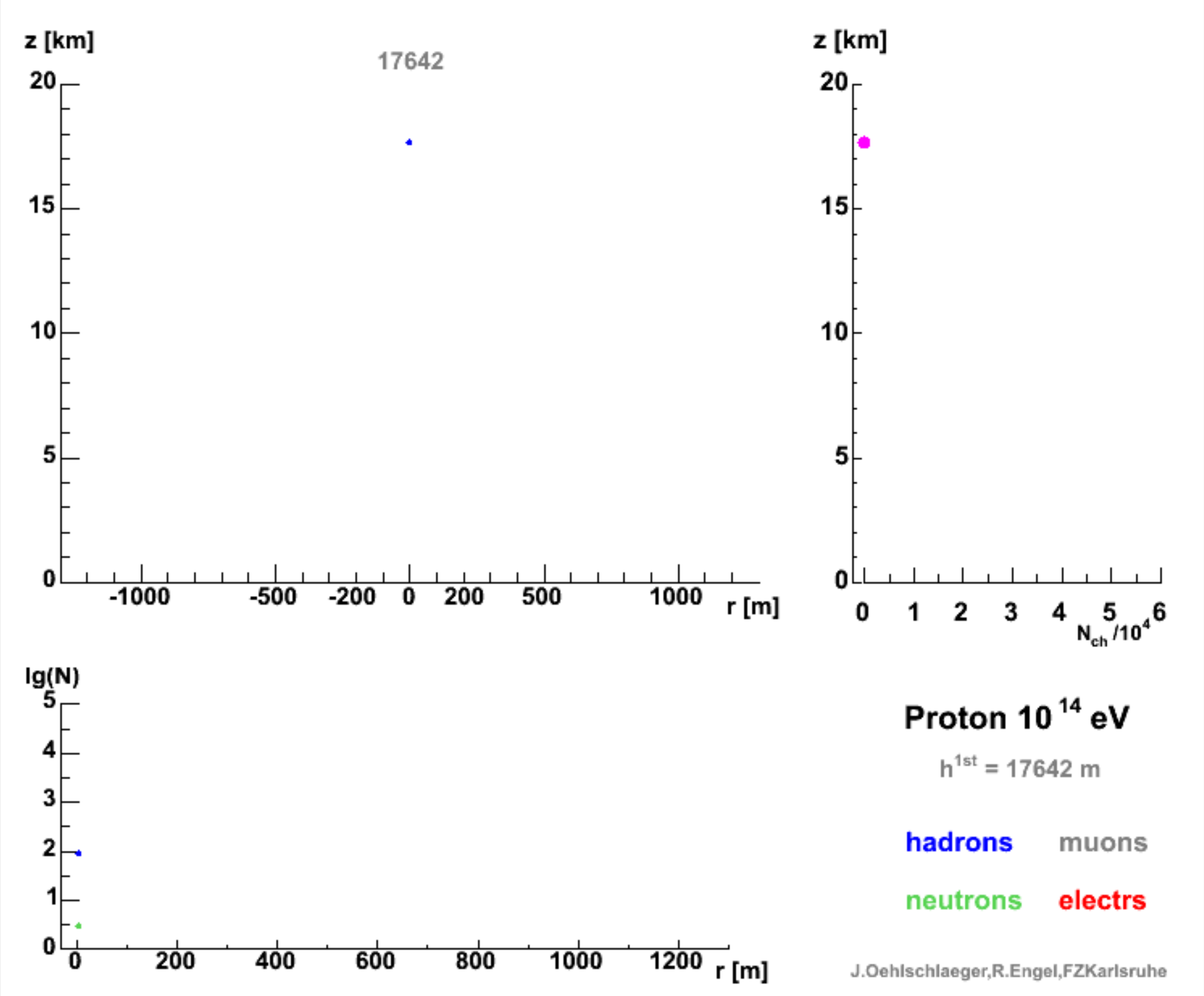
Simulation of shower development (i)



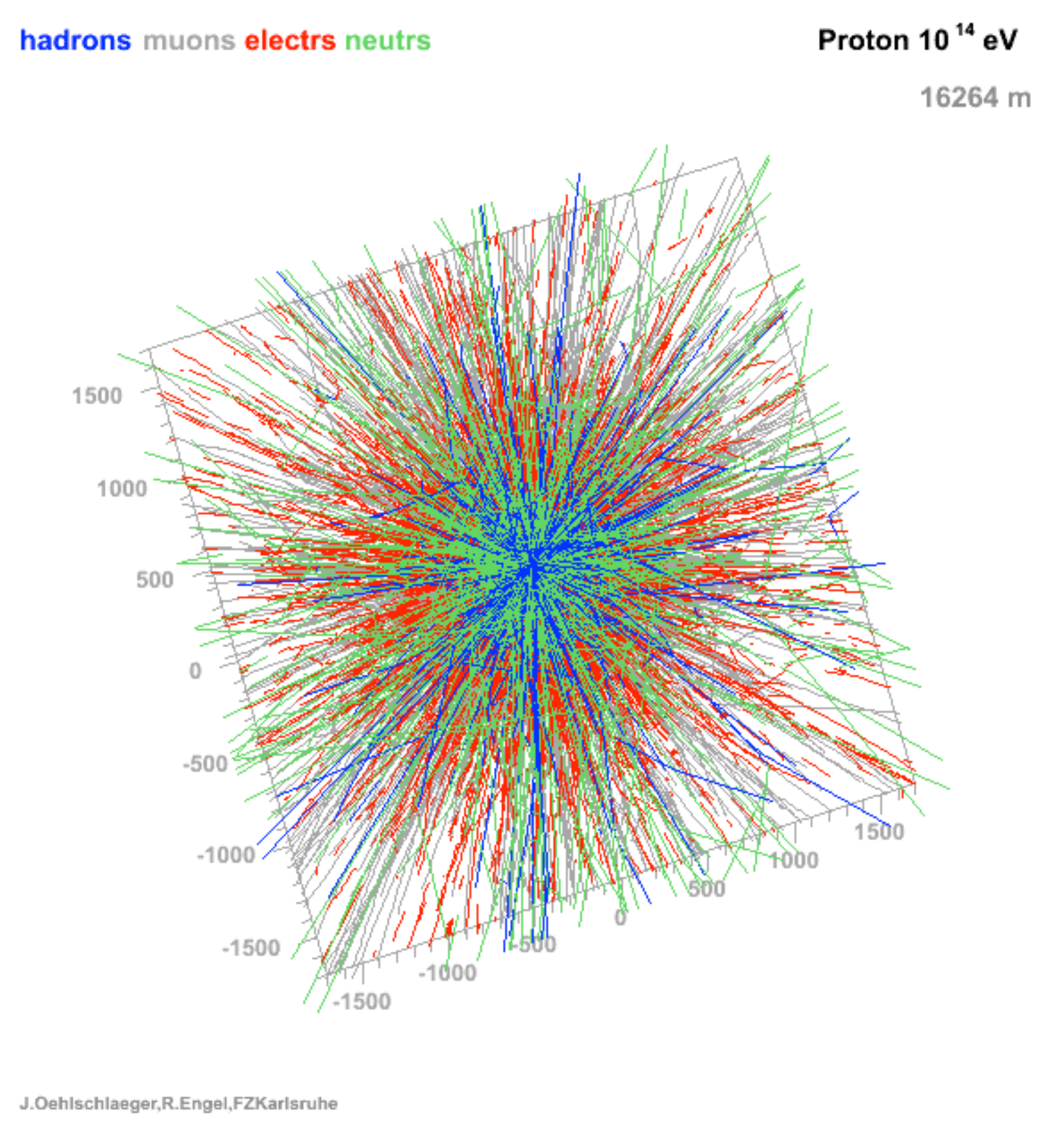
Realistic simulation with CORSIKA

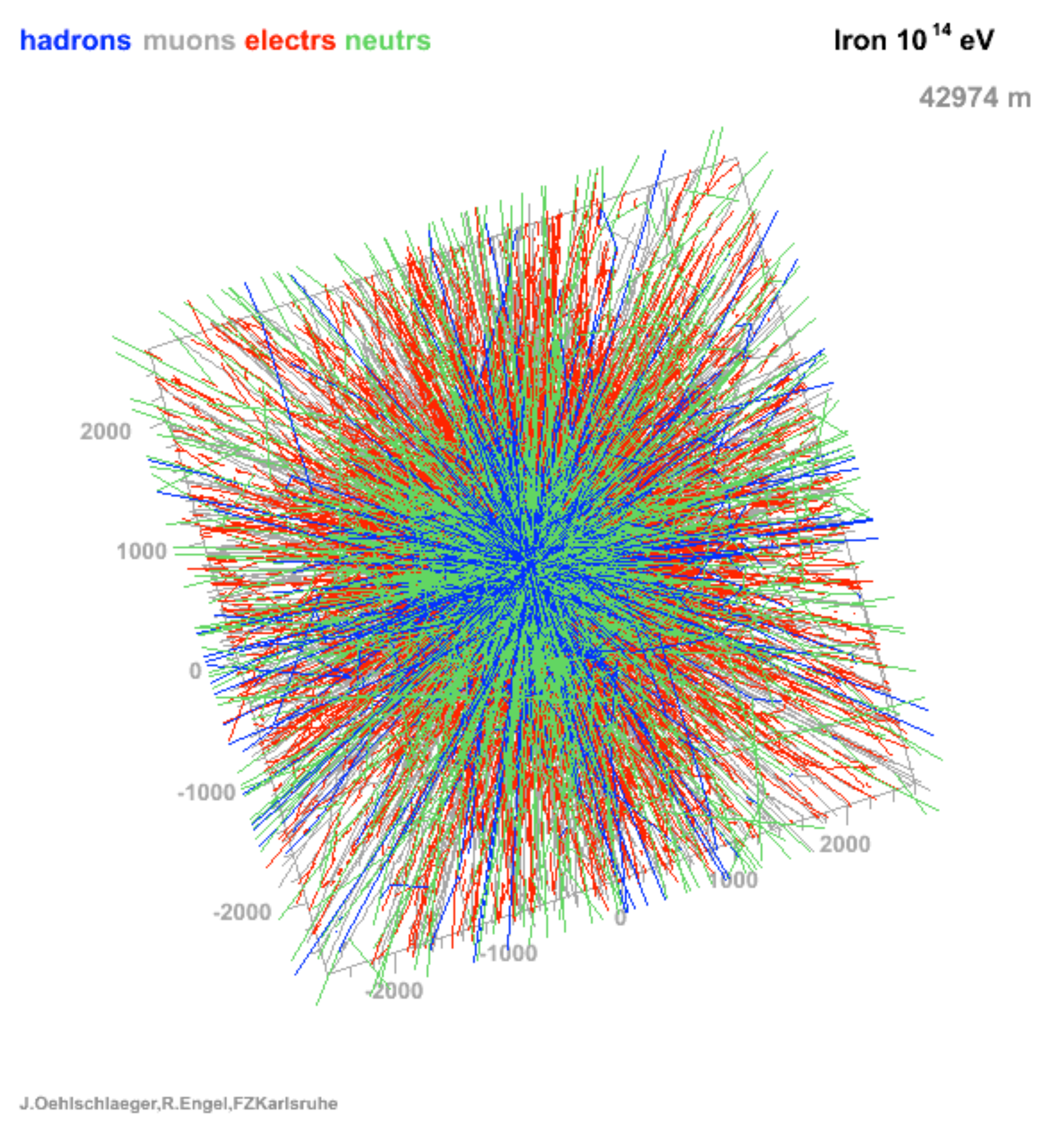
Proton shower of low energy (knee region)

Simulation of shower development (ii)

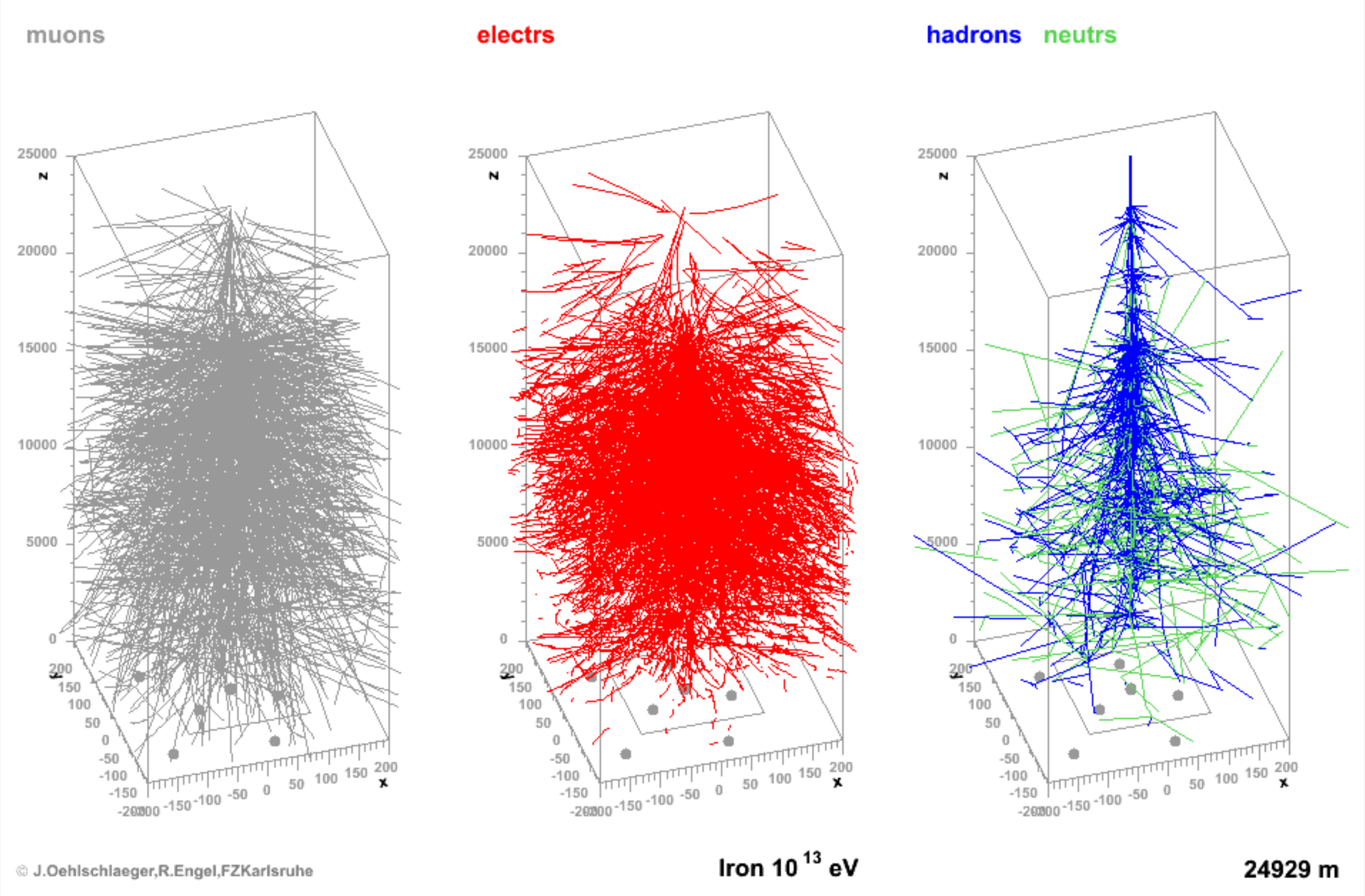


Simulation of air shower tracks (i)

