

Asymptotic safety and the road towards running relational observables

Renata Ferrero

based on work in collaboration with Kevin Falls and Martin Reuter

February 22nd 2024

11th TUX Workshop in Quantum Gravity



FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG

Plan of the talk

Plan of the talk

Asymptotic Safety

Effective Average Action

Functional Renormalization Group

Quantum Einstein Gravity

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Asymptotic Safety

Effective Average Action

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Quantum Einstein Gravity

Running relational observables

Composite operators

Covariant formulation

Flow of the observables

Effective Average Action &
Functional Renormalization Group

Asymptotic Safety

Reuter-Saueressig and Percacci's book

The RG

Callan-Symanzik equation

RG in perturbation theory

only the finitely many beta functions that are
related to the relevant couplings are considered

Callan (1970), Symanzik (1970)

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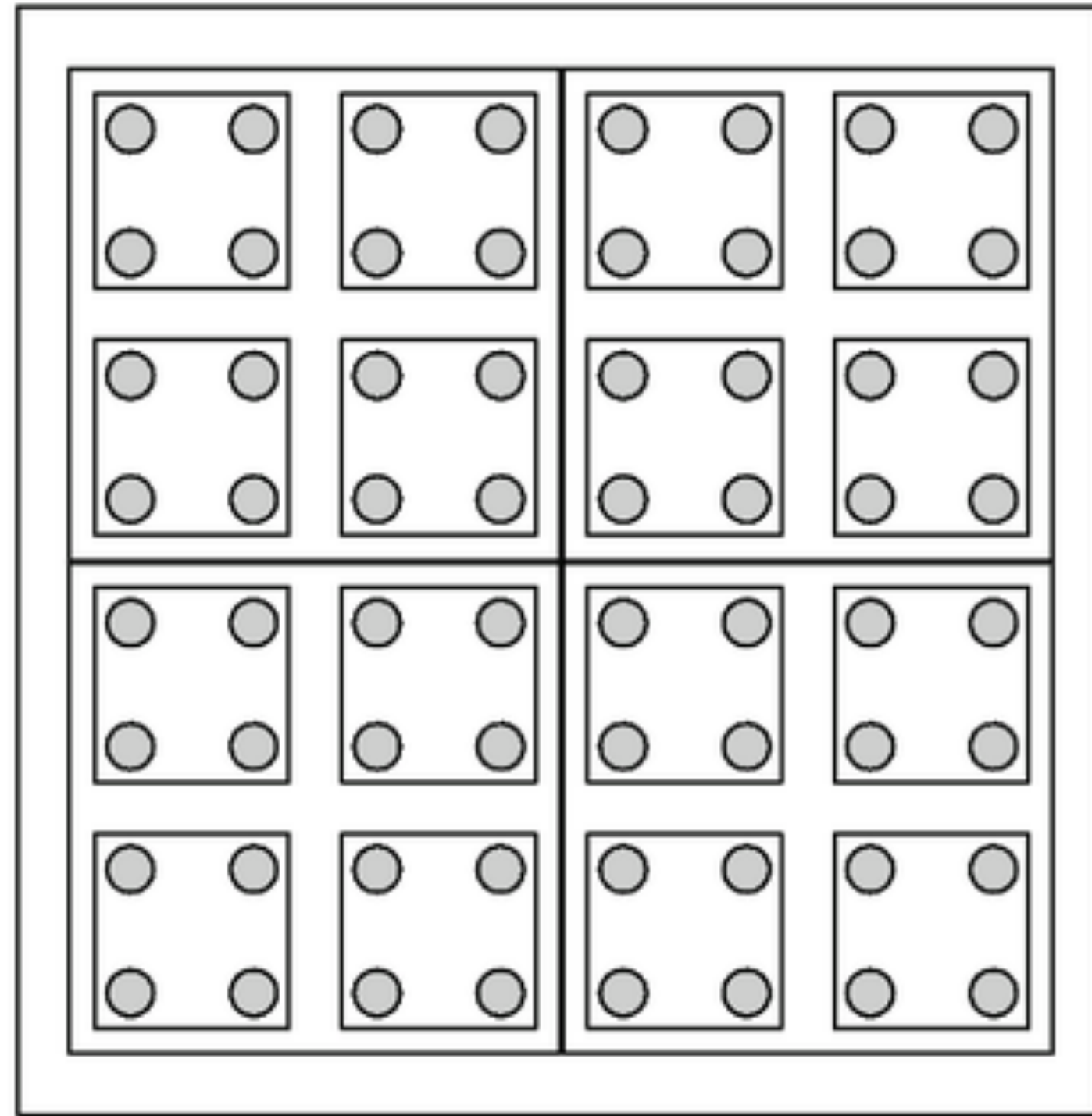
The RG

Exact RGE

Wilsonian Exact RG

the quantum fluctuations in the path integral
can be integrated out progressively

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