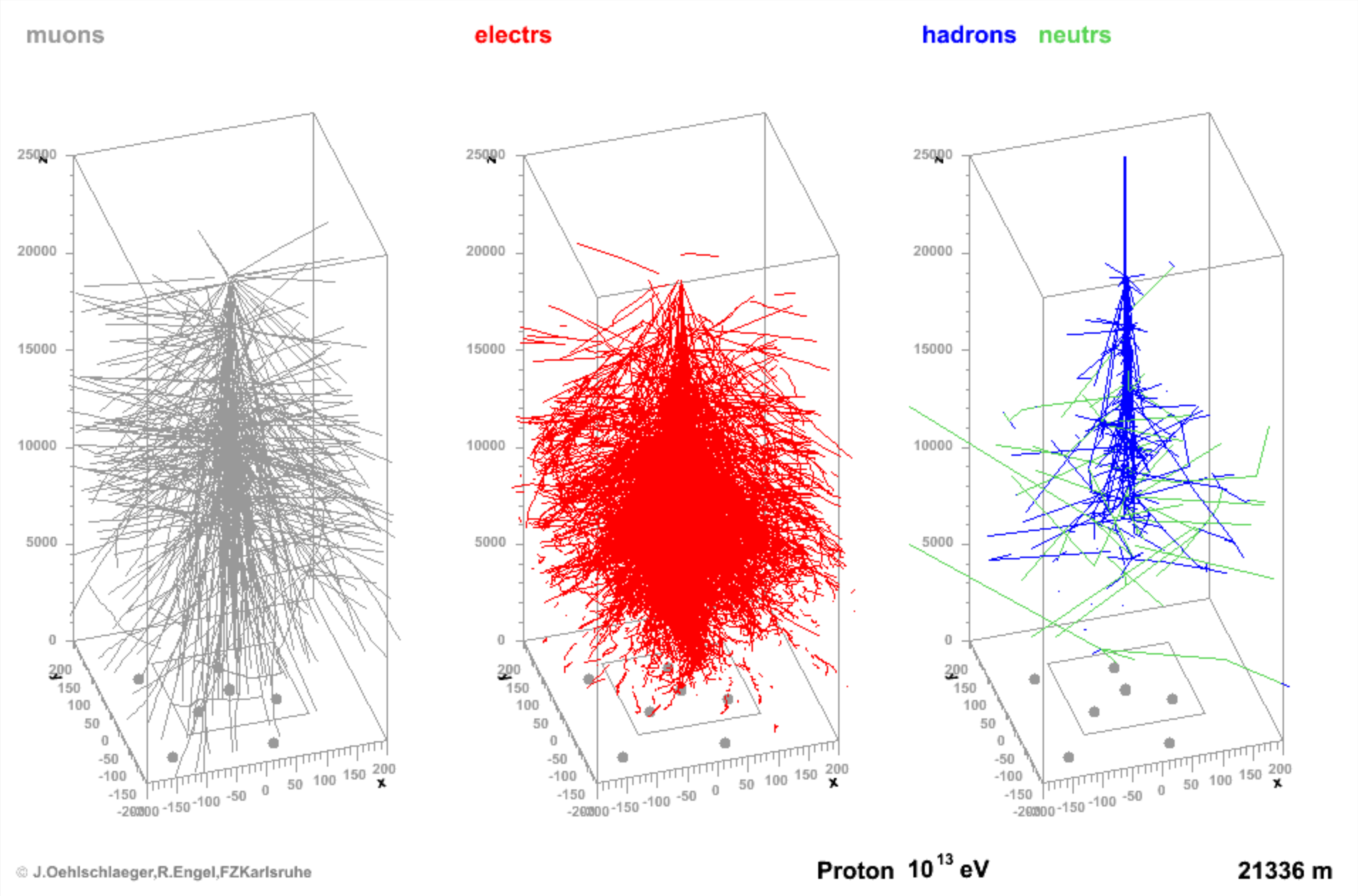




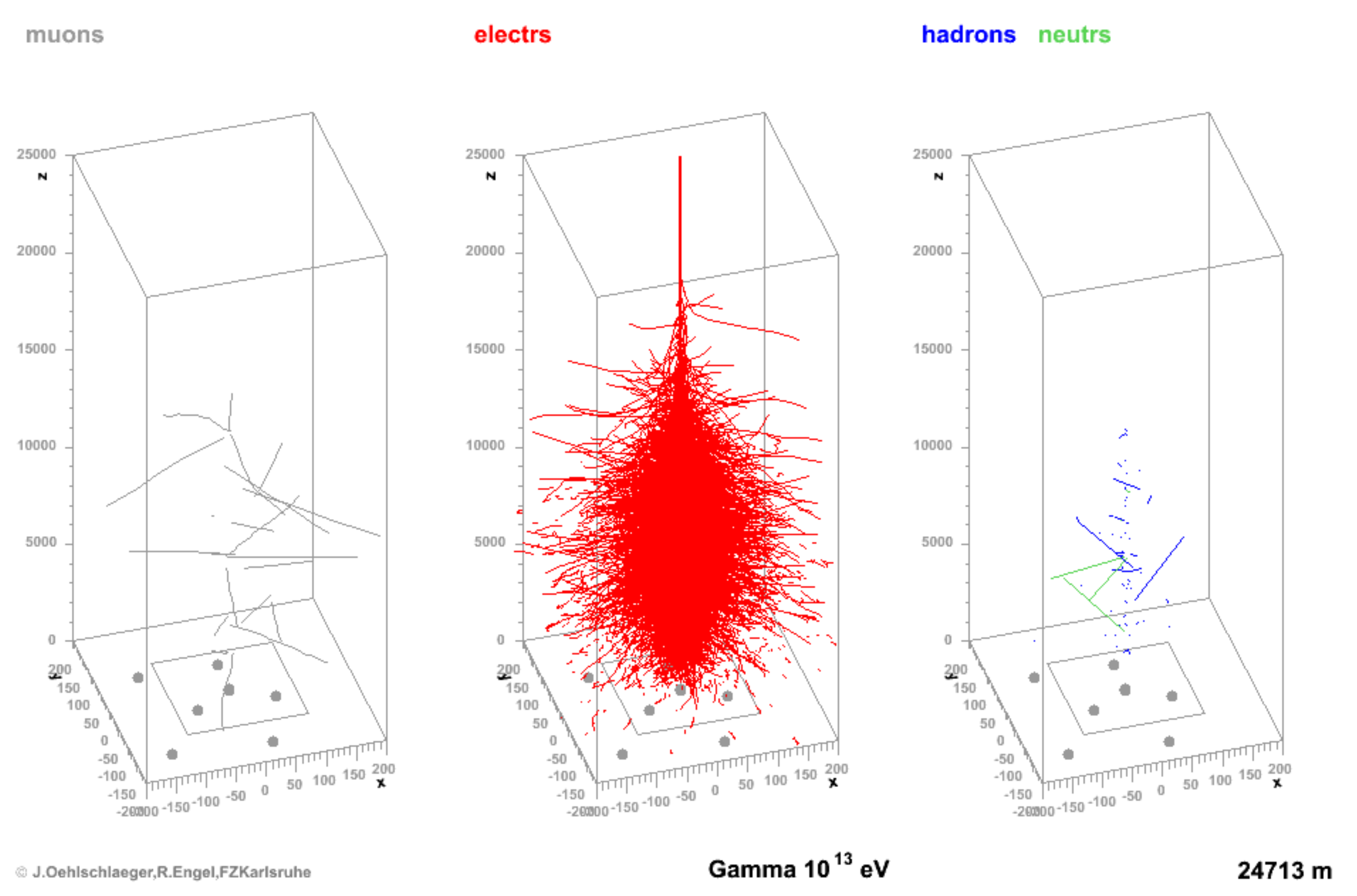
Particles of an iron shower



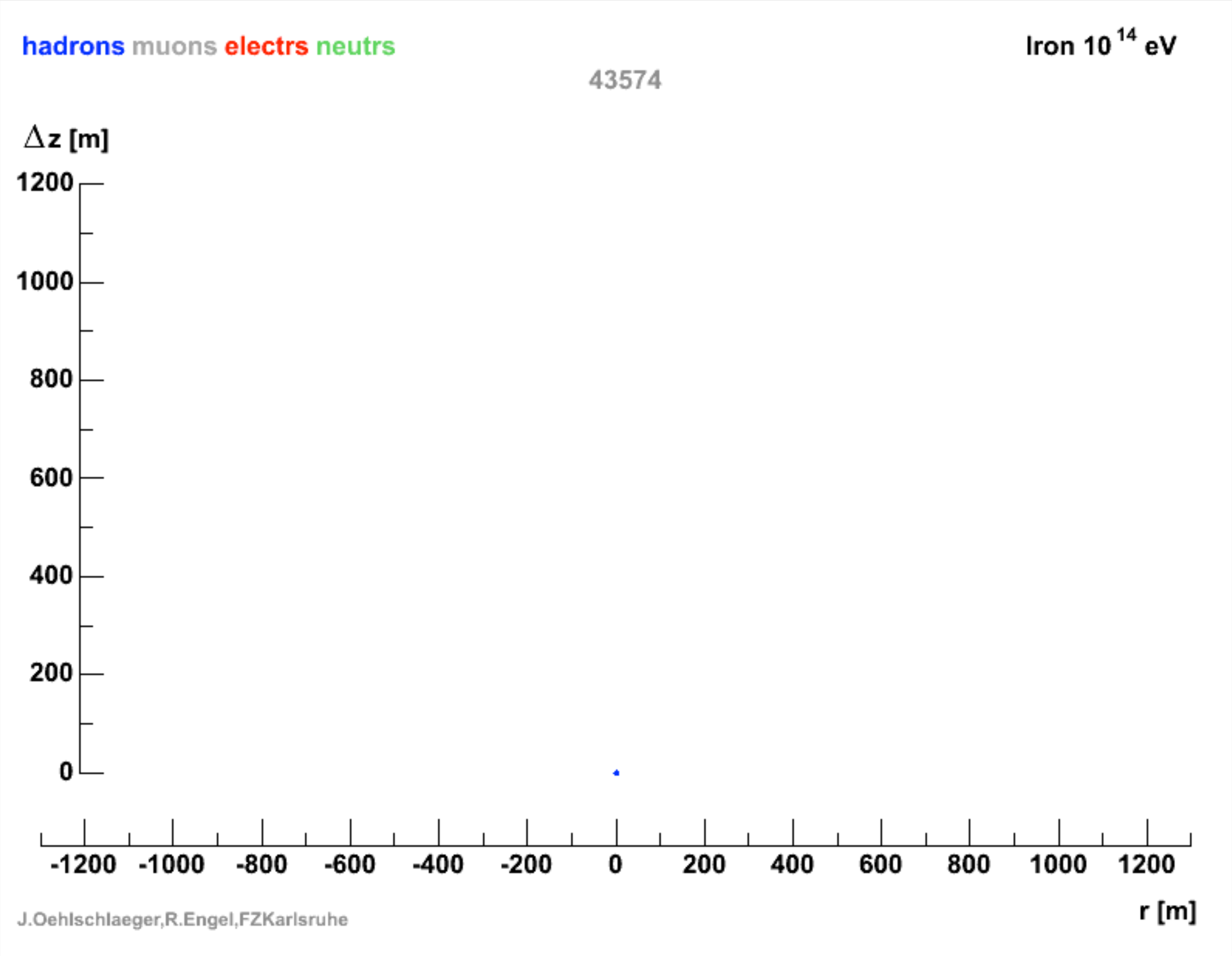
Particles of an proton shower



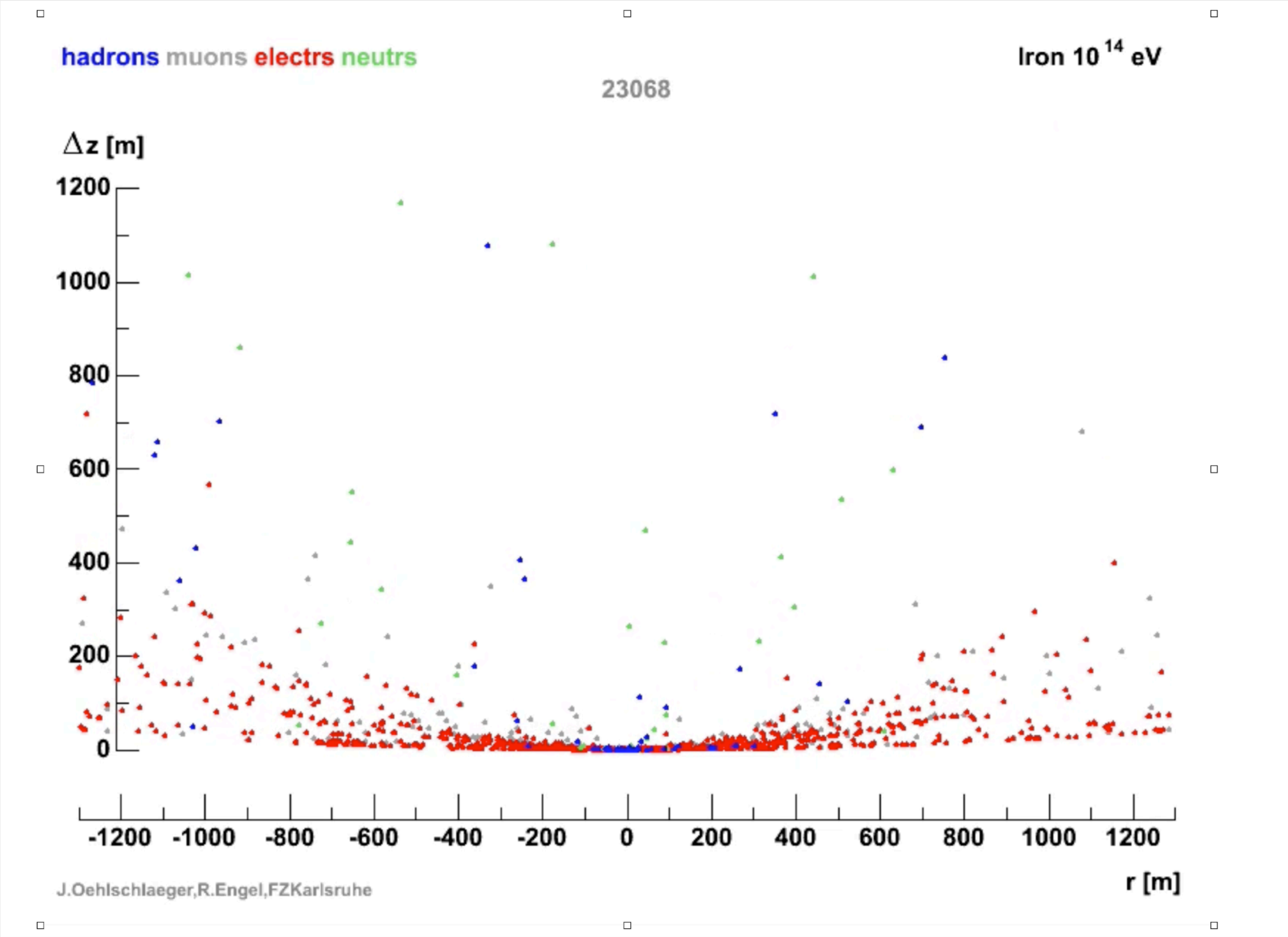
Particles of a gamma-ray shower

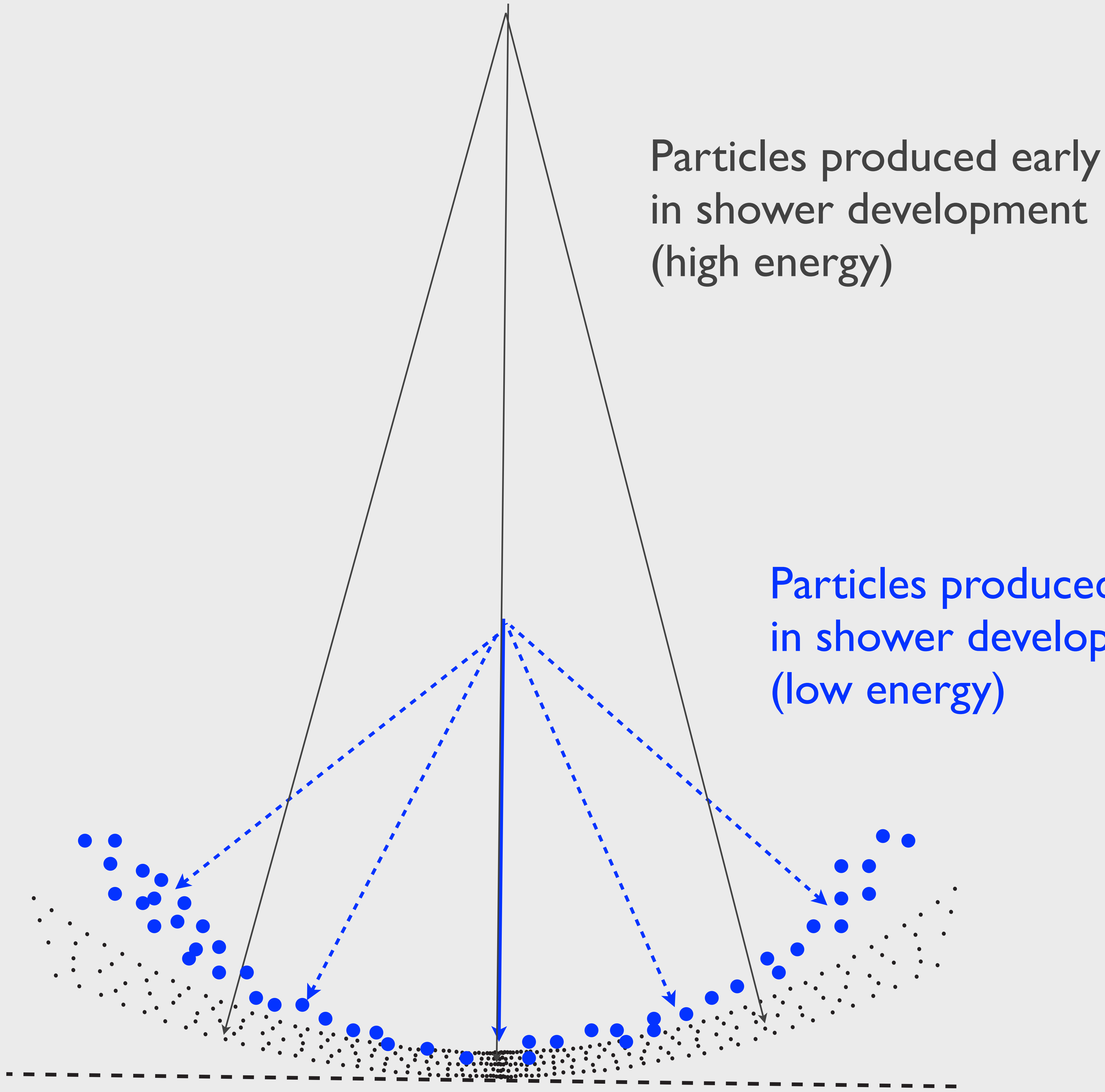


Time structure of shower disk



Time structure of shower disk



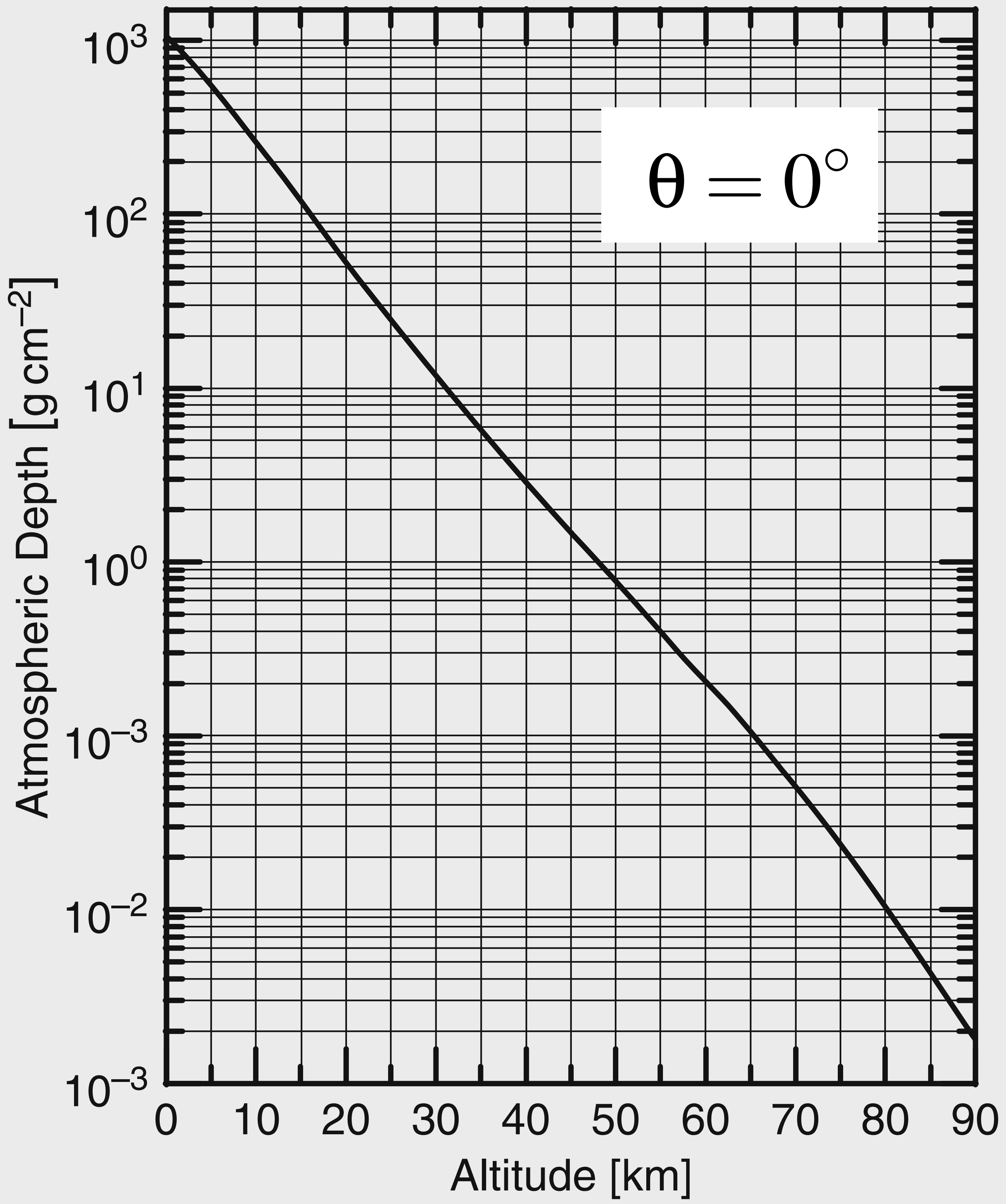


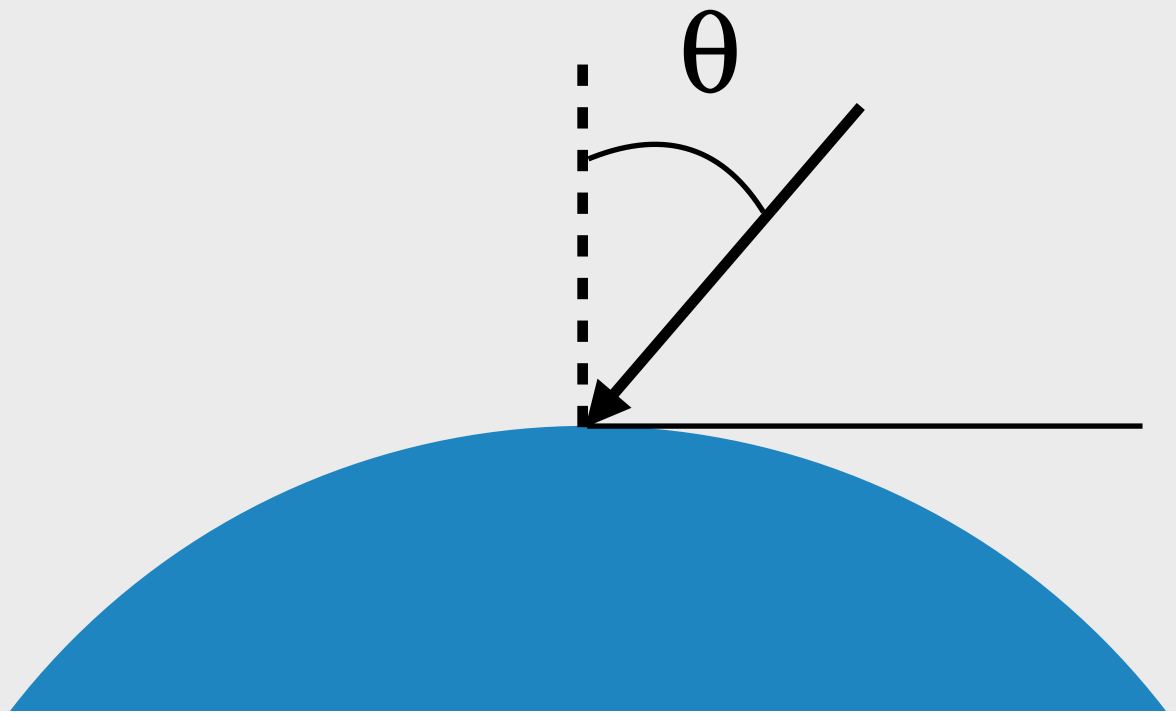
Curvature of shower front
sensitive to early muons

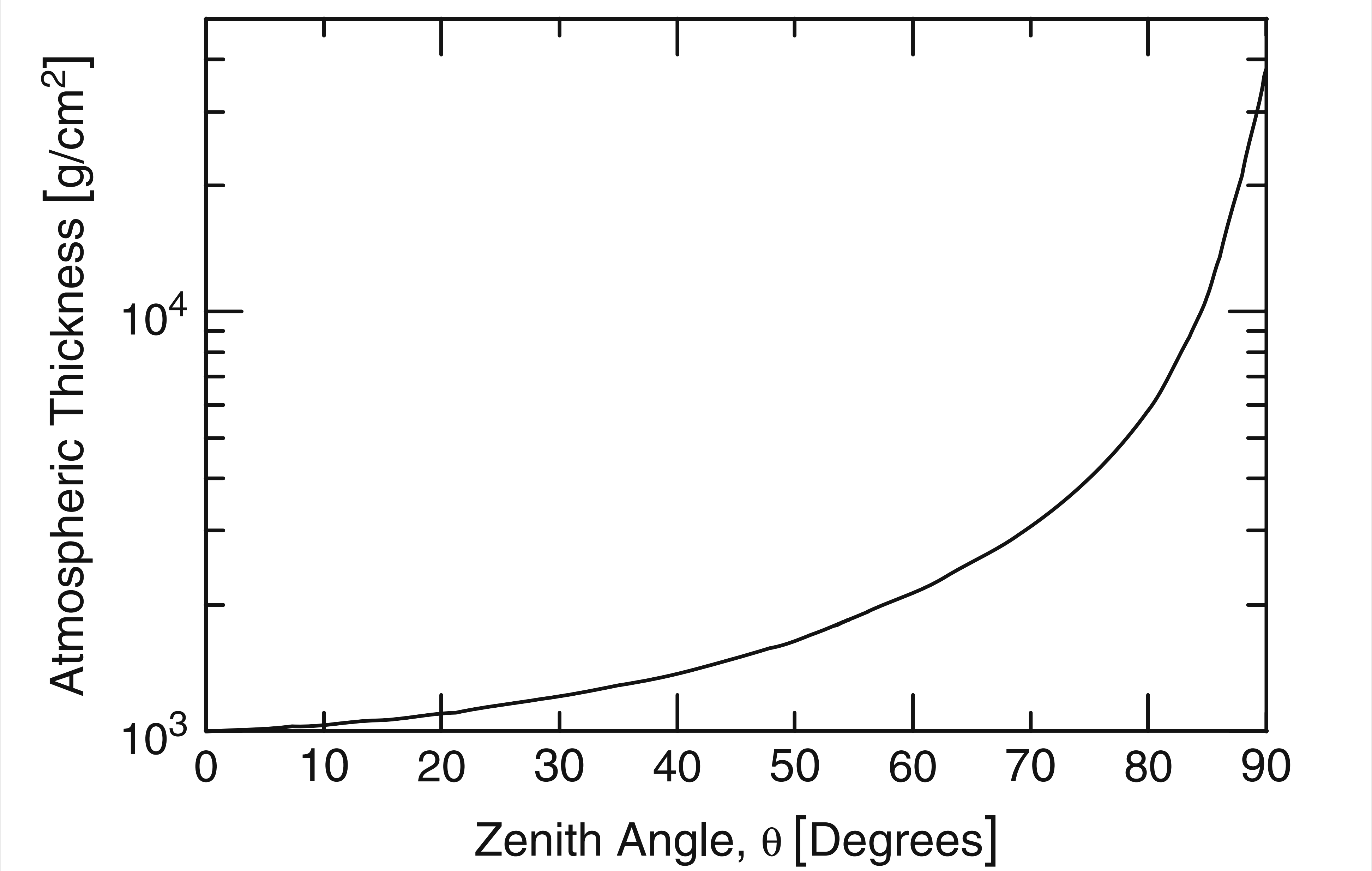
2. Basics

The Earth’s atmosphere

Altitude (km)	Local density (10^{-3} g/cm^3)
40	3.8×10^{-3}
30	1.8×10^{-2}
20	8.8×10^{-2}
15	0.19
10	0.42
5	0.74
3	0.91
1.5	1.06
0.5	1.17
0	1.23







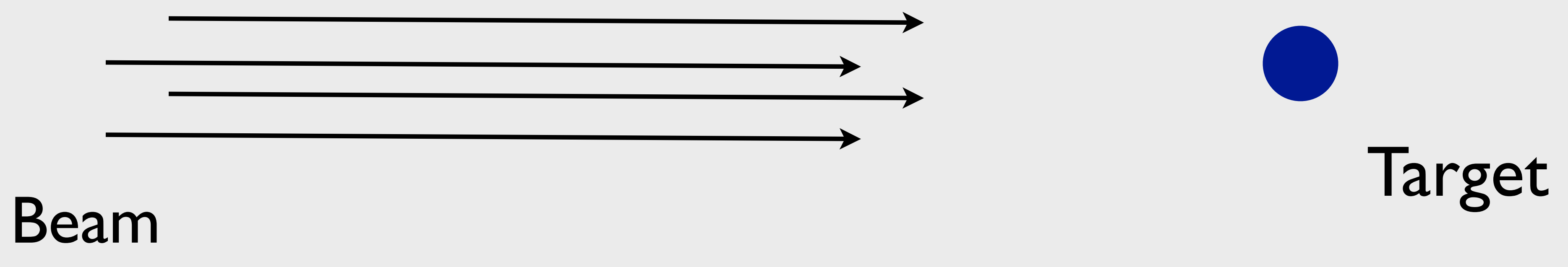
(Grieder EAS, 2010)

Atmospheric **slant depth**
(integral taken along shower axis)

$$\int \rho_{air}\, dl = X$$

Cross section, interaction rate, interaction length

$$\Phi = \frac{\mathrm{d}N_{\mathrm{beam}}}{\mathrm{d}A\ \mathrm{d}t}$$



Definition of cross section

$$\sigma = \frac{1}{\Phi}\frac{\mathrm{d}N_{\mathrm{int}}}{\mathrm{d}t}$$

(Units: 1 barn = 10^{-28} m^2
1 mb = 10^{-27} cm^2)

Flux of particles on single target

Interaction rate

Interaction length (g/cm^2)

$$\lambda_{\mathrm{int}} = \frac{\langle m_{\mathrm{target}}\rangle}{\sigma}$$

$$\frac{\mathrm{d}N_{\mathrm{int}}}{\mathrm{d}t\,\mathrm{d}V} = \frac{\rho_{\mathrm{target}}}{\langle m_{\mathrm{target}}\rangle}\,\sigma\,\Phi$$

$$\mathrm{d}X = \rho_{\mathrm{target}}\,\mathrm{d}l$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}X} = -\frac{\sigma}{\langle m_{\mathrm{target}}\rangle}\,\Phi = -\frac{1}{\lambda_{\mathrm{int}}}\,\Phi$$

Examples of numerical values

Altitude (km)	Vertical depth (g/cm^2)	Local density (10^{-3} g/cm^3)	Molière unit (m)	Electron Cherenkov threshold (MeV)	Cherenkov angle (°)
40	3	3.8×10^{-3}	2.4×10^4	386	0.076
30	11.8	1.8×10^{-2}	5.1×10^3	176	0.17
20	55.8	8.8×10^{-2}	1.0×10^3	80	0.36
15	123	0.19	478	54	0.54
10	269	0.42	223	37	0.79
5	550	0.74	126	28	1.05
3	715	0.91	102	25	1.17
1.5	862	1.06	88	23	1.26
0.5	974	1.17	79	22	1.33
0	1,032	1.23	76	21	1.36

US standard atmosphere

Typical values

$$\lambda_{\gamma\to e^+e^-} \approx 46\,\mathrm{g/cm^2}$$

$$\lambda_\pi \approx \lambda_K \approx 120\,\mathrm{g/cm^2}$$

$$\lambda_p \approx 80\,\mathrm{g/cm^2}$$

$$\lambda_{\mathrm{Fe}} \approx 10\,\mathrm{g/cm^2}$$

Interaction length in air

$$\lambda_{\mathrm{int}} = \frac{\langle m_{\mathrm{air}} \rangle}{\sigma_{\mathrm{int}}} = \frac{24160\,\mathrm{mb\ g/cm^2}}{\sigma_{\mathrm{int}}}$$

3. Electromagnetic Showers

Energy loss of charged particles

Ionization energy loss:
Bethe-Bloch formula

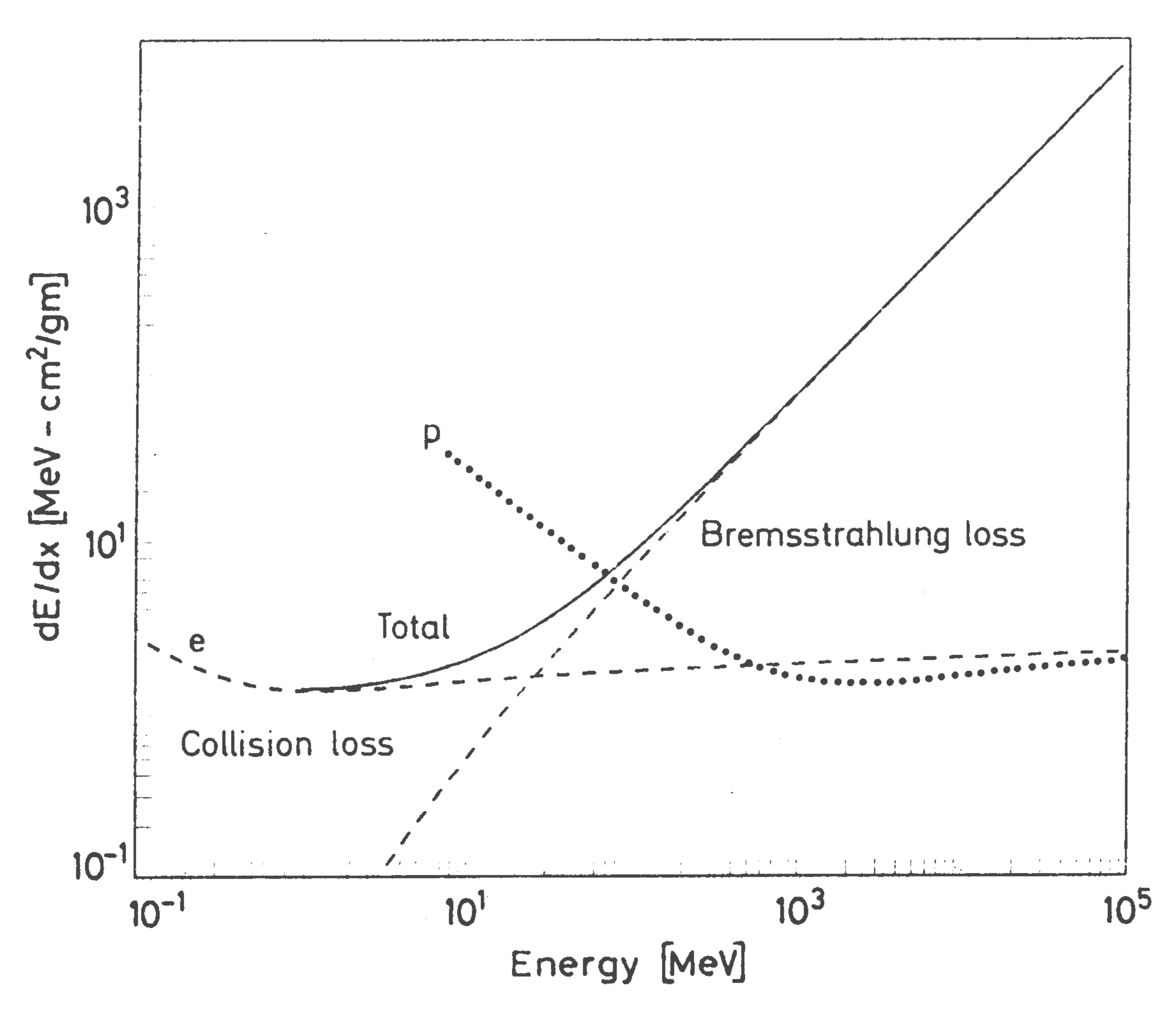
$$\frac{dE_{ion}}{dX} = -\alpha(E)$$

$$\alpha \sim 2.4\,\text{MeV}/(\text{g}/\text{cm}^2)$$

Radiation energy loss:
bremsstrahlung

$$\frac{dE_{rad}}{dX} = -\frac{E}{X_0}$$

Radiation length X_0 $$X_0 \sim 36\,\text{g}/\text{cm}^2$$

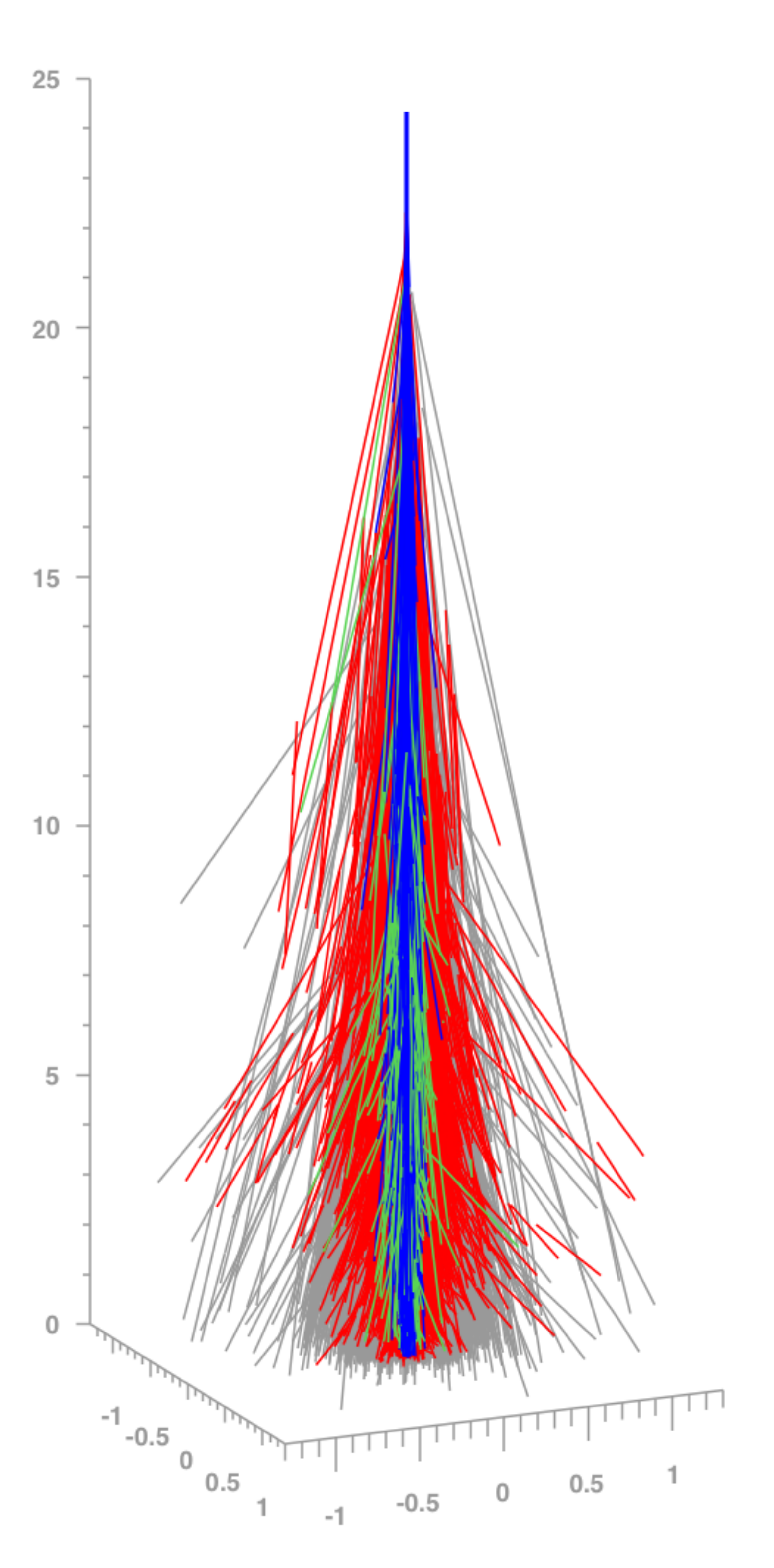


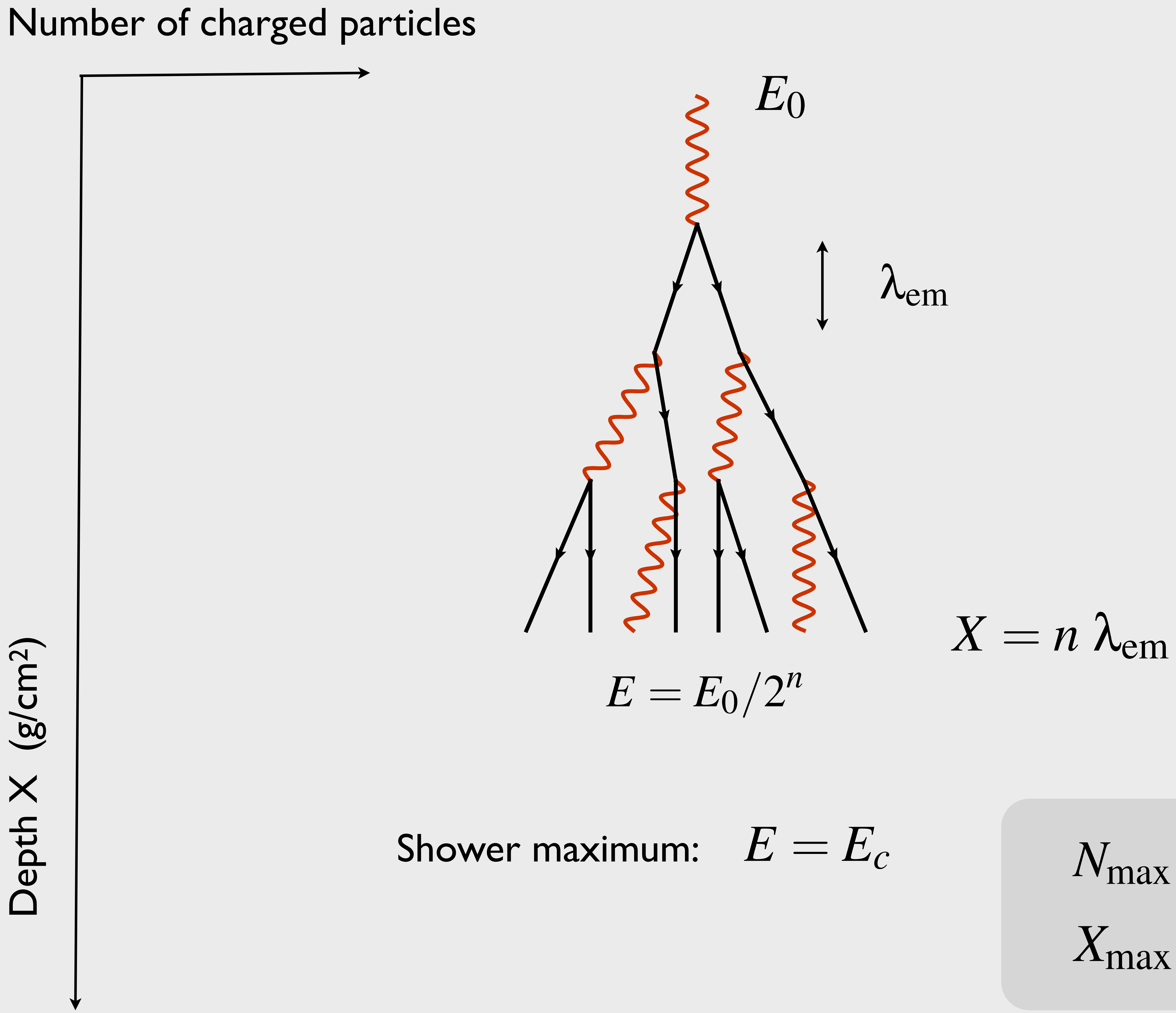
$$\frac{dE}{dX} = -\alpha(E) - \frac{E}{X_0}$$

Critical energy E_c defined as energy at which both losses are equal

$$E_c = \alpha\, X_0 \sim 85\,\text{MeV}$$

Qualitative approach: Heitler model





Shower maximum: $E = E_c$

$$N_{\max} = E_0/E_c$$

$$X_{\max} \sim \lambda_{\text{em}} \ln(E_0/E_c)$$

Cascade equations

Energy loss of electron:

$$\frac{\mathrm{d}E}{\mathrm{d}X} = -\alpha - \frac{E}{X_0}$$

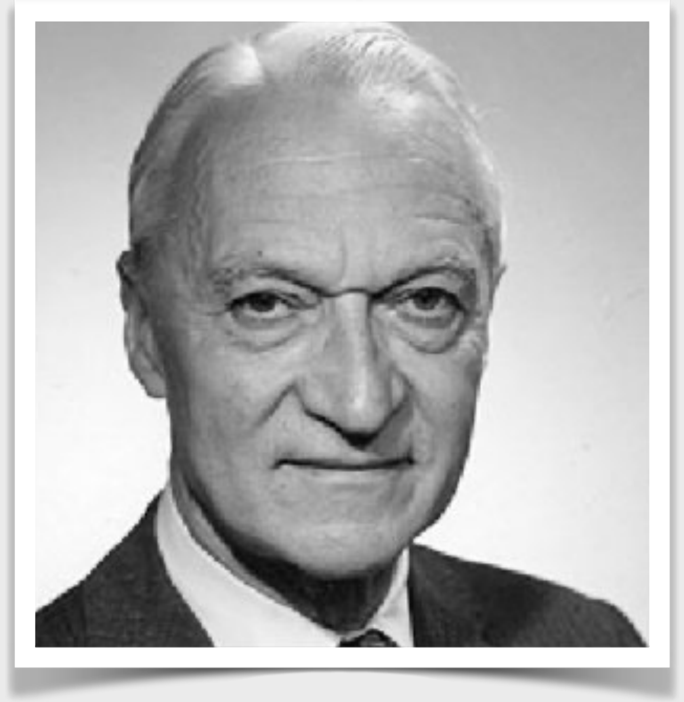
Critical energy: $E_c = \alpha\, X_0 \sim 85\,\mathrm{MeV}$

Radiation length: $X_0 \sim 36\,\mathrm{g/cm^2}$

Cascade equations

$$\frac{\mathrm{d}\Phi_e(E)}{\mathrm{d}X} = -\frac{\sigma_e}{\langle m_{\mathrm{air}}\rangle}\Phi_e(E) + \int_E^\infty \frac{\sigma_e}{\langle m_{\mathrm{air}}\rangle}\Phi_e(\tilde{E})P_{e\to e}(\tilde{E},E)\mathrm{d}\tilde{E}$$

$$+\int_E^\infty \frac{\sigma_\gamma}{\langle m_{\mathrm{air}}\rangle}\Phi_\gamma(\tilde{E})P_{\gamma\to e}(\tilde{E},E)\mathrm{d}\tilde{E} \;+\; \alpha\frac{\partial\Phi_e(E)}{\partial E}$$



Bruno Rossi

$$X_{\mathrm{max}} \approx X_0 \ln\left(\frac{E_0}{E_c}\right) \qquad N_{\mathrm{max}} \approx \frac{0.31}{\sqrt{\ln(E_0/E_c) - 0.33}}\frac{E_0}{E_c}$$

(Rossi & Greisen, Rev. Mod. Phys. 13 (1940) 240)

Shower age and Greisen formula

Longitudinal profile

$$N_e(X) \approx \frac{0.31}{[\ln E_0/E_c]^{1/2}} \exp\left\{ \frac{X}{X_0} \left(1 - \frac{3}{2} \ln s \right) \right\}$$

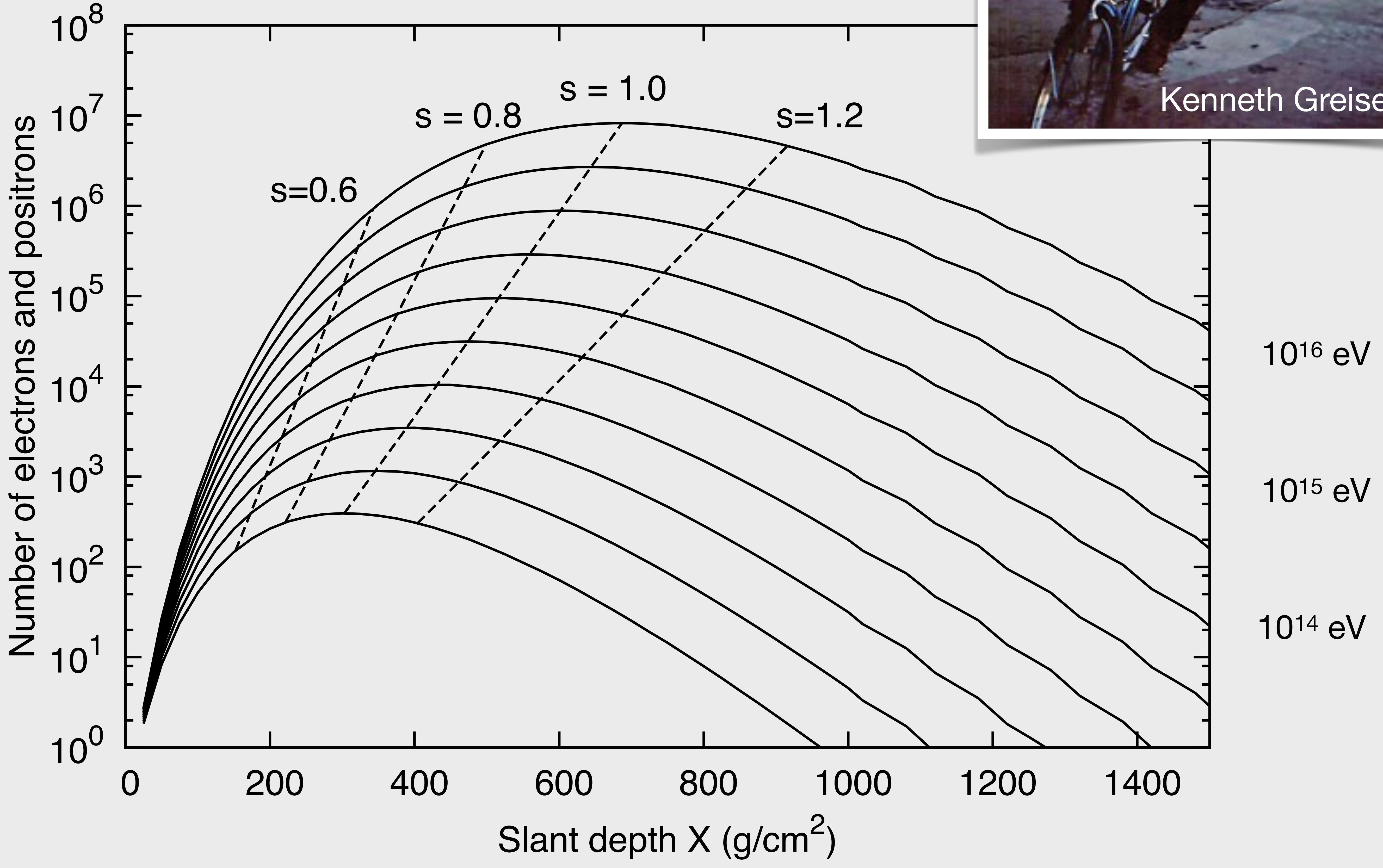
Shower age

$$s = \frac{3X}{X + 2X_{max}}$$

Energy spectrum particles

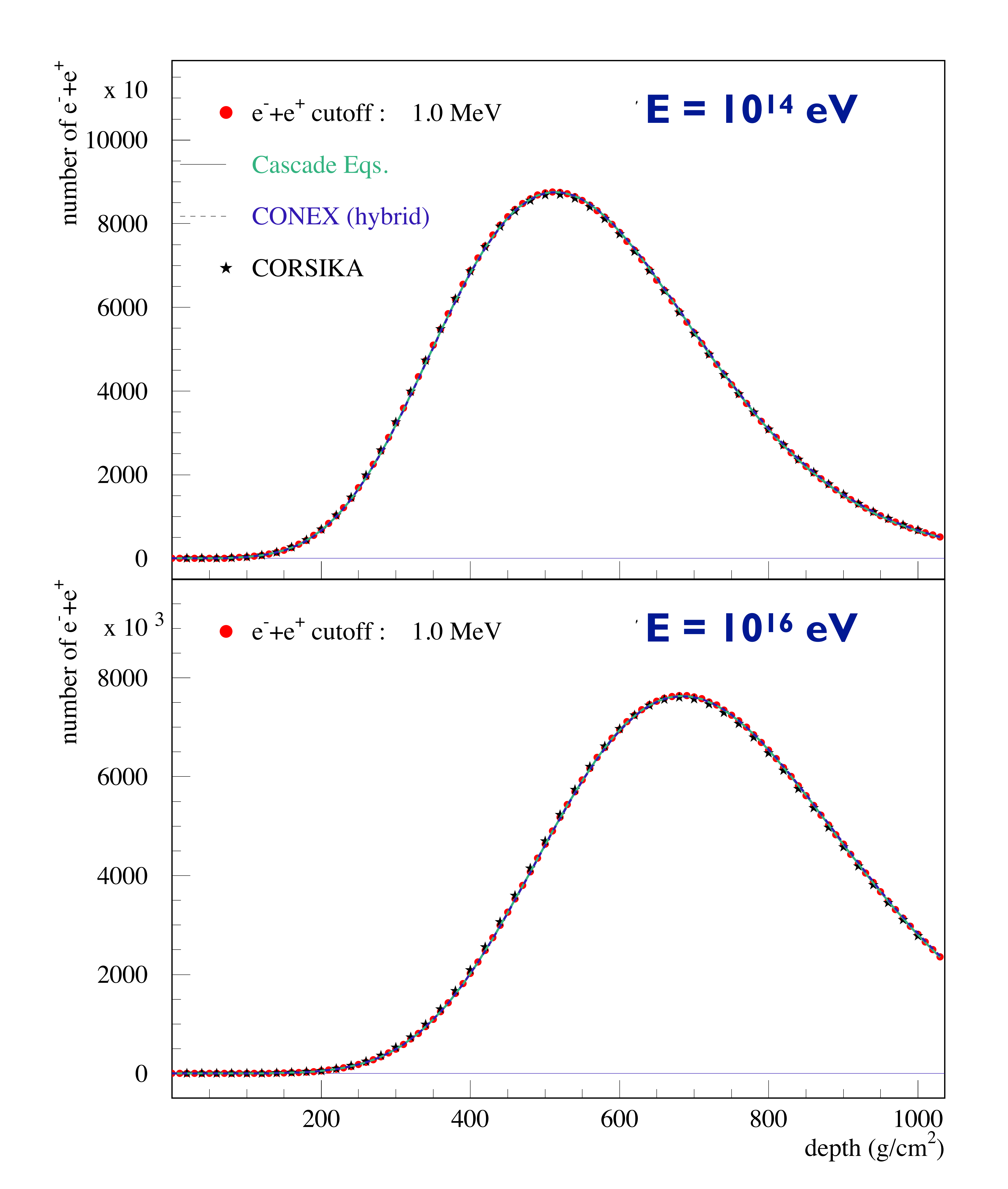
$$\frac{dN_e}{dE} \sim \frac{1}{E^{1+s}}$$

Electrons in photon-initiated shower



(Greisen 1956, see also Lipari PRD 2009)

Mean longitudinal shower profile



Calculation with cascade Eqs.

Photons

- Pair production
- Compton scattering

Electrons

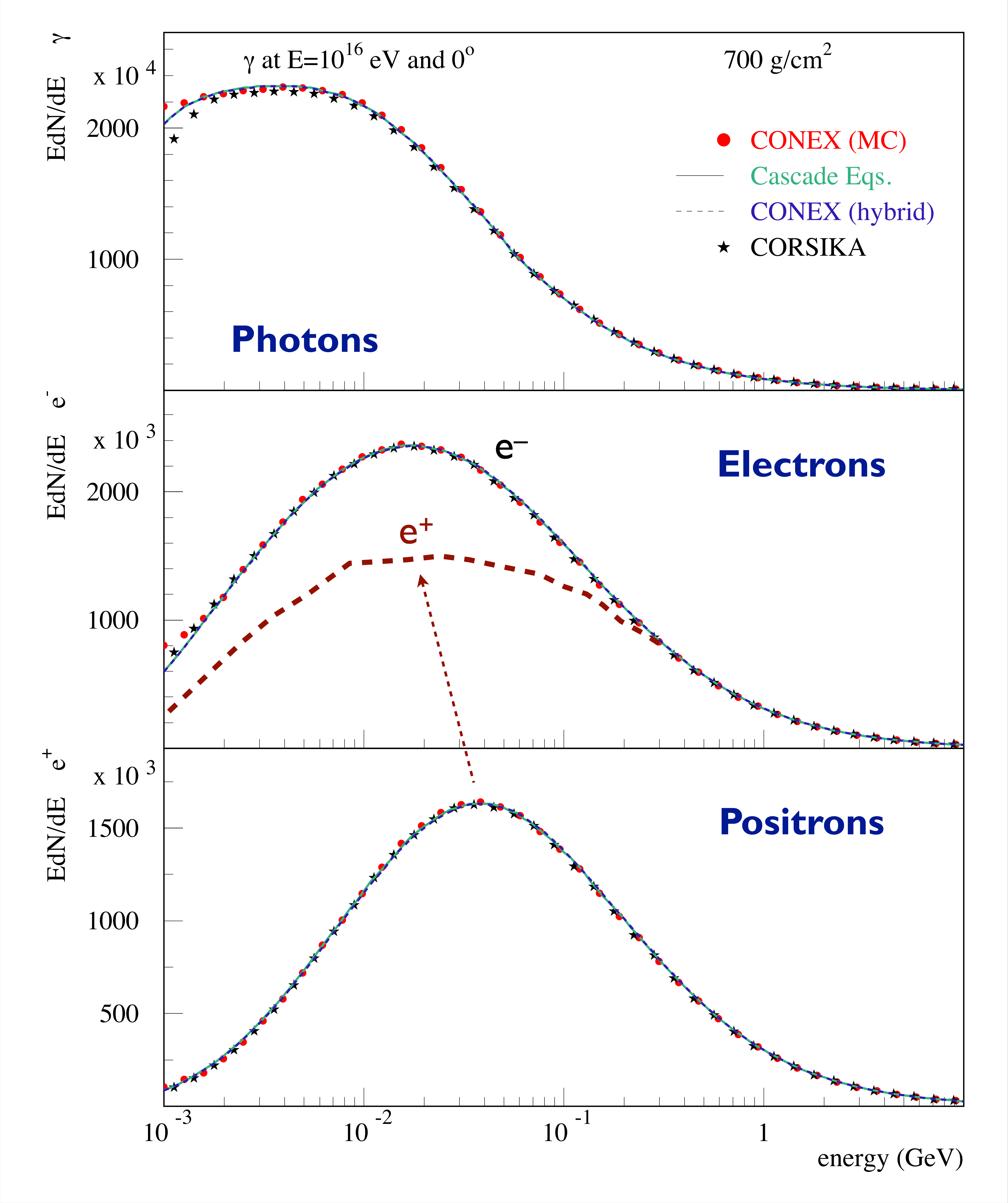
- Bremsstrahlung
- Moller scattering

Positrons

- Bremsstrahlung
- Bhabha scattering

(Bergmann et al., Astropart.Phys. 26 (2007) 420)

Energy spectra of secondary particles



Number of photons divergent, energy threshold applied in calculation

- Typical energy of electrons and positrons $E_c \sim 80$ MeV
- Electron excess of 20 - 30%
- Pair production symmetric
- Excess of electrons in target

(Bergmann et al., Astropart.Phys. 26 (2007) 420)

Lateral distribution of shower particles

Coulomb scatting

$$\frac{dN}{d\Omega} = \frac{1}{64\pi} \frac{1}{\ln(191 Z^{-1/3})} \left(\frac{E_s}{E}\right)^2 \frac{1}{\sin^4 \theta/2}$$

$$E_s \approx 21\,\mathrm{MeV}$$

Expectation value

$$\int \theta^2 \frac{dN}{d\Omega} d\Omega$$

$$\langle \theta^2 \rangle \sim \left(\frac{E_s}{E}\right)^2$$

$$r_1 = r_M = \left(\frac{E_s}{E_c}\right) \frac{X_0}{\rho_{air}}$$

Displacement of particle

$$r \sim \left(\frac{E_s}{E}\right) \frac{X_0}{\rho_{air}} = \left(\frac{E_c}{E}\right) r_1$$

Moliere unit
(78 m at sea level)

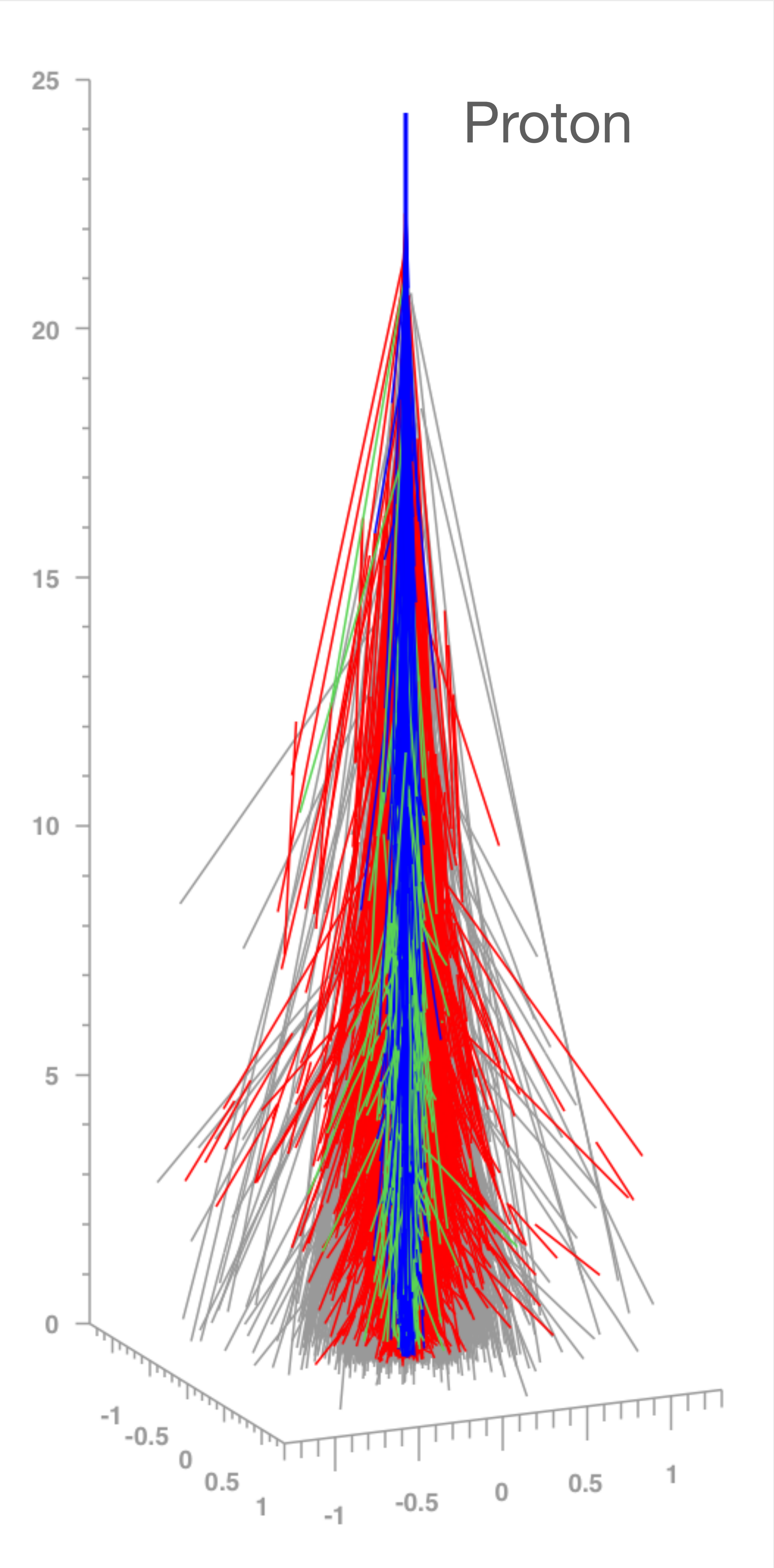
$$\frac{dN_e}{dE} \sim \left(\frac{E_c}{E}\right)^{1+s}$$

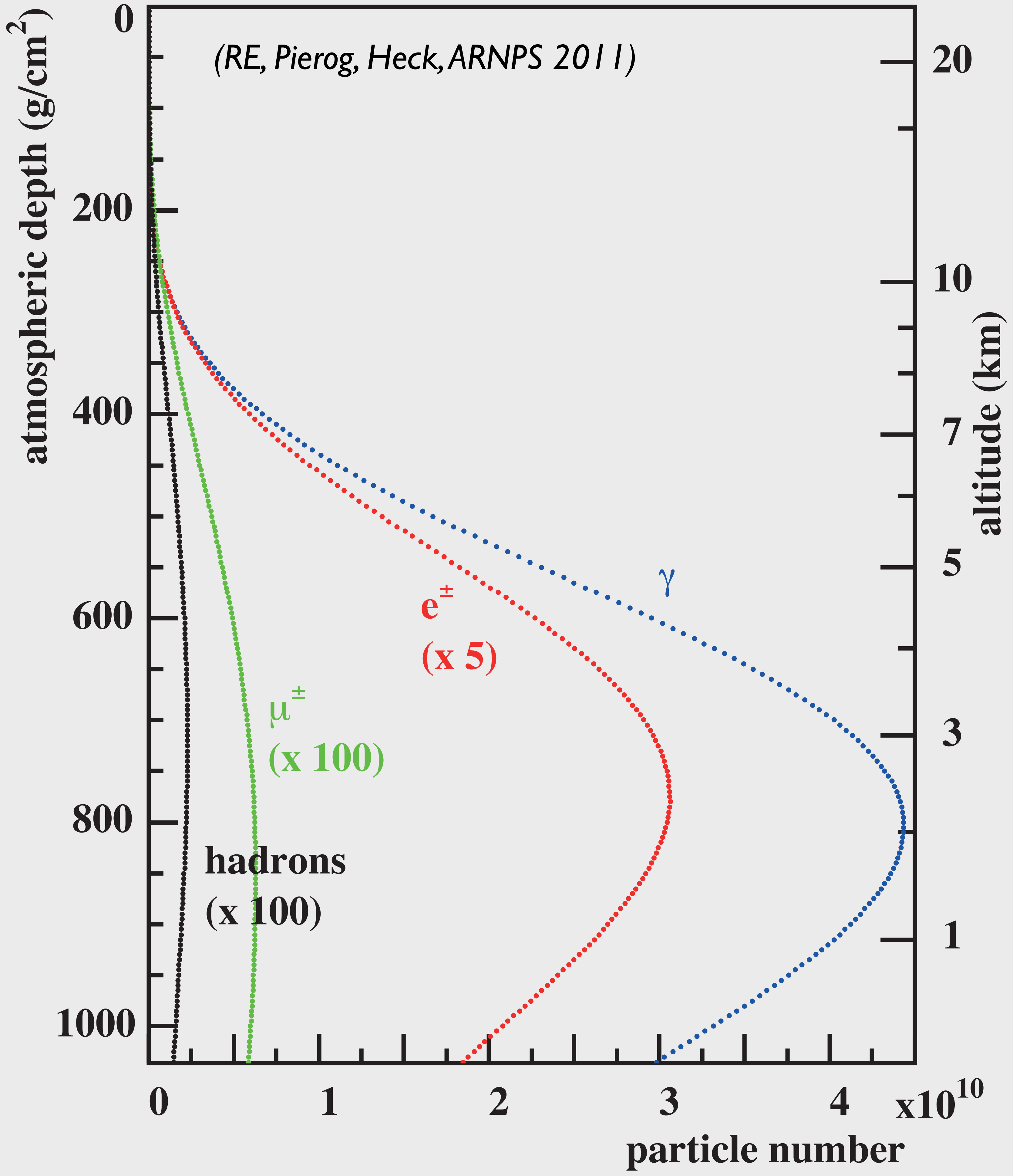
$$\frac{dN_e}{r\,dr} \sim \left(\frac{r}{r_1}\right)^{s-2} \left(1 + \frac{r}{r_1}\right)^{s-4.5}$$

Nishimura-Kamata-Greisen (NKG)
lateral distribution function

4. Hadronic showers

Expectation from simulations



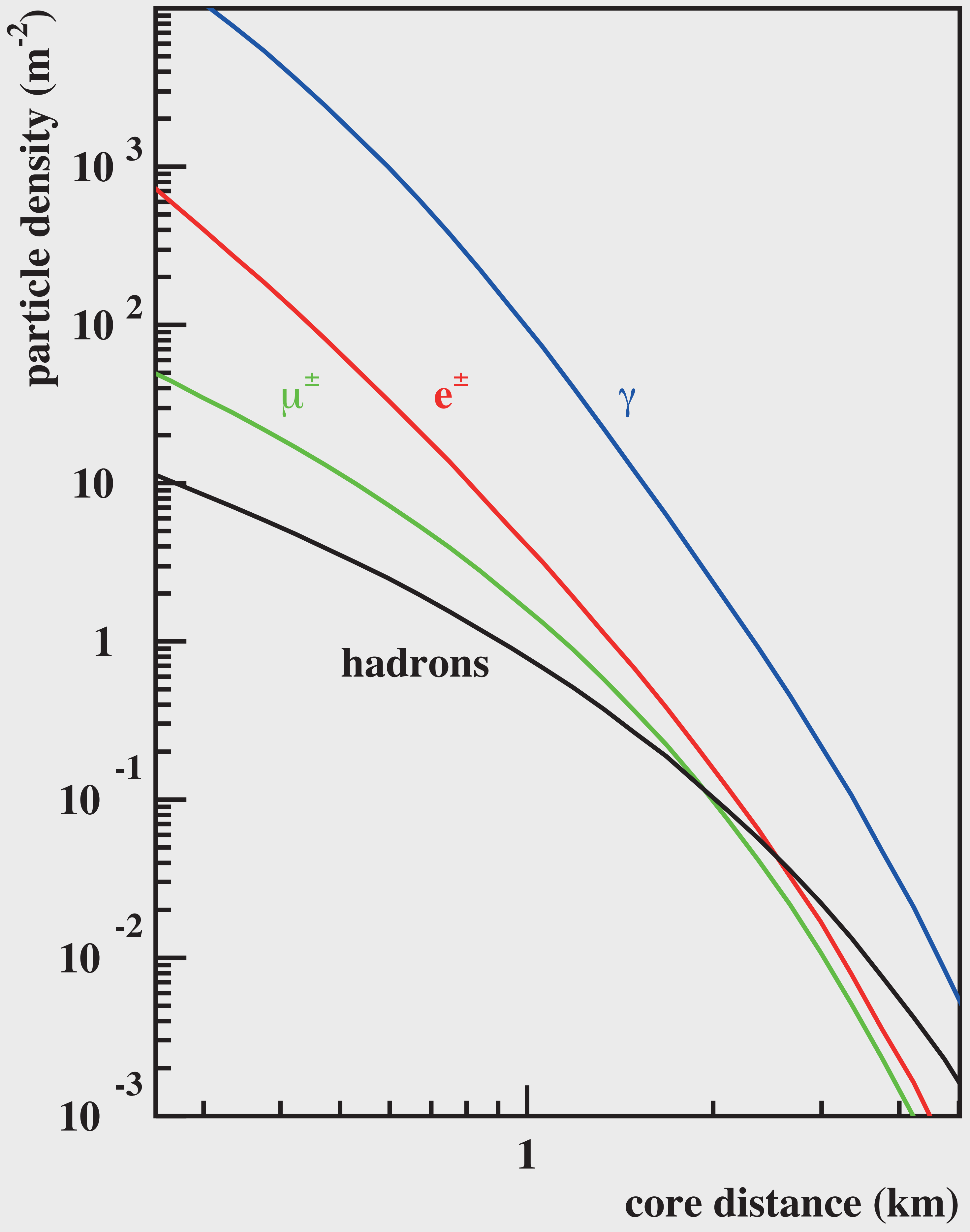


Longitudinal profile:

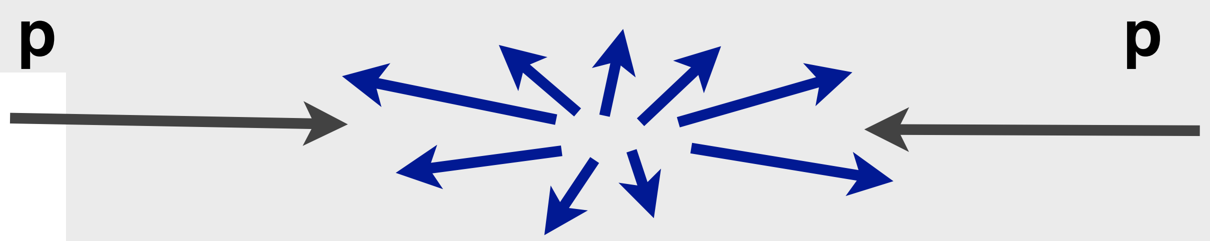
Cherenkov light
Fluorescence light
(bulk of particles measured)

Lateral profiles:

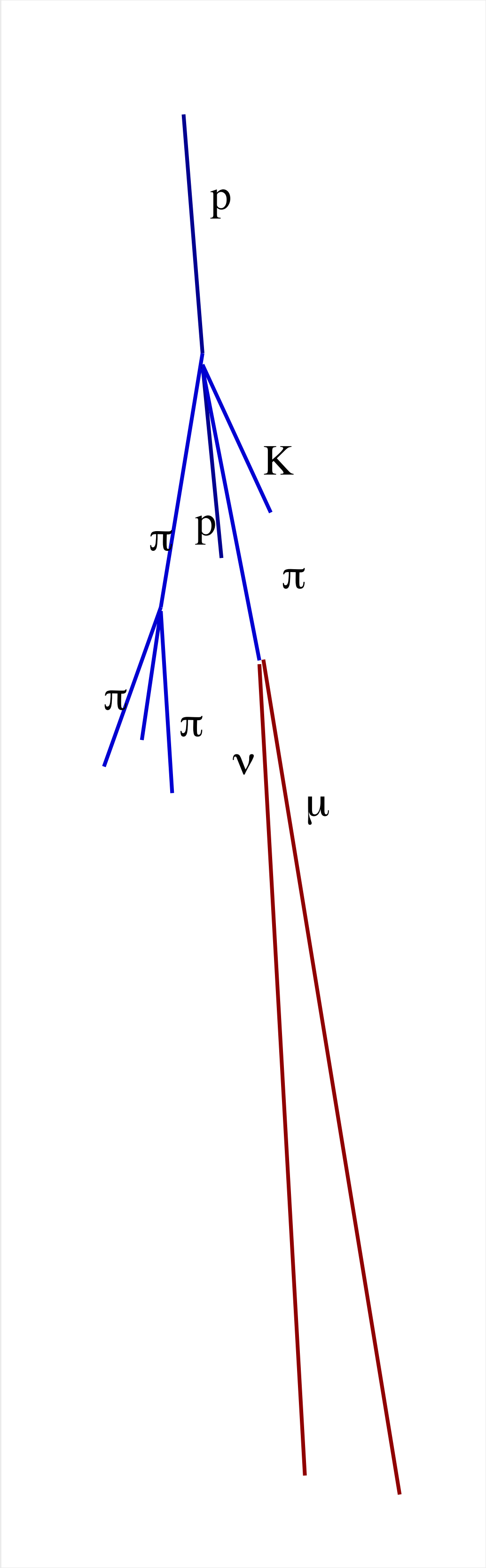
particle detectors at ground
(very small fraction of particles sampled)

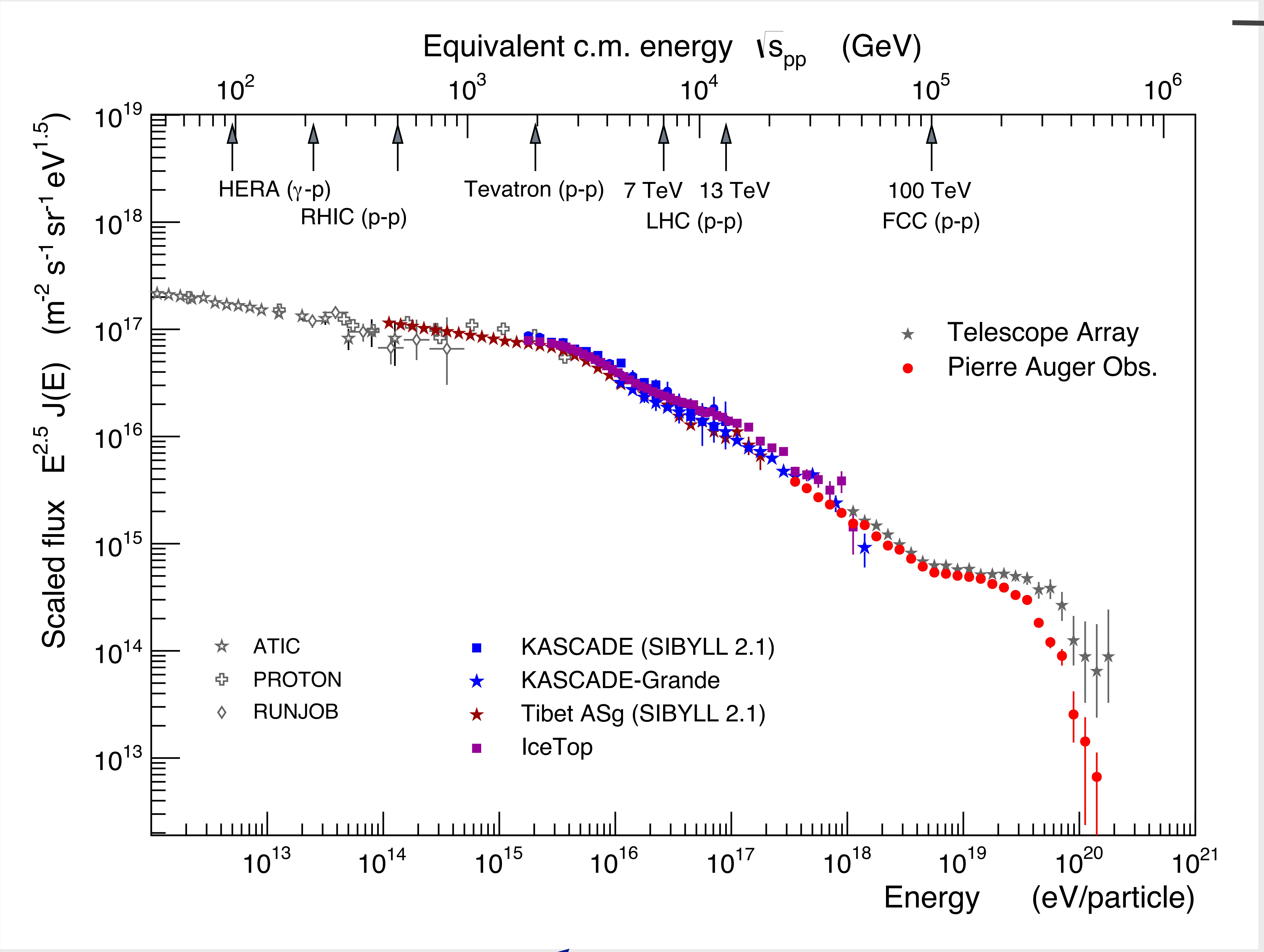


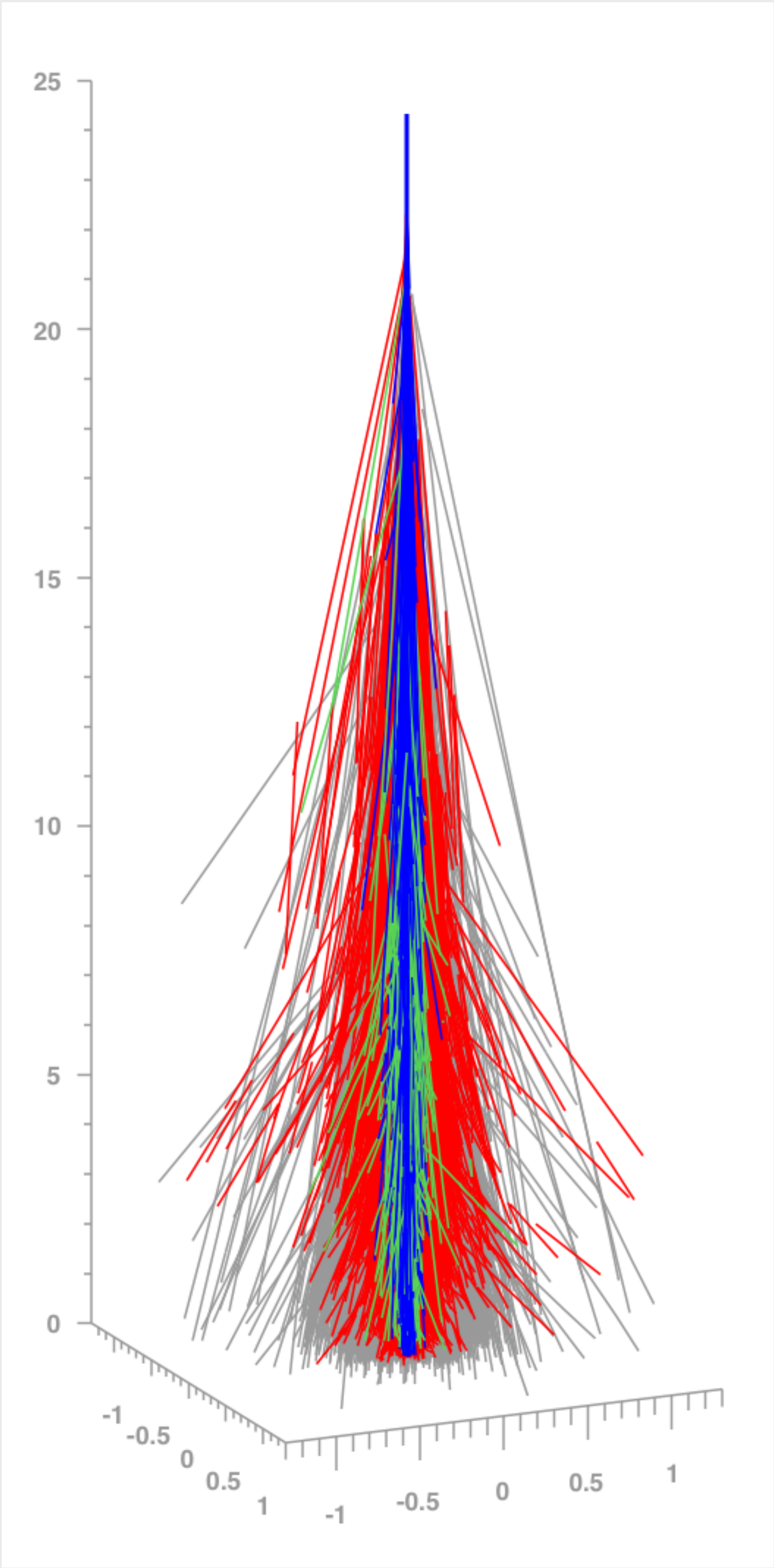
Cosmic ray flux and interaction energies



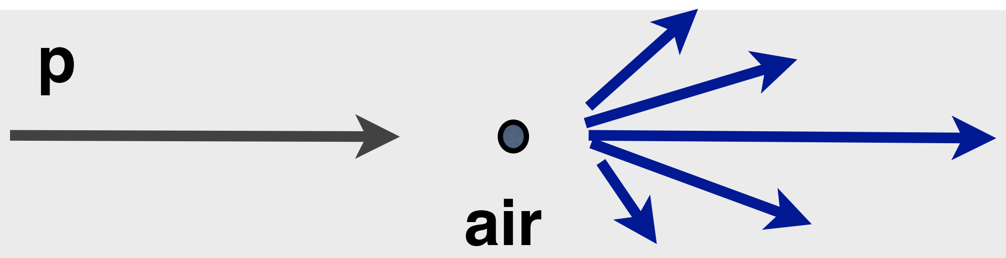
Center-of-mass energy







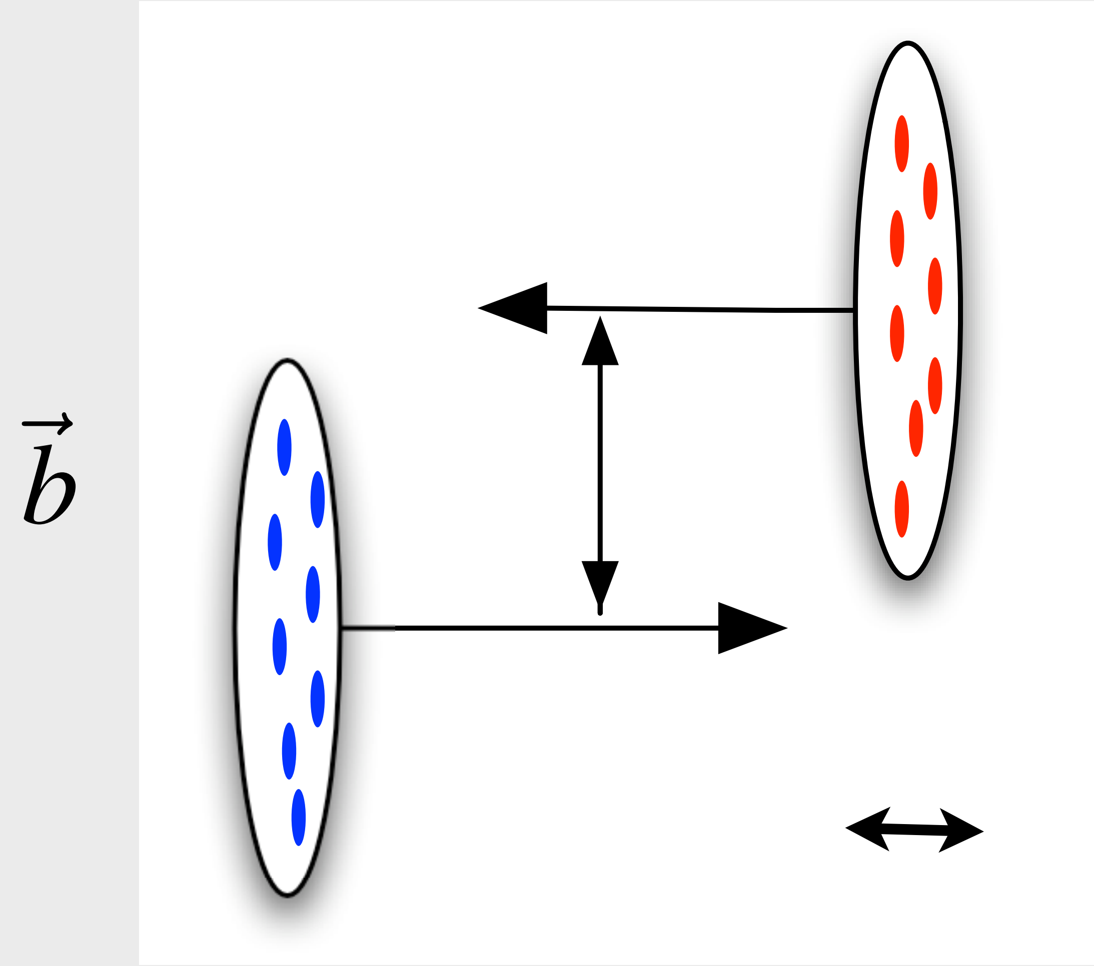
Laboratory energy



Expectations from uncertainty relation

Assumptions:

- hadrons built up of partons
- partons deflected/liberated in collision process, small momentum
- partons fragment into hadrons (pions, kaons,...) after interaction
- interaction viewed in c.m. system (other systems equally possible)



$$R \approx 1\,\mathrm{fm} \approx 5\,\mathrm{GeV}^{-1}$$

$$R' = R/\Gamma = R\,\frac{m_p}{E_p}$$

$$\Gamma = E_p/m_p$$

Heisenberg uncertainty relation

$$\Delta x\,\Delta p_x \simeq 1$$

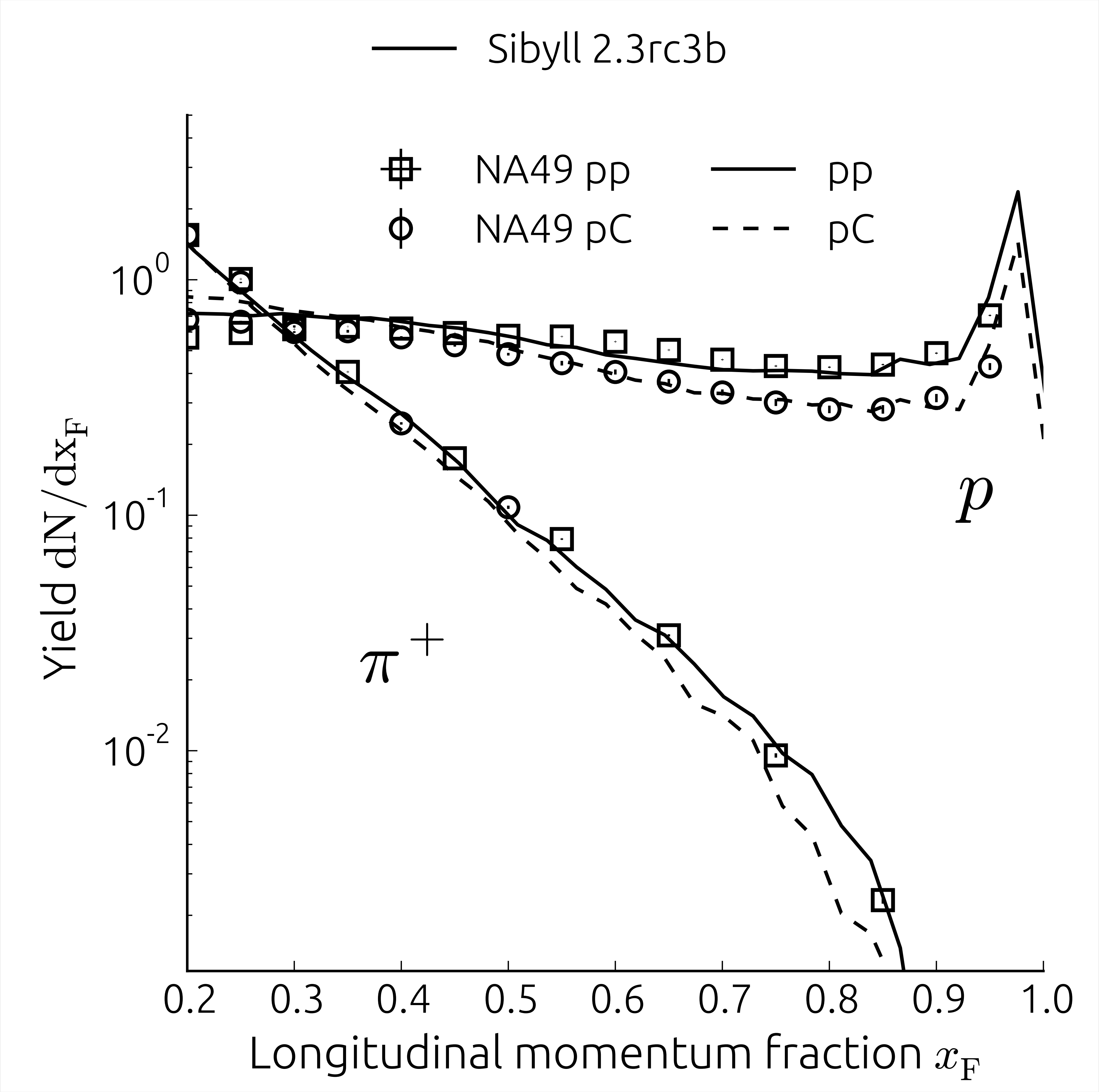
Longitudinal momenta of secondaries

$$\langle p_\parallel \rangle \sim \Delta p_\parallel \approx \frac{1}{R'} \approx \frac{1}{5} E_p$$

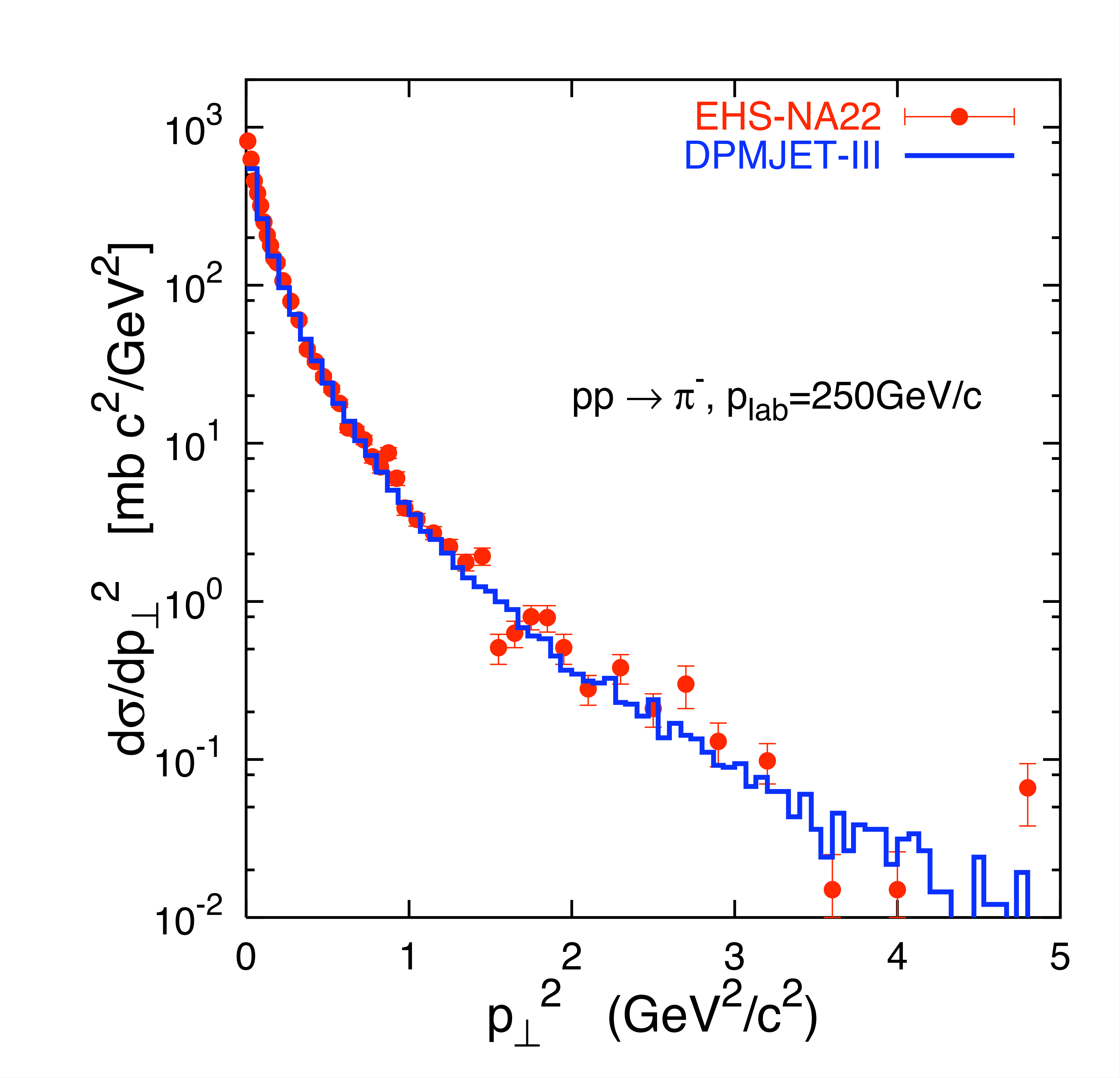
Transverse momenta of secondaries

$$\langle p_\perp \rangle \sim \Delta p_\perp \sim \frac{1}{R} \approx 200\,\mathrm{MeV}$$

Typical hadronic final states



NA49 p-p and p-C at 158 GeV



Feynman-x $x_F = \left(\frac{p_\parallel}{p_{\max}}\right)_{\text{CMS}}$

Transverse momentum

(Riehn et al. ICRC 2017)

Secondary particle multiplicities

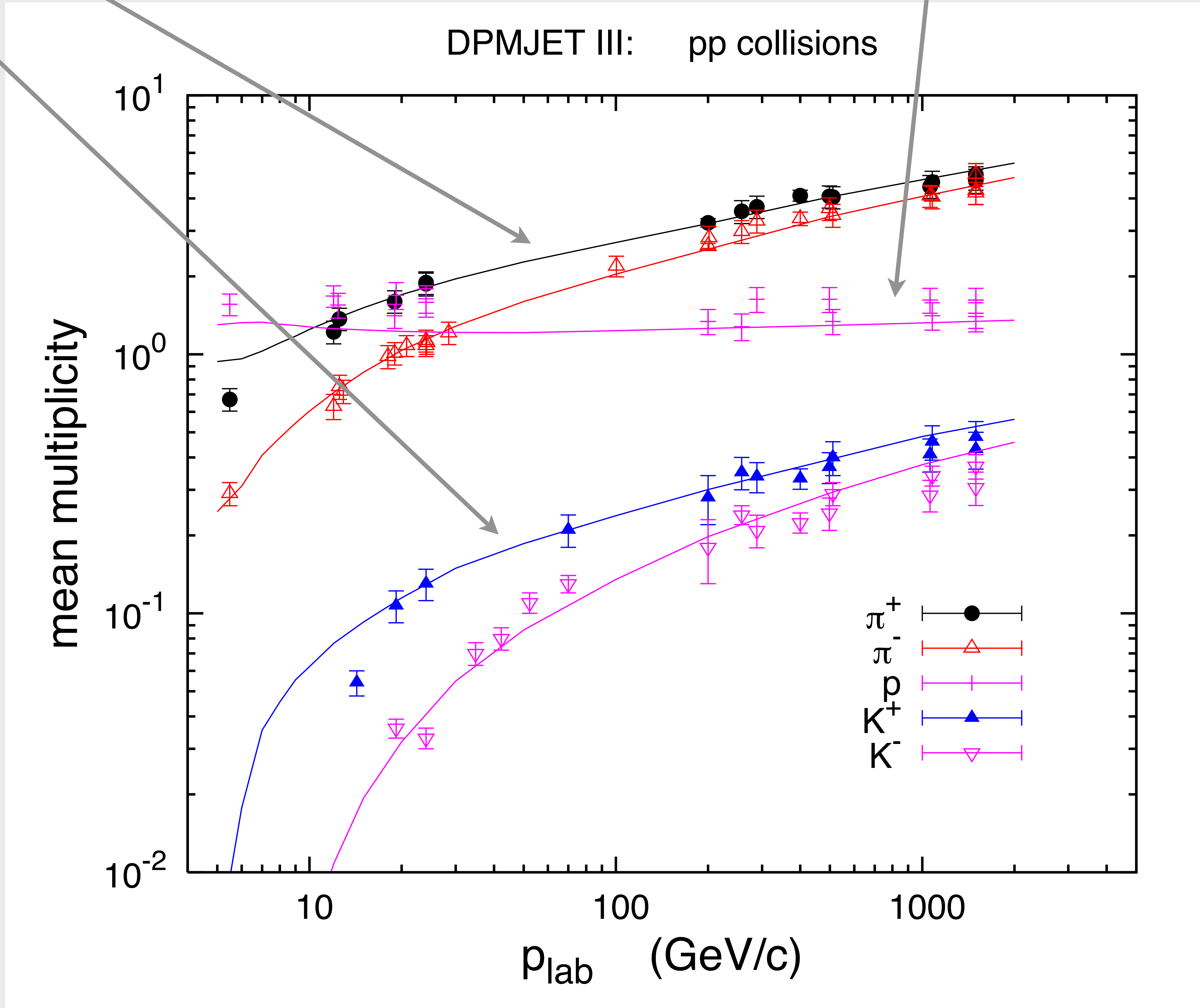
Power-law increase of number of secondary

Leading particles (multiplicity const.)

$$n_{\text{ch}} \sim s^{0.1}$$

proton - proton, $E_{\text{lab}} = 2 \times 10^{11}$ eV

	Exp.	DPMJET-III
charged	7.69 ± 0.06	7.64
neg.	2.85 ± 0.03	2.82
p	1.34 ± 0.15	1.26
n	0.61 ± 0.30	0.66
π^+	3.22 ± 0.12	3.20
π^-	2.62 ± 0.06	2.55
K^+	0.28 ± 0.06	0.30
K^-	0.18 ± 0.05	0.20
Λ	0.096 ± 0.01	0.10
$\bar{\Lambda}$	0.0136 ± 0.004	0.0105



Competing processes of interaction and decay

Interaction length

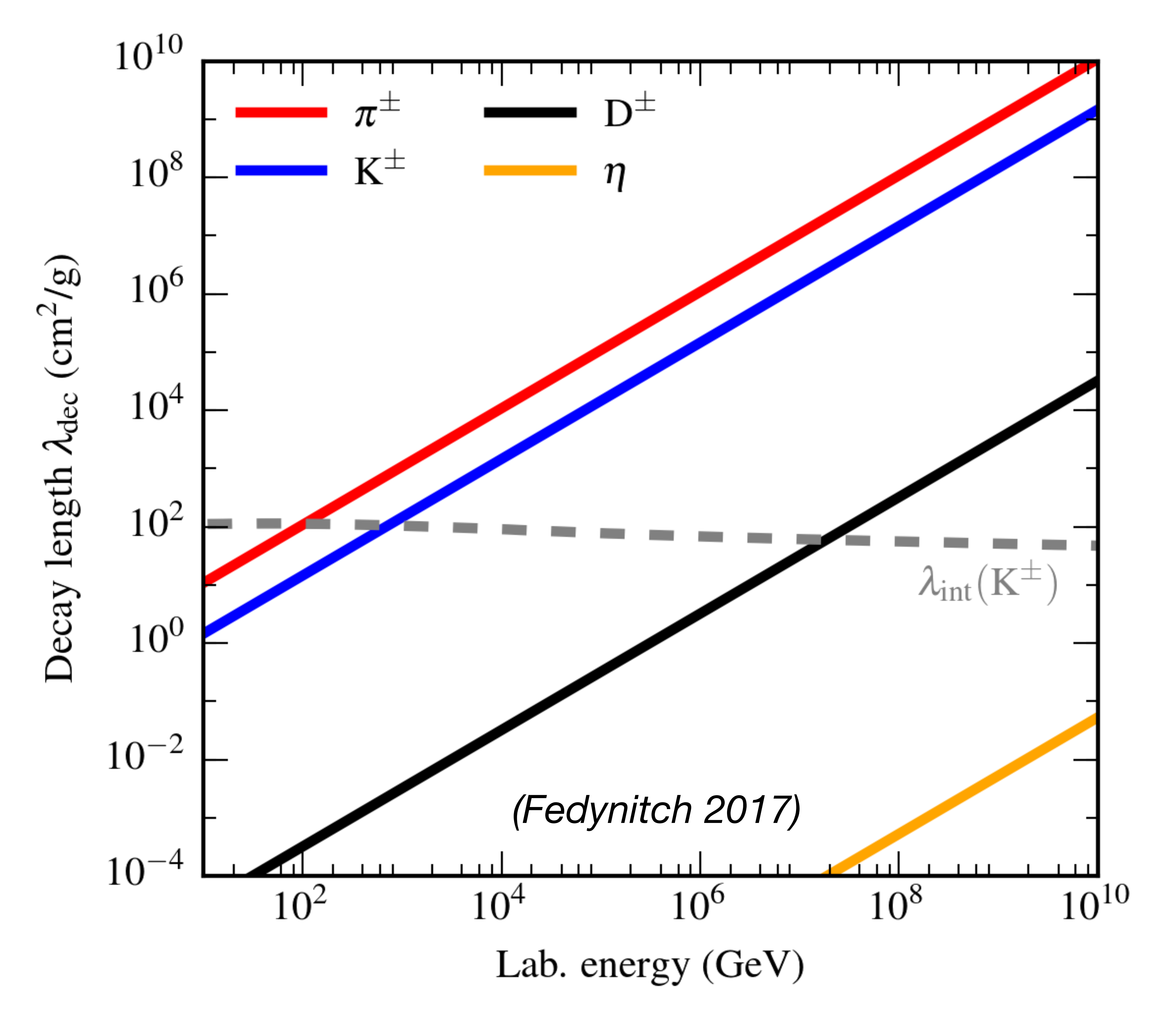
$$\lambda_{\rm int} = \frac{\langle m_{\rm air}\rangle}{\sigma_{\rm int}}$$

$$\lambda_\pi \approx \lambda_K \approx 120\,{\rm g/cm^2}$$

Decay length

$$\lambda_{\rm dec} = \rho\, l_{\rm dec} \approx c\tau\rho\frac{E}{m}$$

air density



Air at altitude of 8 km

$$c\tau_{\pi^\pm} = 7.8\,{\rm m}$$

$$c\tau_{\pi^0} = 25.1\,{\rm nm}$$

$$\pi^+ \rightarrow \mu^+\nu_\mu$$

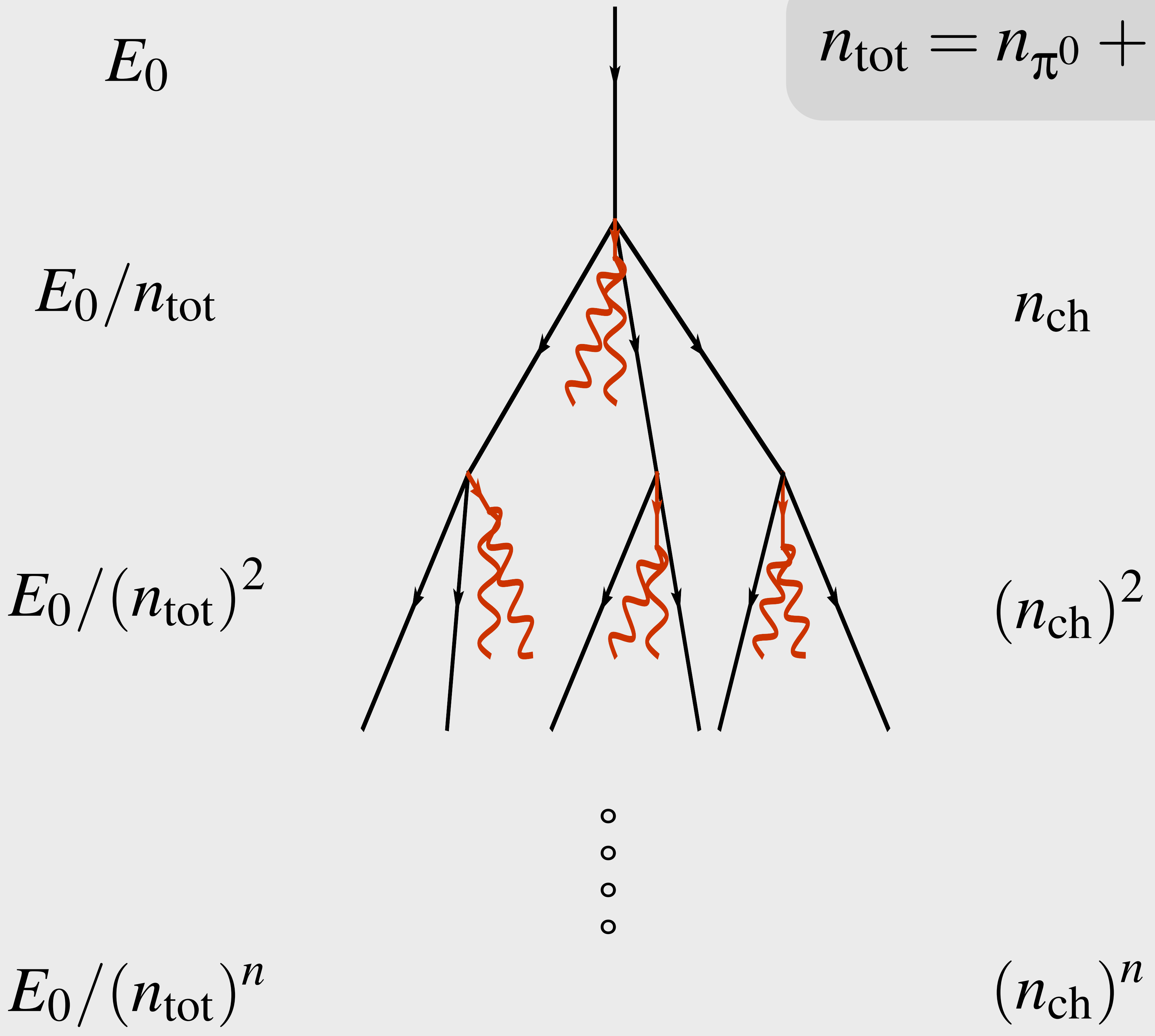
$$\pi^0 \rightarrow \gamma\gamma$$

Charged pions interact E > 30 GeV

Neutral pions always decay

Qualitative approach: Heitler-Matthews model

$$n_{\mathrm{tot}} = n_{\pi^0} + n_{\mathrm{ch}}$$



Primary particle proton

π^0 decay immediately

$\pi^\pm$ initiate new hadronic cascades

$$N_\mu = \left(\frac{E_0}{E_{\mathrm{dec}}}\right)^\alpha$$

$$\alpha = \frac{\ln n_{\mathrm{ch}}}{\ln n_{\mathrm{tot}}} \approx 0.85 \ldots 0.95$$

Assumptions:

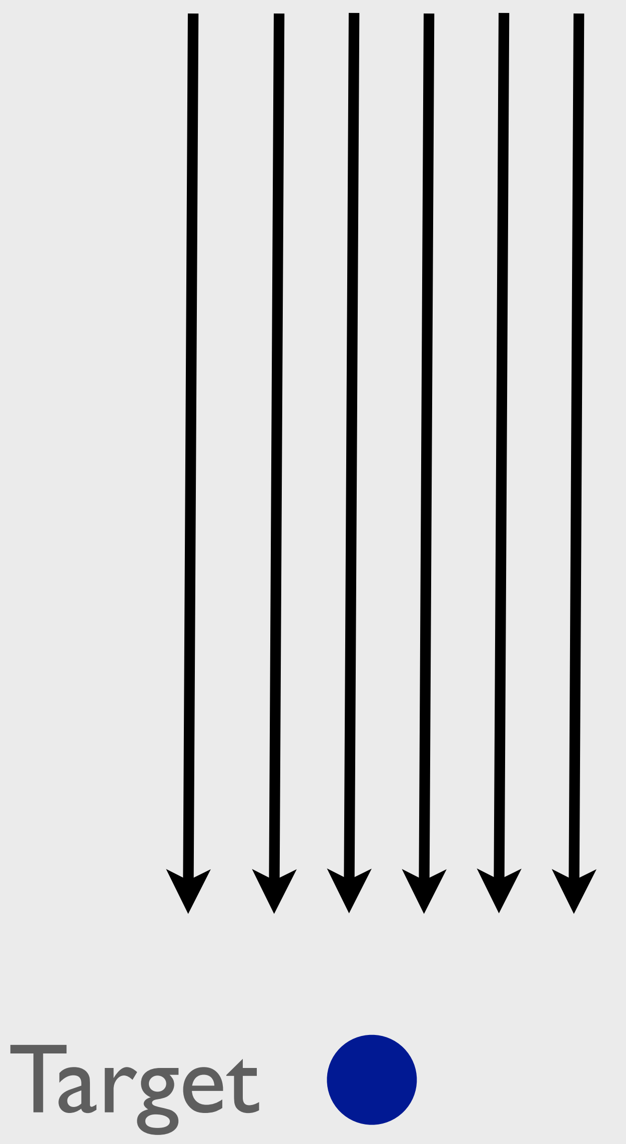
- cascade stops at $E_{\mathrm{part}} = E_{\mathrm{dec}}$
- each hadron produces one muon

(Matthews, Astropart.Phys. 22, 2005)

Superposition model – particle numbers

Nucleus
(binding energy ~5 MeV/nuc)

$$E_i = E_0/A$$



Proton-induced shower

$$N_{\max} \sim E_0/E_c$$

$$N_\mu = \left(\frac{E_0}{E_{\text{dec}}}\right)^\alpha \qquad \alpha \approx 0.9$$

Assumption:
nucleus of mass *A* and energy E_0 corresponds to *A* nucleons (protons) of energy $E_n = E_0/A$

$$N^A_{\max} \sim A\left(\frac{E_0}{AE_c}\right) = N_{\max}$$

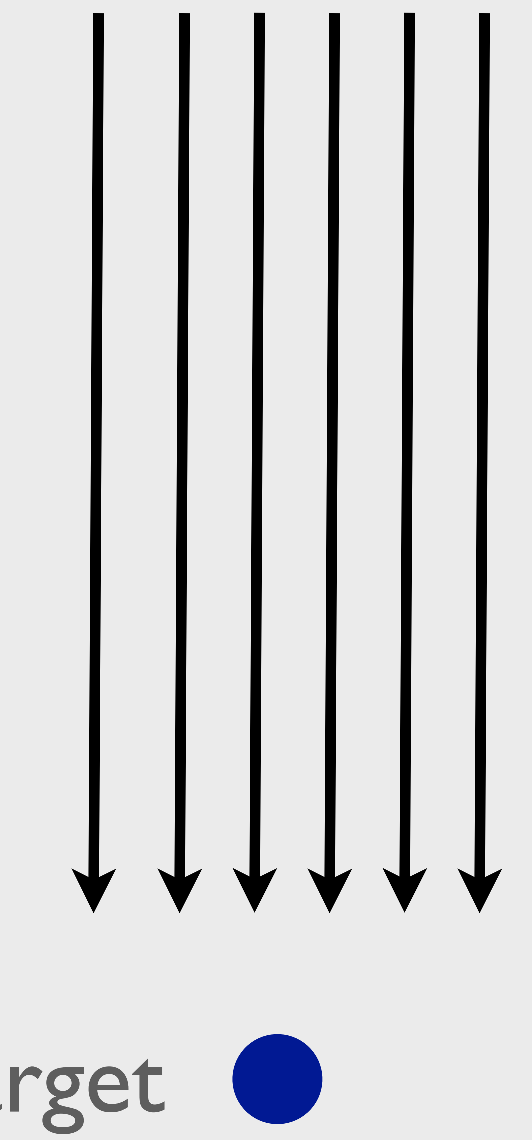
$$N^A_\mu = A\left(\frac{E_0}{AE_{\text{dec}}}\right)^\alpha = A^{1-\alpha} N_\mu$$

Iron showers ~40% more muons than proton showers

Superposition model – depth of shower maximum

Nucleus
(binding energy ~5 MeV/nuc)

$$E_i = E_0/A$$



Target

Proton-induced shower

$$X_{\max} \sim \lambda_{\text{eff}} \ln(E_0)$$

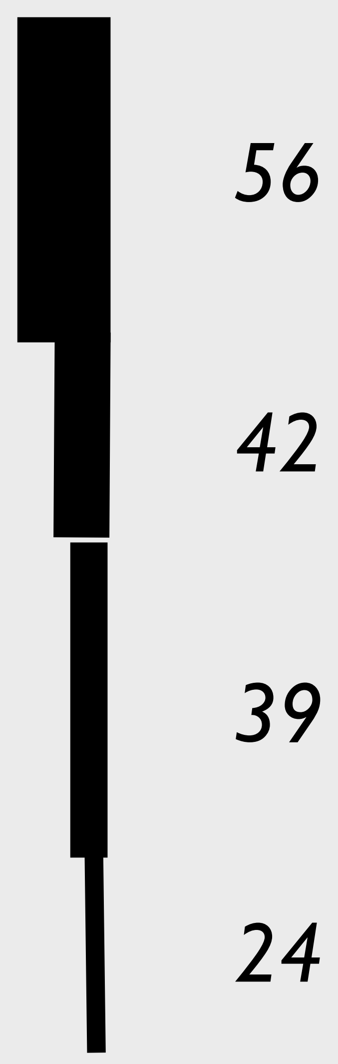
Assumption:
nucleus of mass A and energy E_0 corresponds to A nucleons (protons) of energy $E_n = E_0/A$

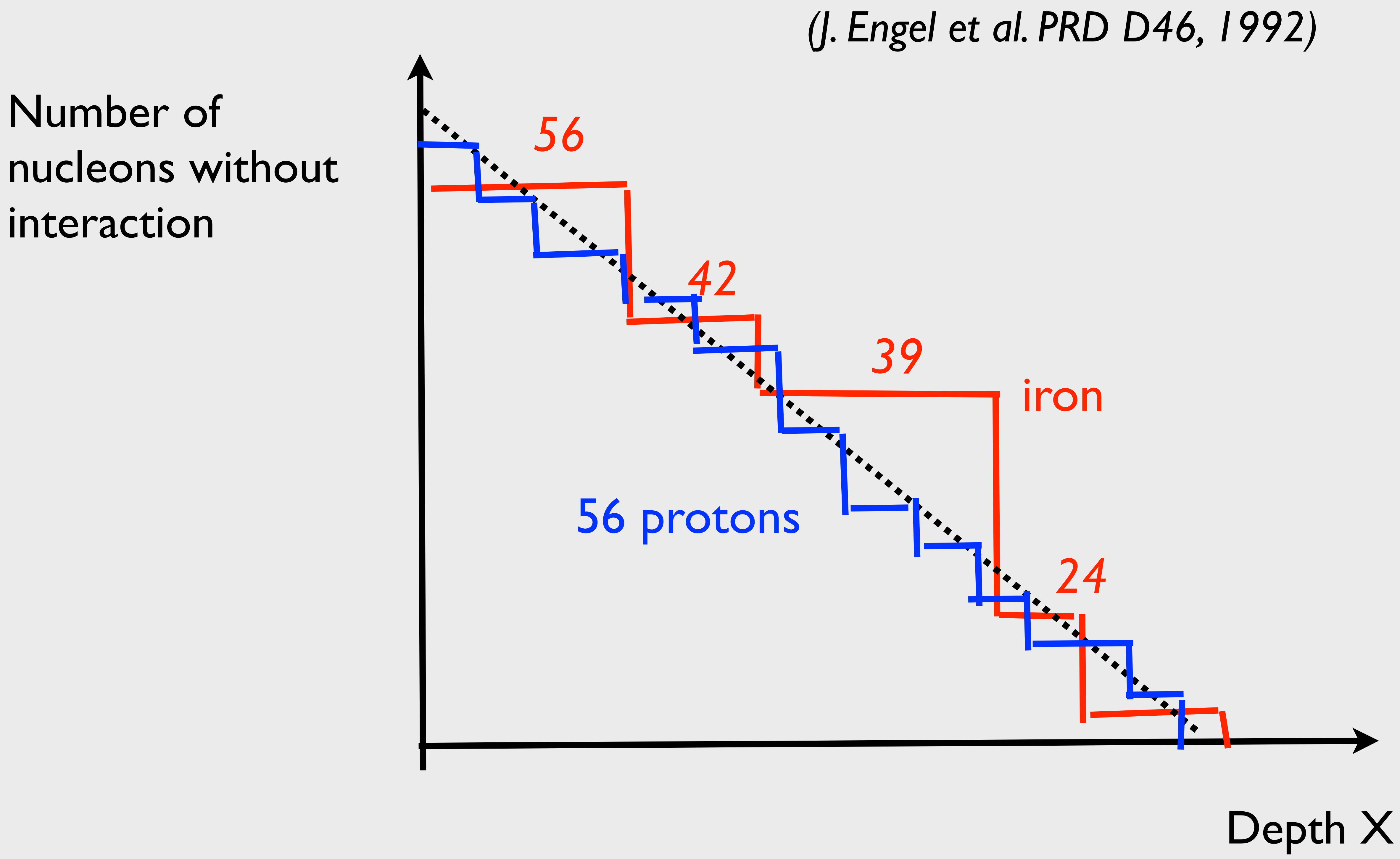
Proton showers penetrate deeper than iron showers ~ ln(A)

$$X^A_{\max} \sim \lambda_{\text{eff}} \ln(E_0/A)$$

Superposition and semi-superposition models

iron nucleus





Used in Sibyll interaction model

Jonathan ≠ Ralph

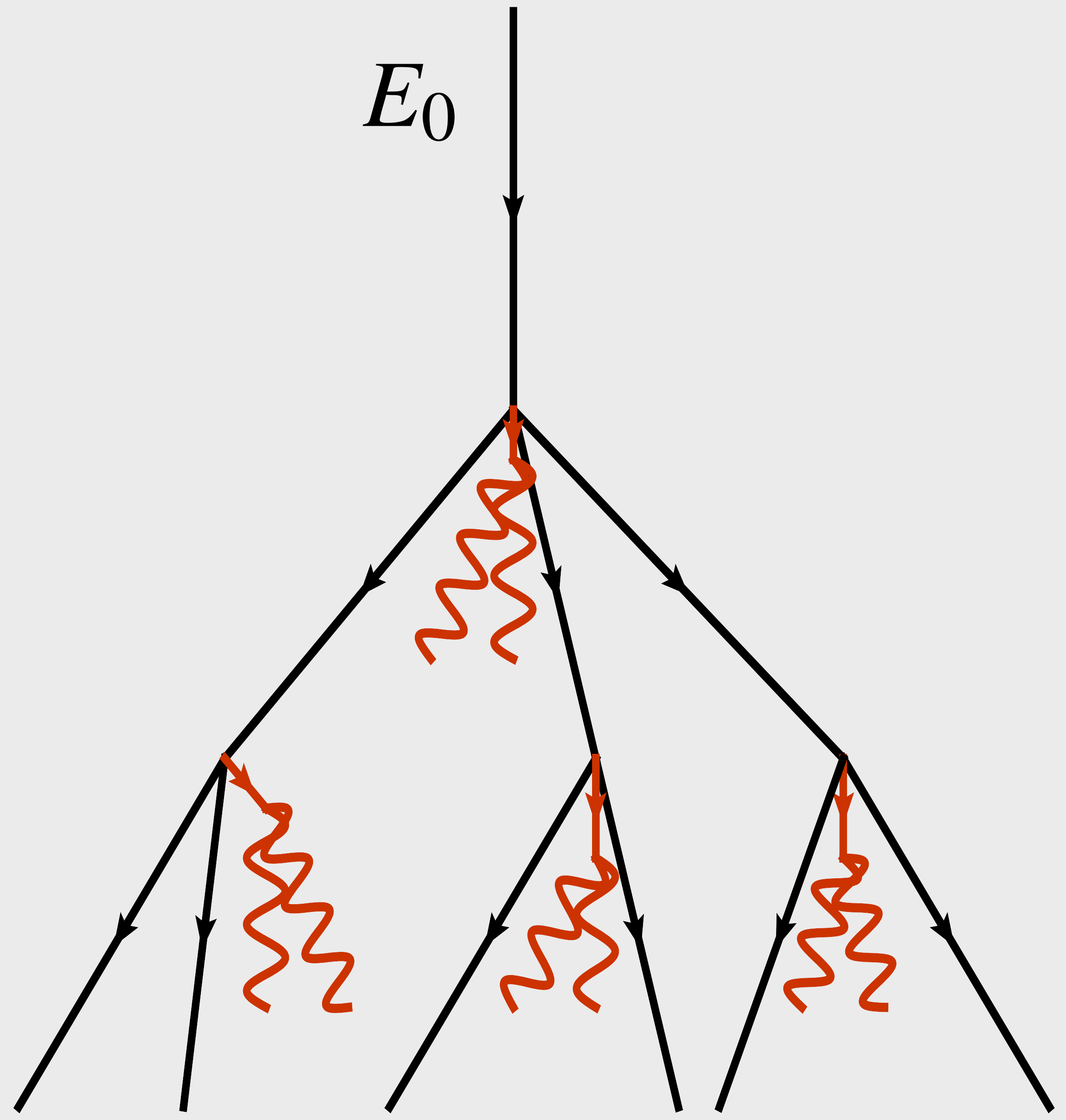
Glauber approximation (unitarity)

$$\sigma_{\mathrm{Fe-air}} = \left(\frac{A}{n_{\mathrm{part}}}\right) \sigma_{\mathrm{p-air}}$$

Average depth distribution of nucleon interaction points correctly described

5. Energy transfer to em. component

Electromagnetic energy and energy transfer



Hadronic energy

$$\frac{2}{3}E_0$$

$$\frac{2}{3}\left(\frac{2}{3}E_0\right)$$

$$\vdots$$

$$E_{\rm had} = \left(\frac{2}{3}\right)^n E_0$$

Electromagnetic energy

$$\frac{1}{3}E_0$$

$$\frac{1}{3}E_0 + \frac{1}{3}\left(\frac{2}{3}E_0\right)$$

$$\vdots$$

$$E_{\rm em} = \left[1 - \left(\frac{2}{3}\right)^n\right] E_0$$

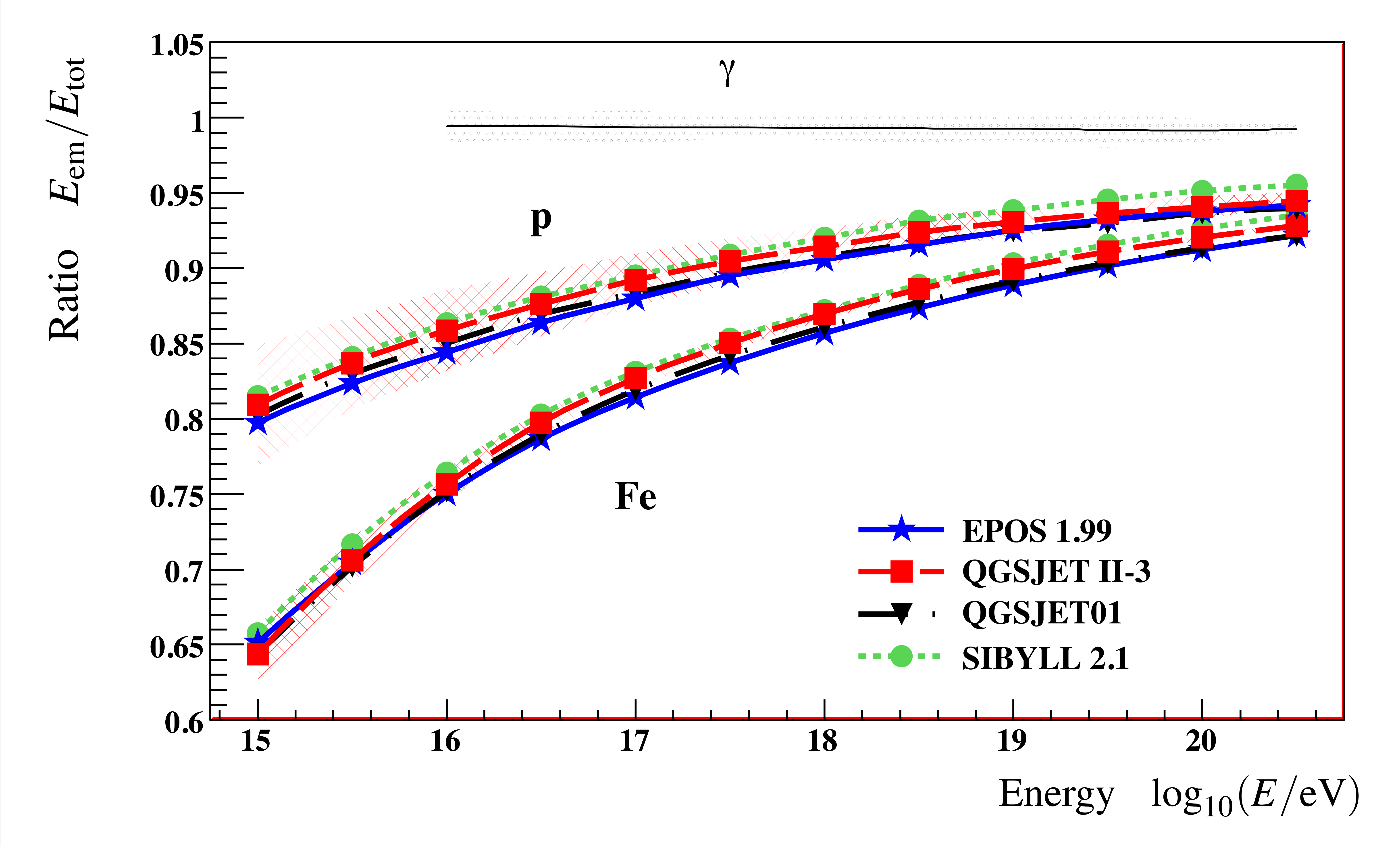
After n generations ...

$n = 5, \quad E_{\rm had} \sim 12\%$

$n = 6, \quad E_{\rm had} \sim 8\%$

Energy transferred to electromagnetic component

(RE, Pierog, Heck, ARNPS 2011)



Ratio of em. to total shower energy

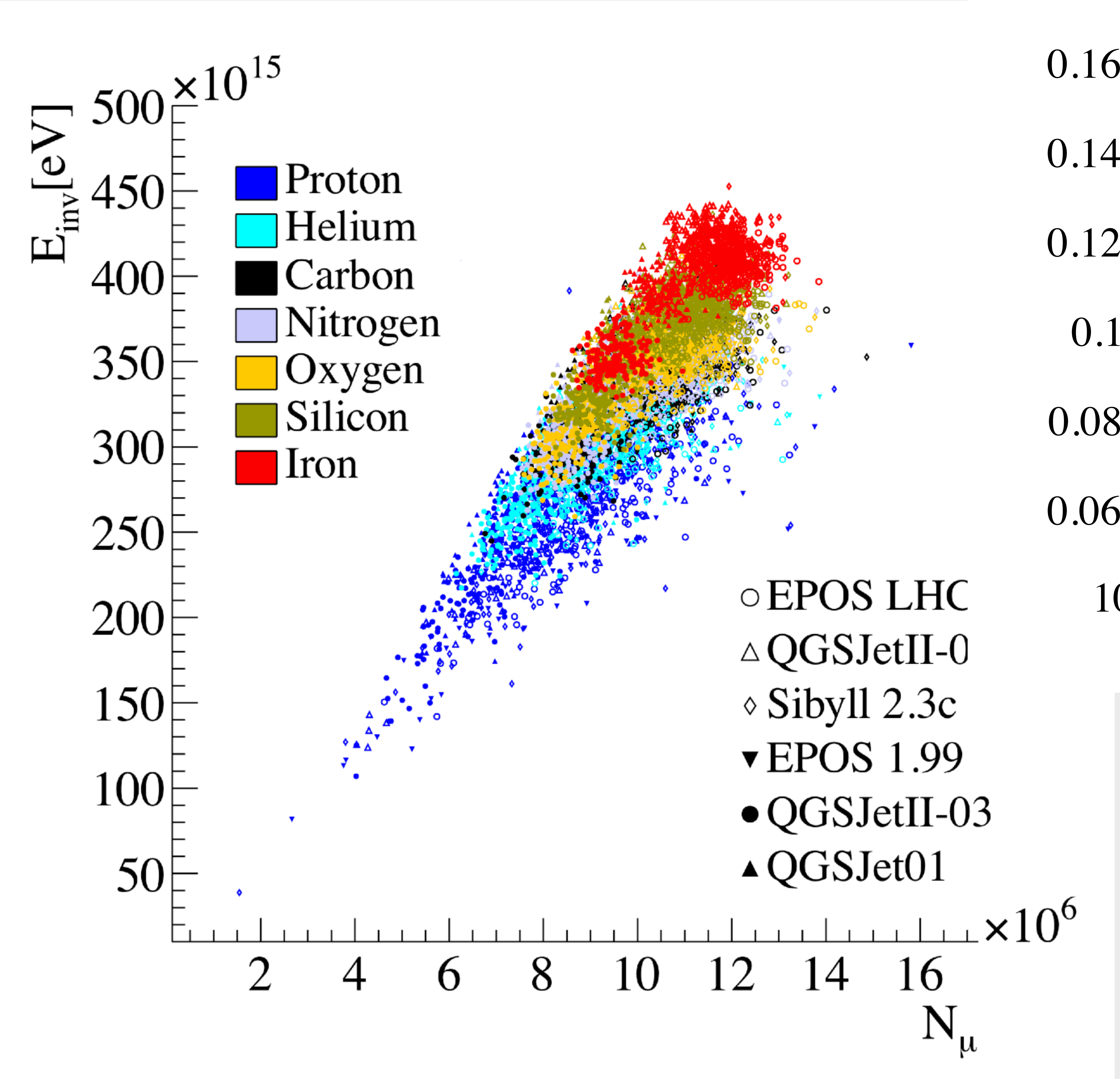
Detailed Monte Carlo simulation with CONEX

$$E_{inv} = E_{tot} - E_{em}$$

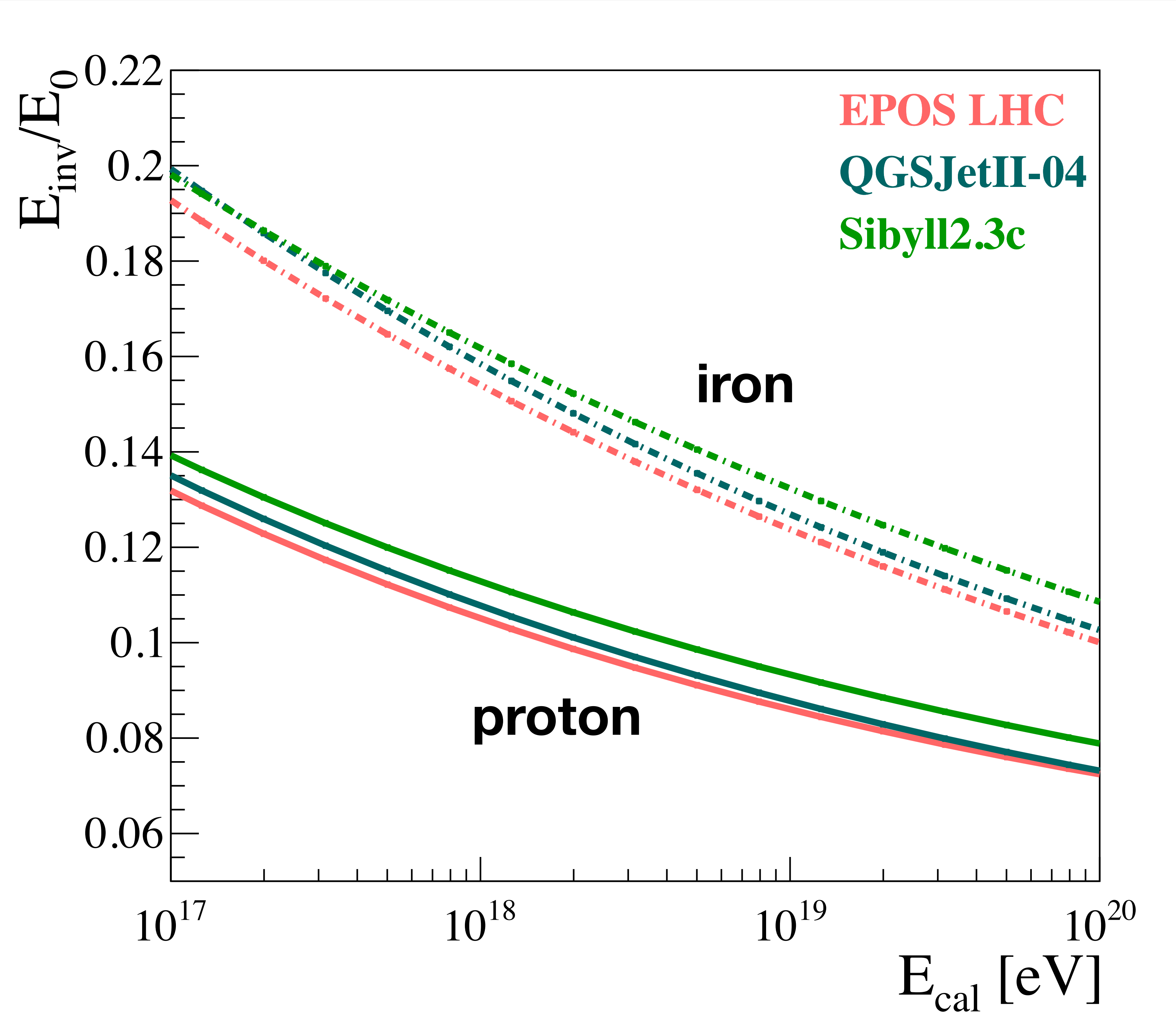
At high energy: model dependence of correction to obtain total energy small

Muons as tracers of the hadronic core

Very good correlation between muon number and invisible energy

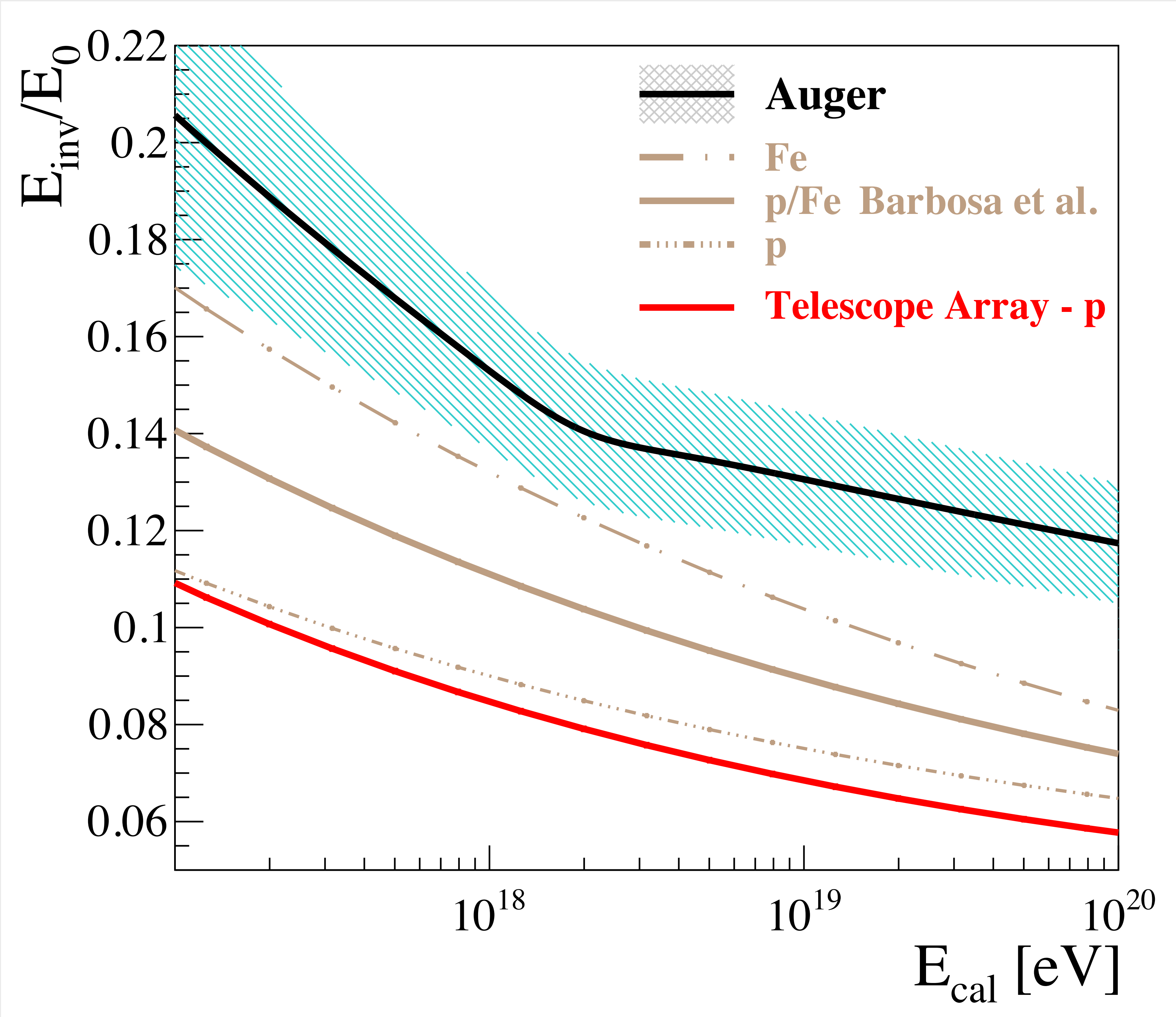


Most recent model predictions



$$E_{\rm inv} = E_{\rm tot} - E_{\rm em}$$

(Auger, PRD 2019)

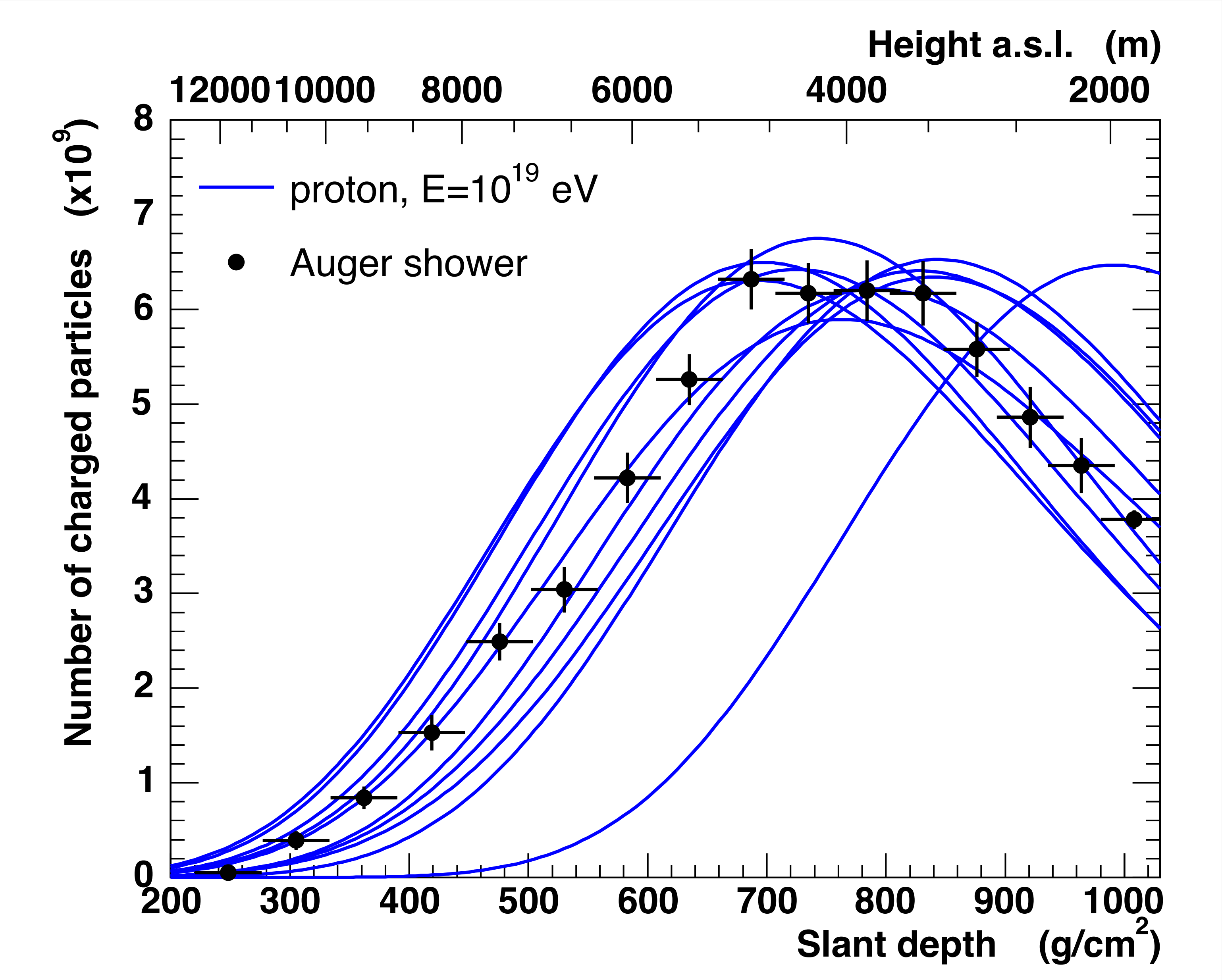


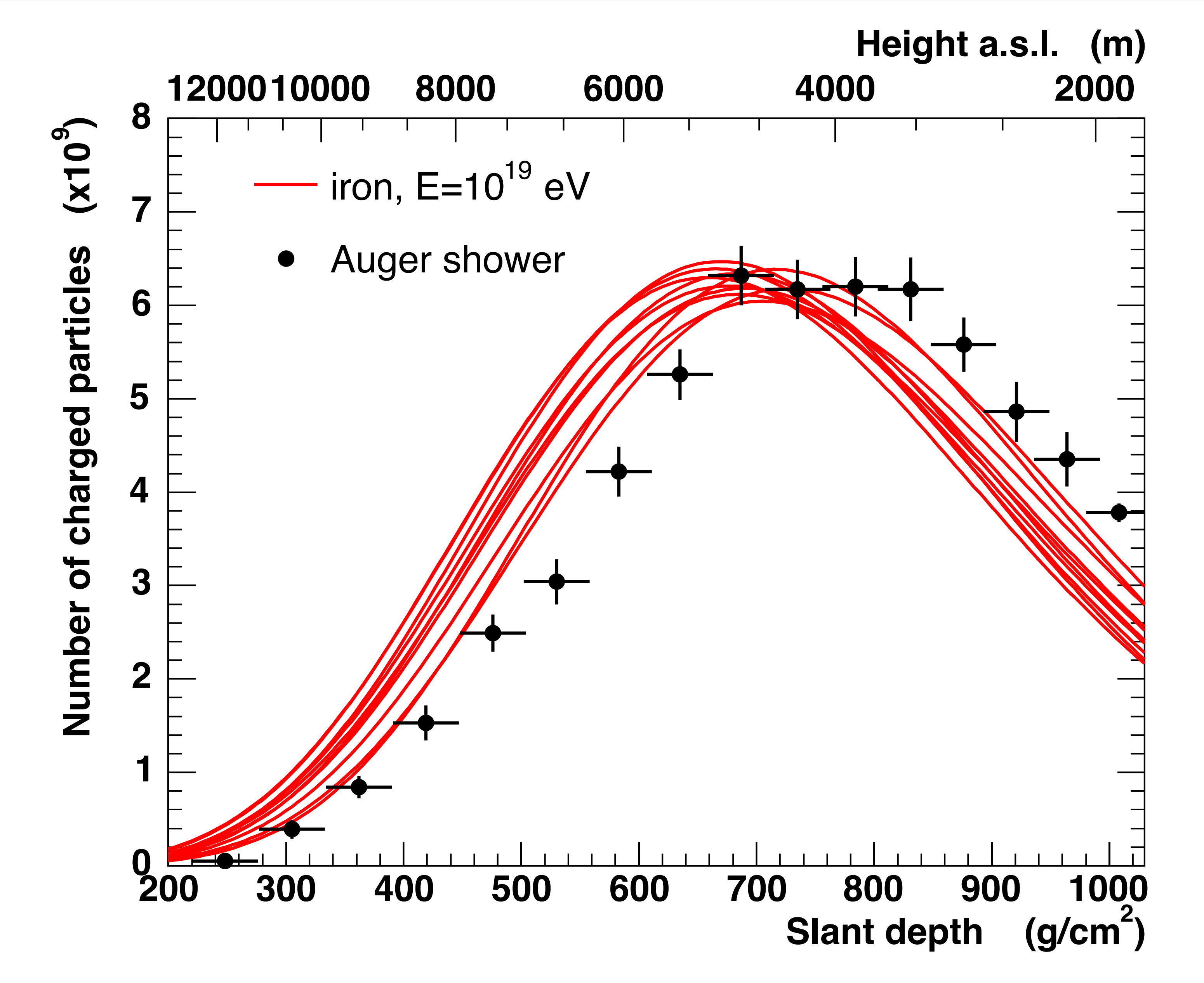
Muon-data based correction for invisible energy used in Auger

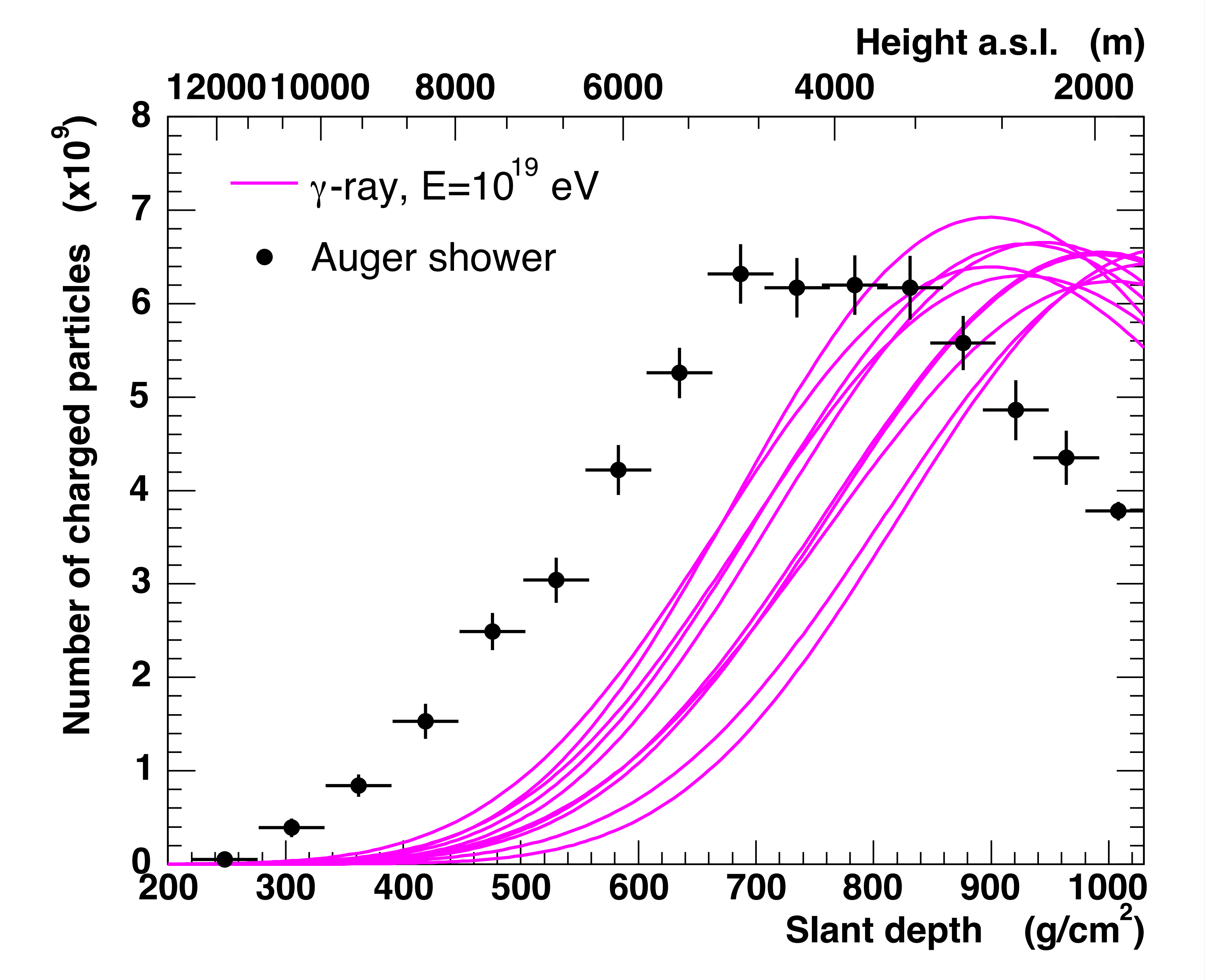
6. Elongation rate theorem

Longitudinal shower profiles: simulations and data

Comparison to event observed by Auger







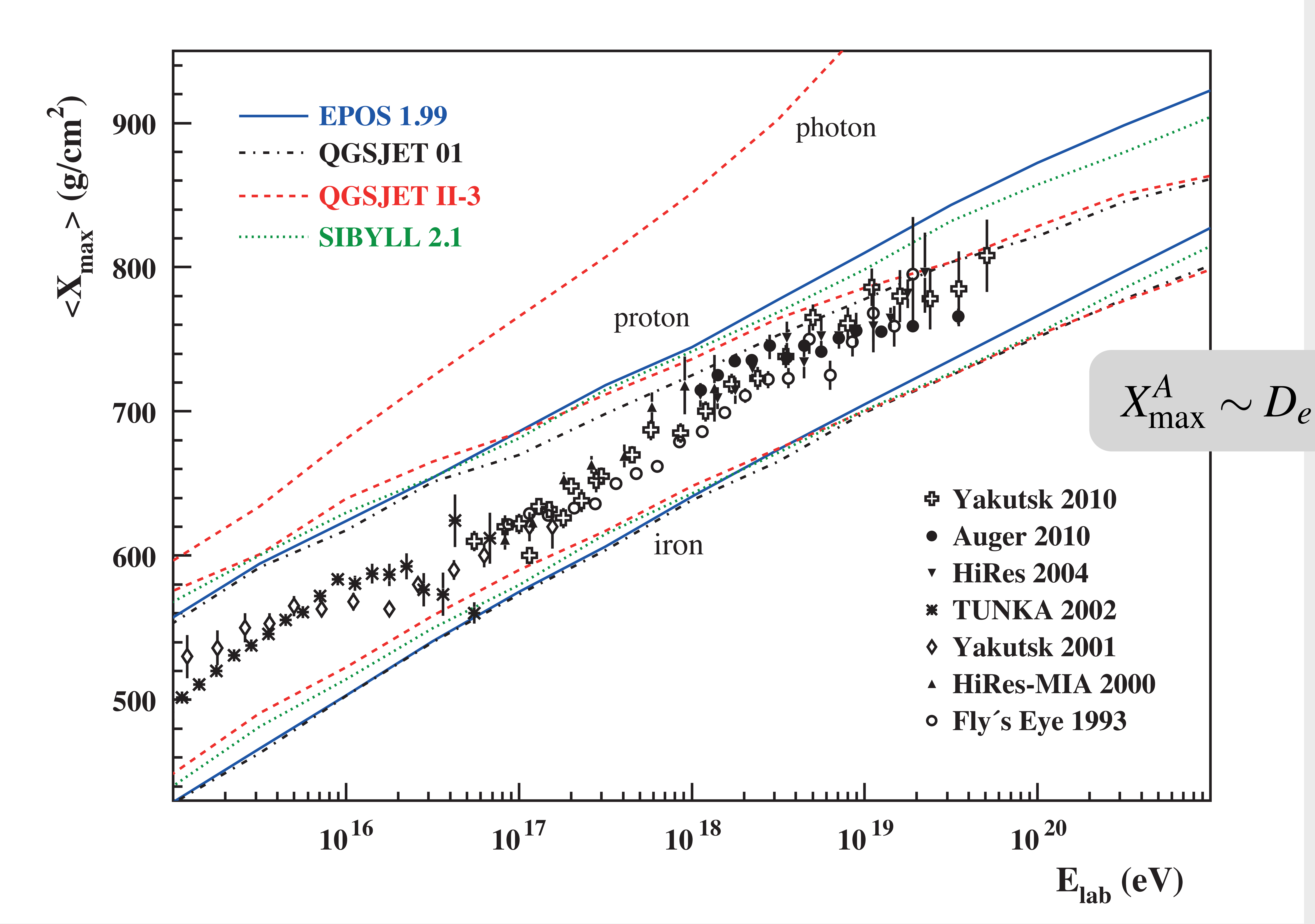
$$N_{\max} = E_0/E_c$$

$$X_{\max} \sim D_e \ln(E_0/E_c)$$

Superposition model:

$$X_{\max}^A \sim D_e \ln(E_0/AE_c)$$

Mean depth of shower maximum

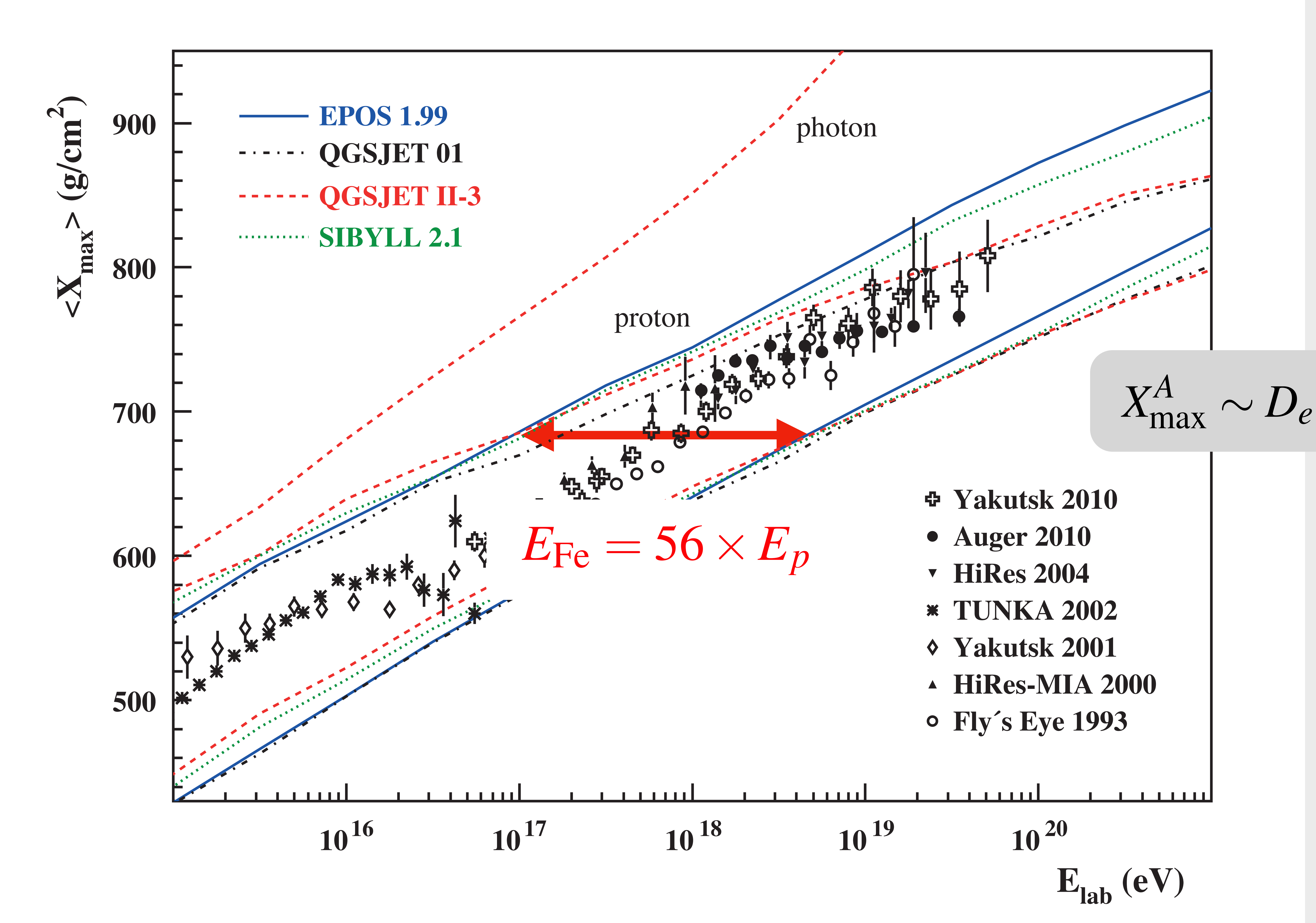


$$X^A_{\rm max} \sim D_e \ln(E_0/AE_c)$$

Note: old data and model predictions (just for clarity)

(RE, Pierog, Heck, ARNPS 2011)

Mean depth of shower maximum



$E_{\rm Fe} = 56 \times E_p$

$X^A_{\rm max} \sim D_e \ln(E_0/AE_c)$

Note: old data and model predictions (just for clarity)

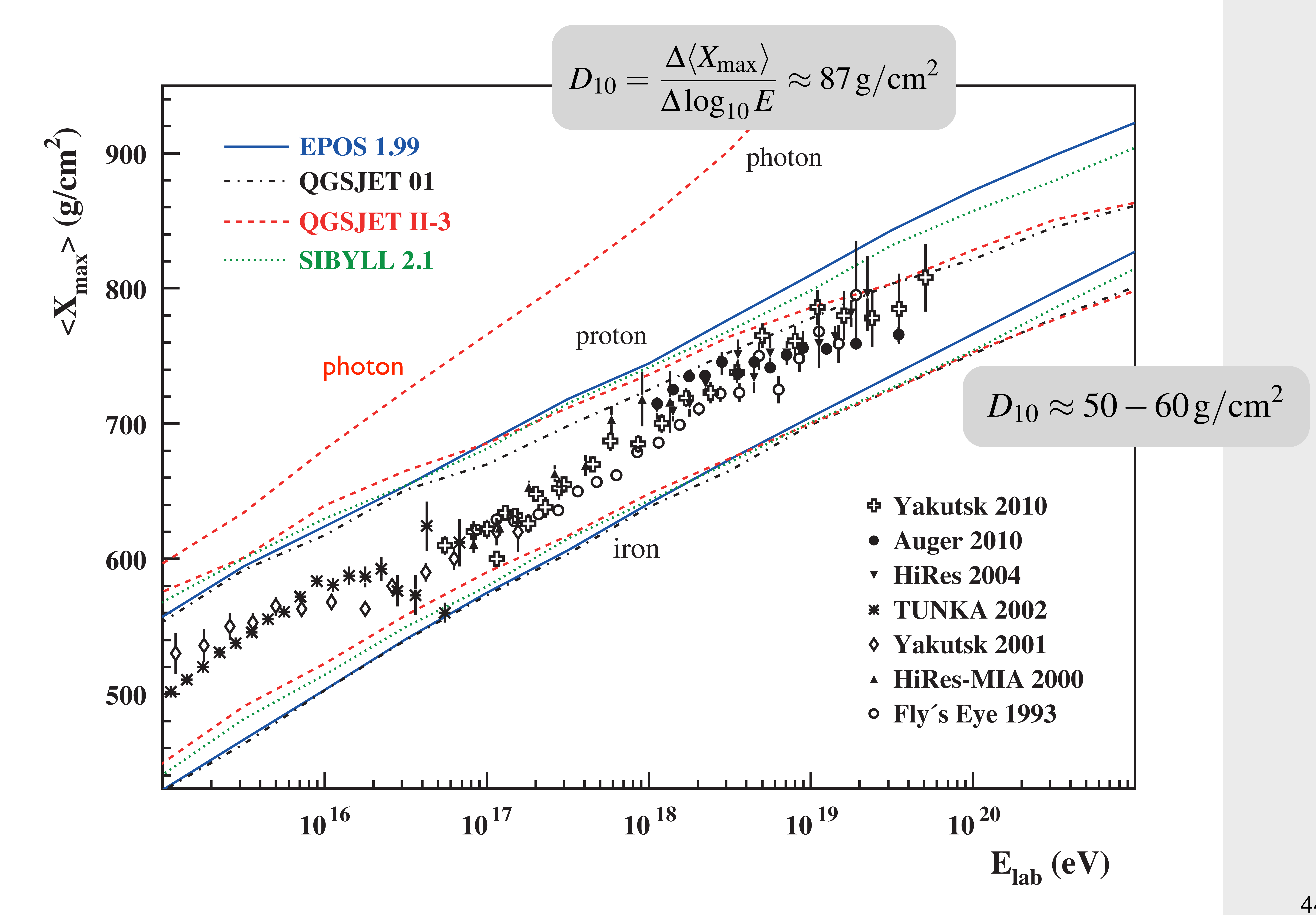
(RE, Pierog, Heck, ARNPS 2011)

Shower elongation rate

$$D_{10} = \frac{\Delta \langle X_{max} \rangle}{\Delta \log_{10} E}$$

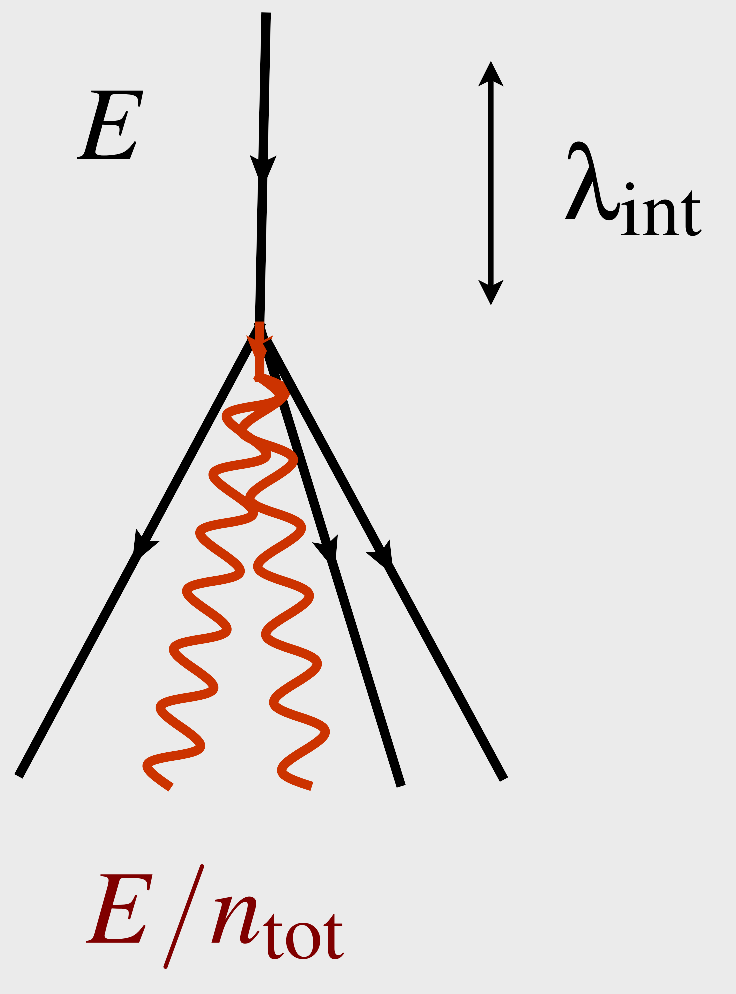
$$D_e = \frac{\Delta \langle X_{max} \rangle}{\Delta \ln E}$$

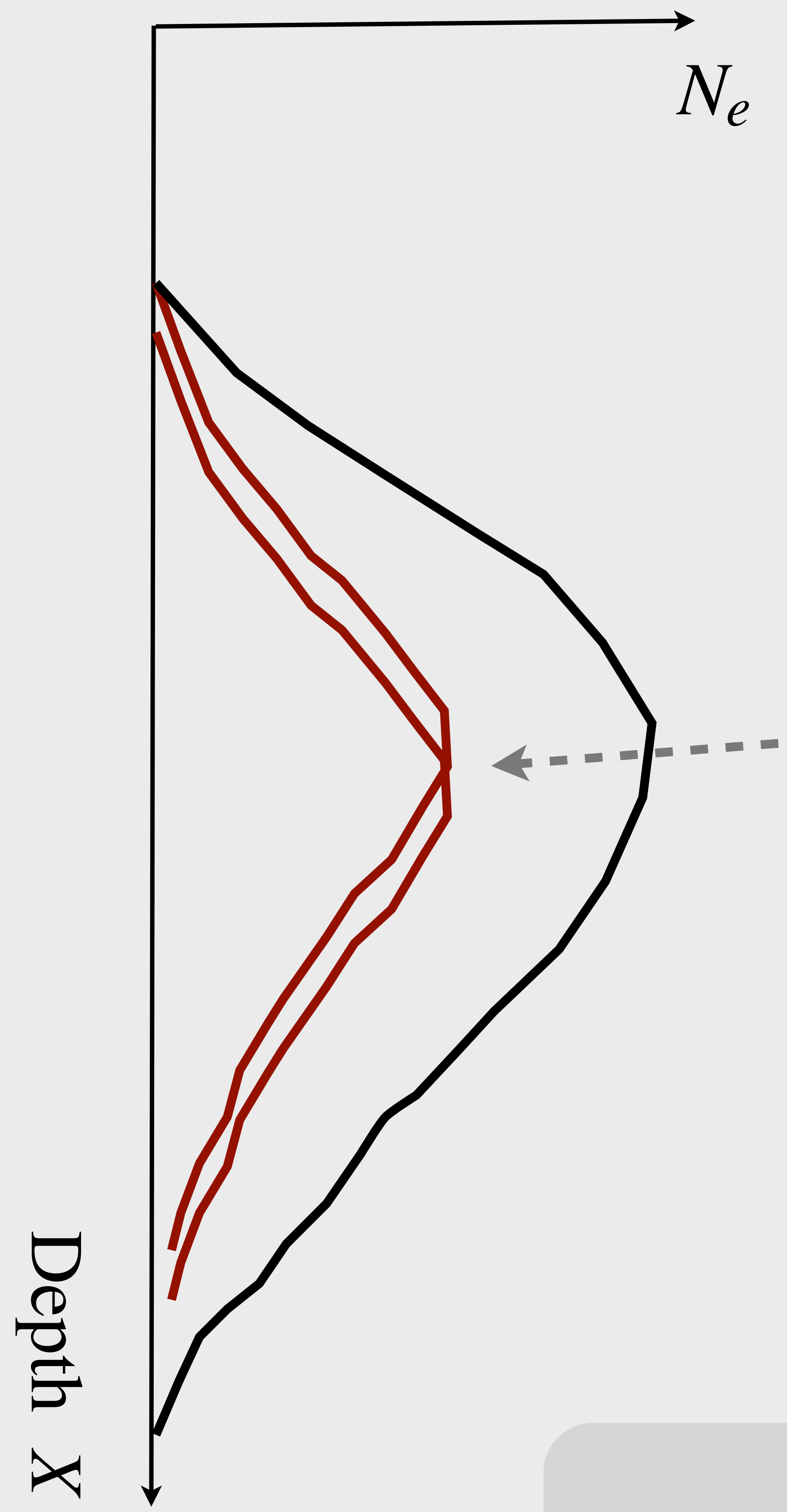
$$D_{10} = \log(10) D_e$$



(RE, Pierog, Heck, ARNPS 2011)

Derivation of elongation rate theorem





$$\langle X_{max}(E)\rangle \;=\; \langle X^{em}_{max}(E/n_{tot})\rangle \;+\; \lambda_{int}$$

em. cascade theory

$$\langle X^{em}_{max}\rangle \sim X_0 \ln(E/n_{tot})$$

$$\langle X_{max}(E)\rangle \;=\; X_0 \;\ln(E/n_{tot}) + c \;+\; \lambda_{int}$$

taking derivative $\log E$

$$D_e = \frac{\mathrm{d}\langle X_{max}(E)\rangle}{\mathrm{d}\ln E} \;\leq\; X_0 \;-\; X_0 \frac{\mathrm{d}\,\ln n_{tot}}{\mathrm{d}\ln E} \;+\; \frac{\mathrm{d}\lambda_{int}}{\mathrm{d}\ln E}$$

Elongation rate of em. shower

Elongation rate theorem

X_0 = 36 g/cm²

$$D_e^{\mathrm{had}} = X_0(1 - B_n - B_\lambda)$$

(Linsley, Watson PRL46, 1981)

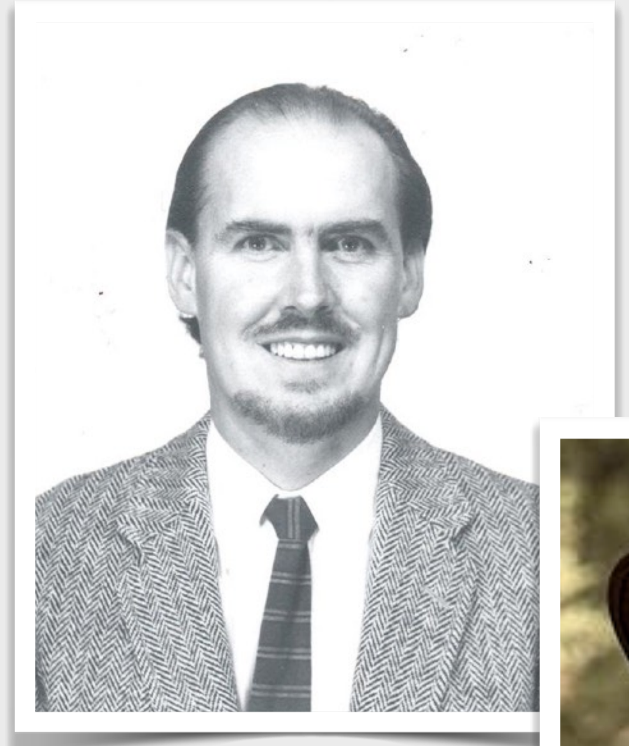
$$B_n = \frac{d \ln n_{\mathrm{tot}}}{d \ln E}$$

Large if multiplicity of high energy particles rises very fast, **zero in case of scaling**

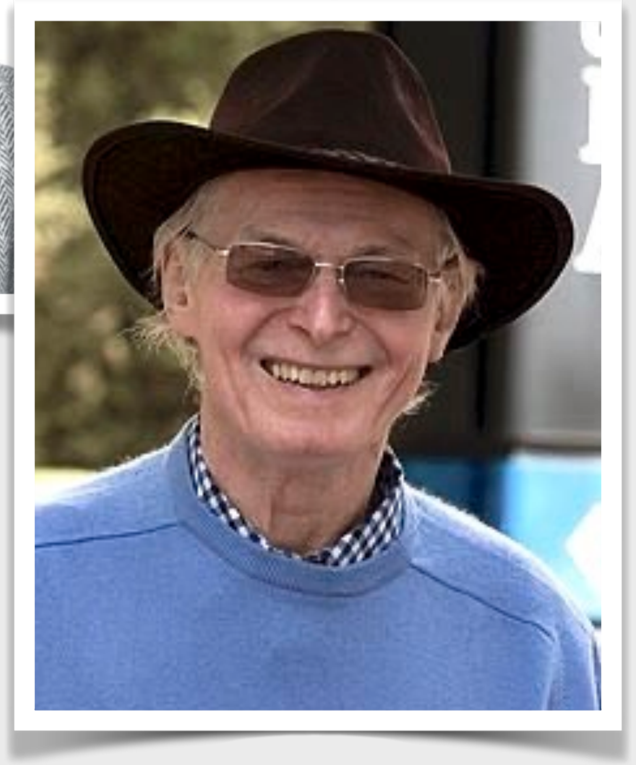
$$B_\lambda = -\frac{1}{X_0}\frac{d\lambda_{\mathrm{int}}}{d \ln E}$$

Large if cross section rises rapidly with energy

Note: $D_{10} = \log(10) D_e$

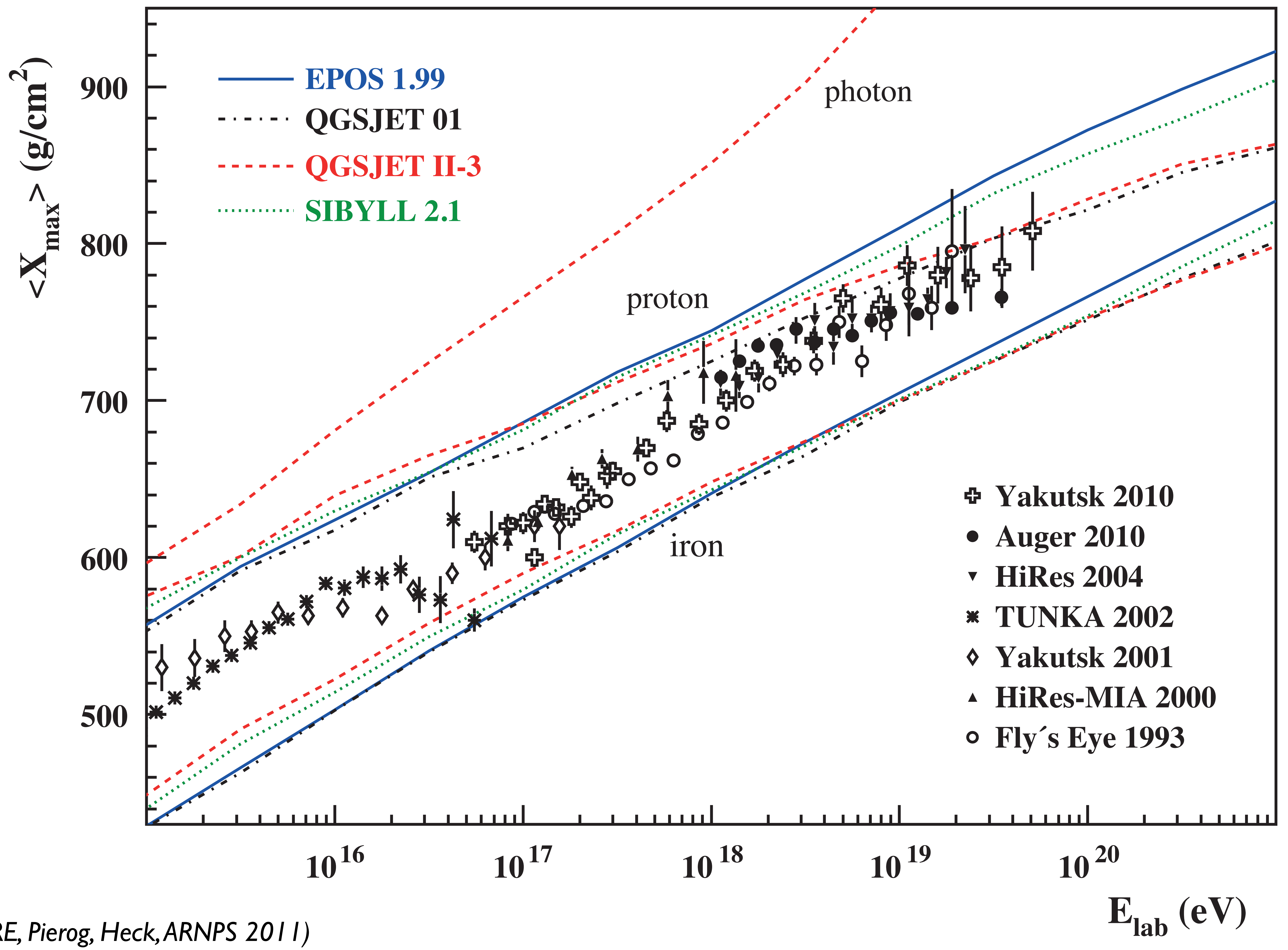


John Linsley



Alan Watson

Mean depth of shower maximum



(RE, Pierog, Heck, ARNPS 2011)

QGSJET predicts very strong scaling violations

Elongation rates and model features

Elongation rate theorem

$$D_{10}^{\text{had}} = \ln 10\, X_0 (1 - B_n - B_\lambda)$$

(Linsley, Watson PRL46, 1981)

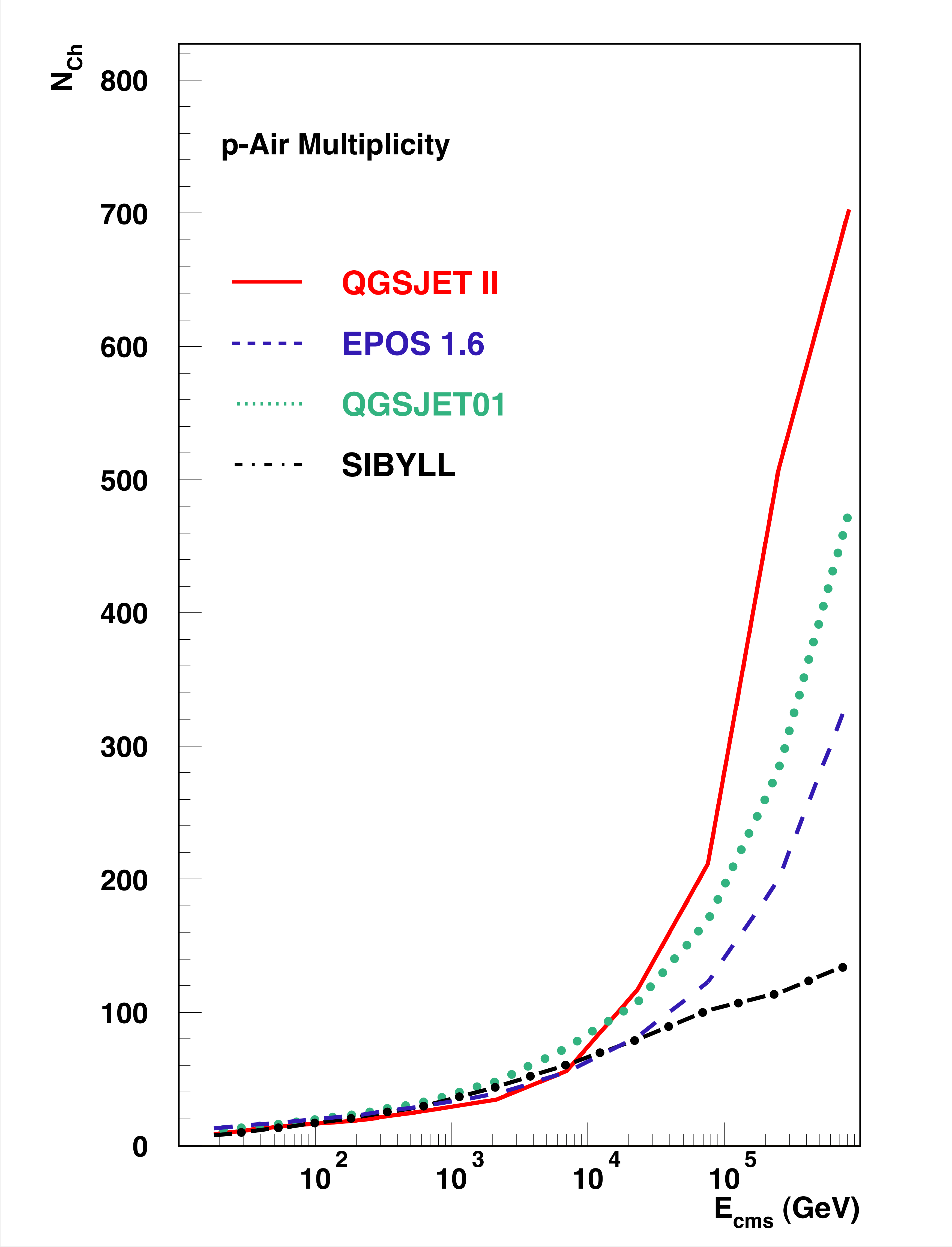
factor ~ 87 g/cm²

$$B_n = \frac{d \ln n_{\text{tot}}}{d \ln E}$$

Large if multiplicity of high energy particles rises very fast, **zero in case of scaling**

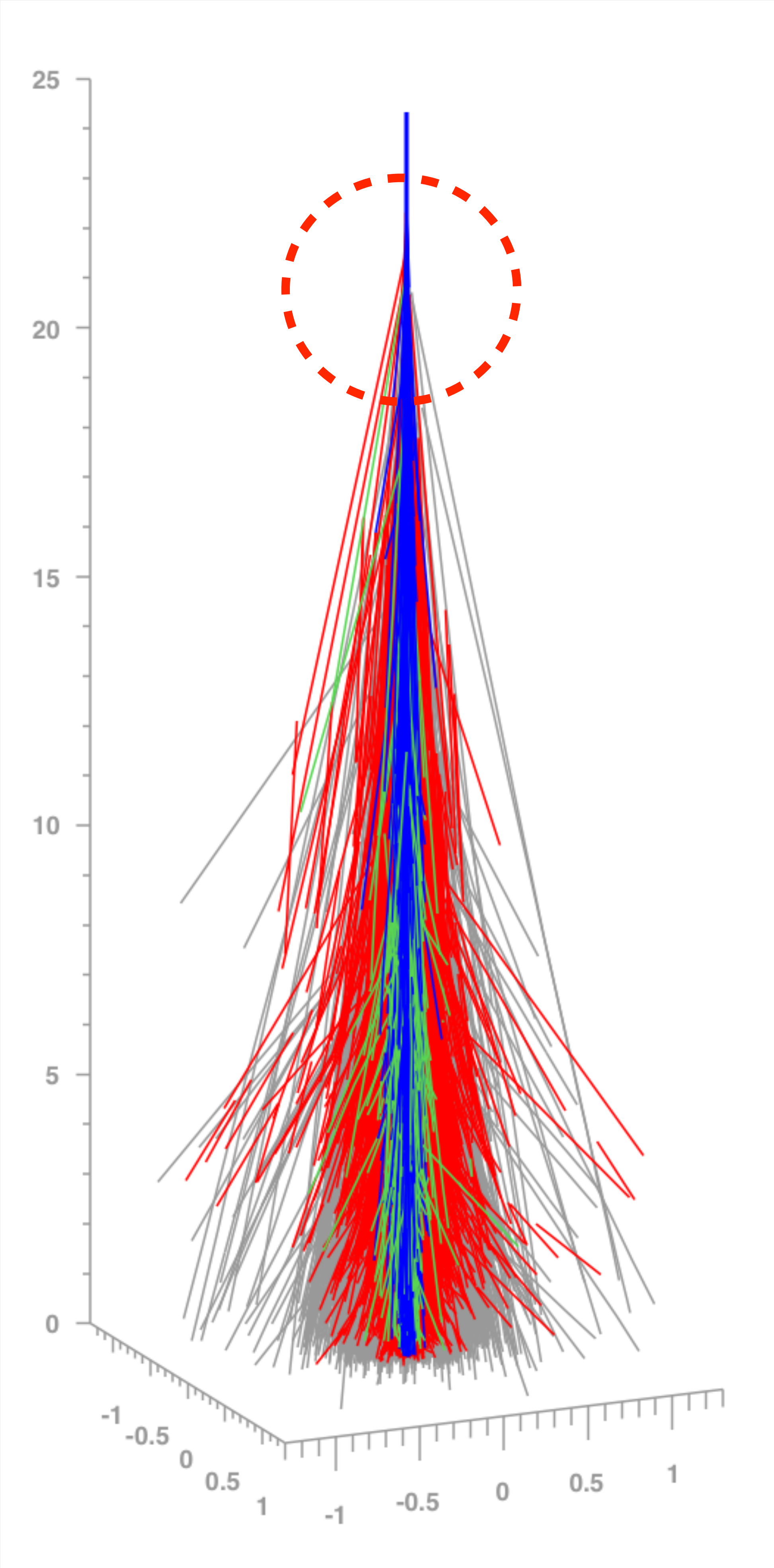
$$B_\lambda = -\frac{1}{X_0} \frac{d\lambda_{\text{int}}}{d \ln E}$$

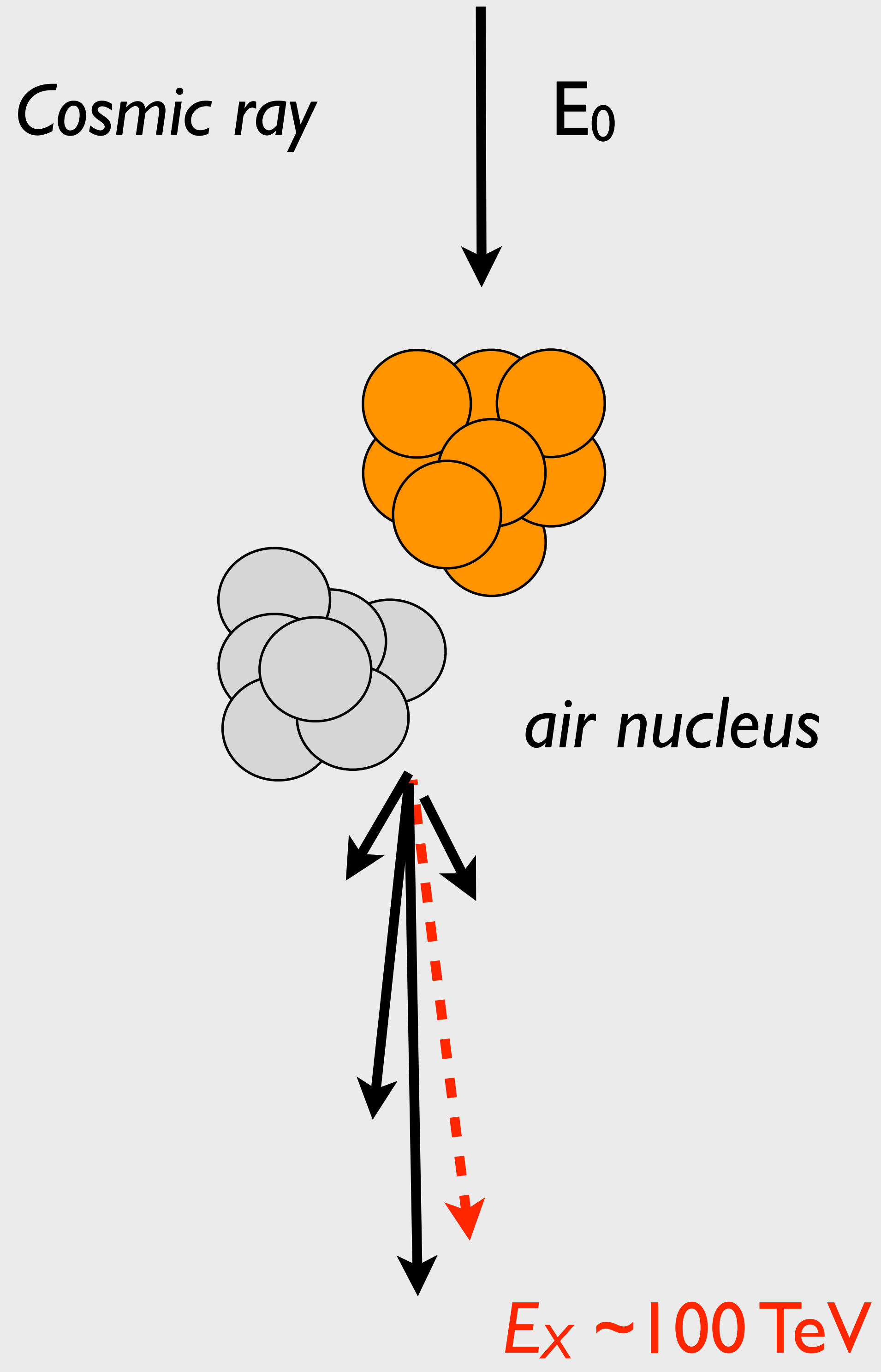
Large if cross section rises rapidly with energy



Backup slides

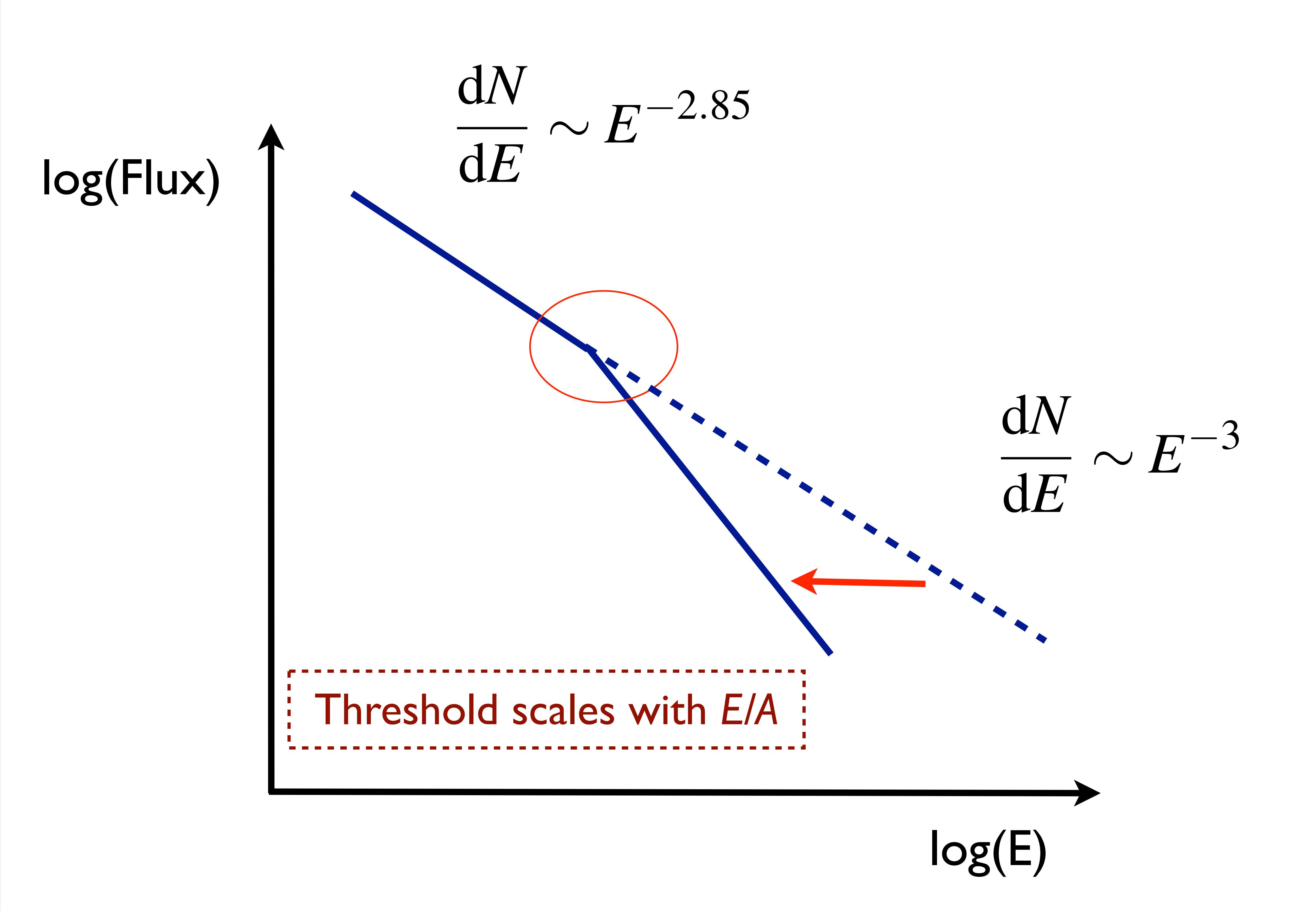
Exotic models for the knee of cosmic ray spectrum





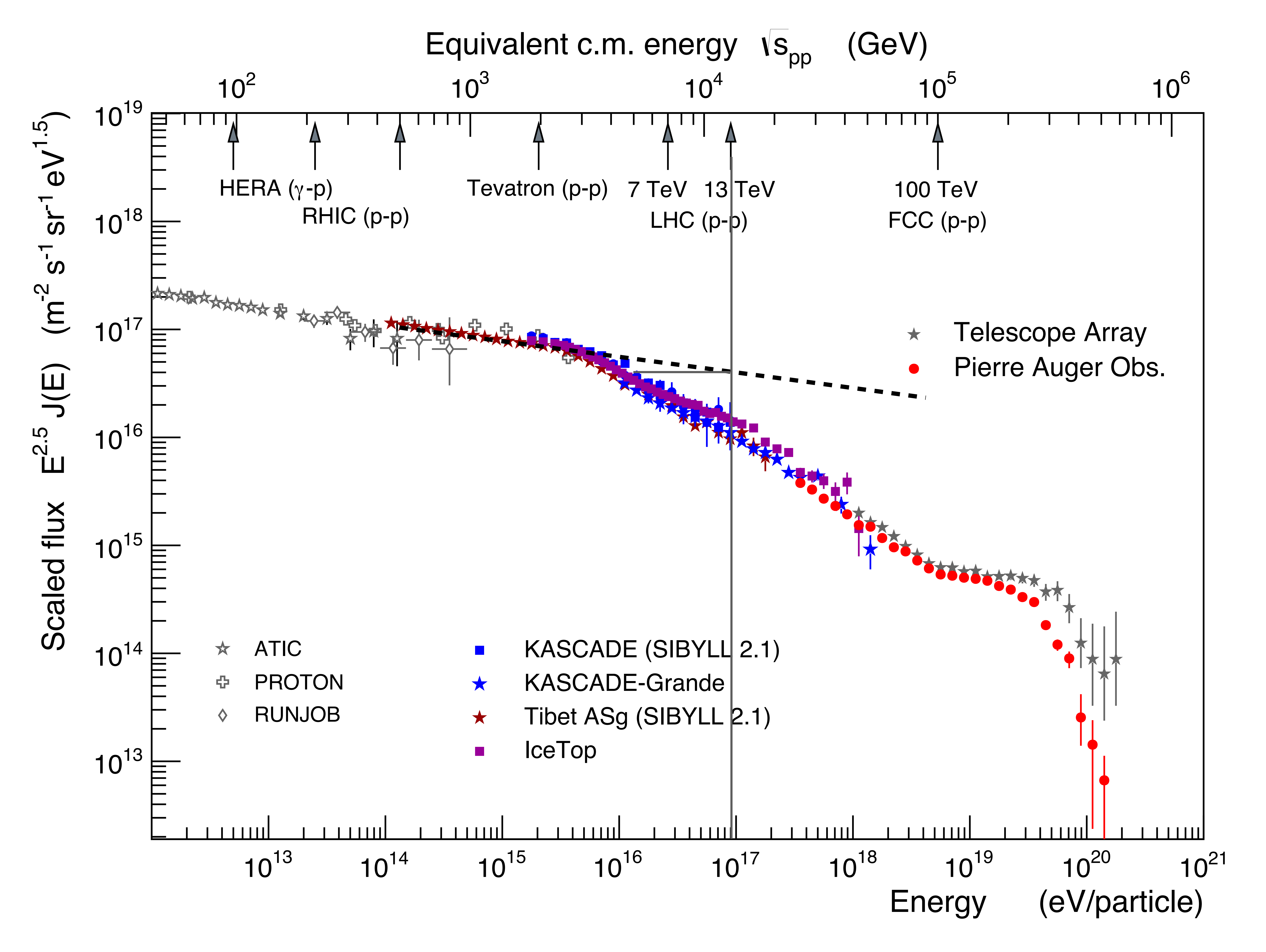
Petrukhin, NPB 151 (2006) 57
Barcelo at al. JACP 06 (2009) 027
Dixit et al. EPJC 68 (2010) 573
Petrukhin NPB 212 (2011) 235

Knee due to wrong energy reconstruction of showers?



New physics: scaling with nucleon-nucleon cms energy

Cosmic ray flux and interaction energies



LHC at 13 TeV cms

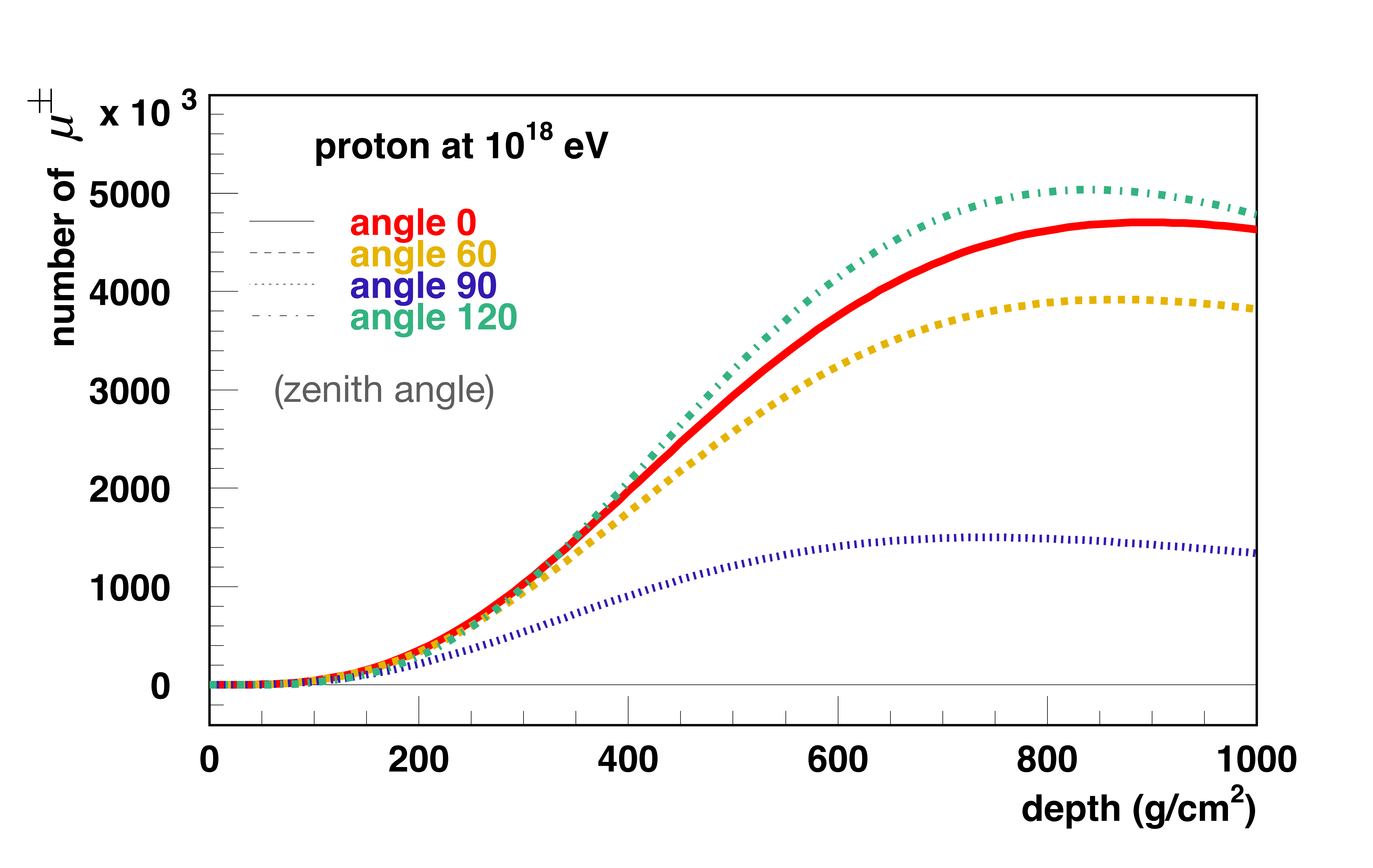
~~About 70%~~ of energy has to be transferred to invisible particles

No sign for change of hadronic interactions seen at LHC

Effect of air density (number of generations)

(Bergmann et al, APP 26, 2007)

$$N_\mu = \left(\frac{E_0}{E_{\text{dec}}}\right)^\alpha$$



Pion decay energy depends on air density, low density corresponds to large E_{dec}

Electromagnetic showers are independent of air density, hadronic showers not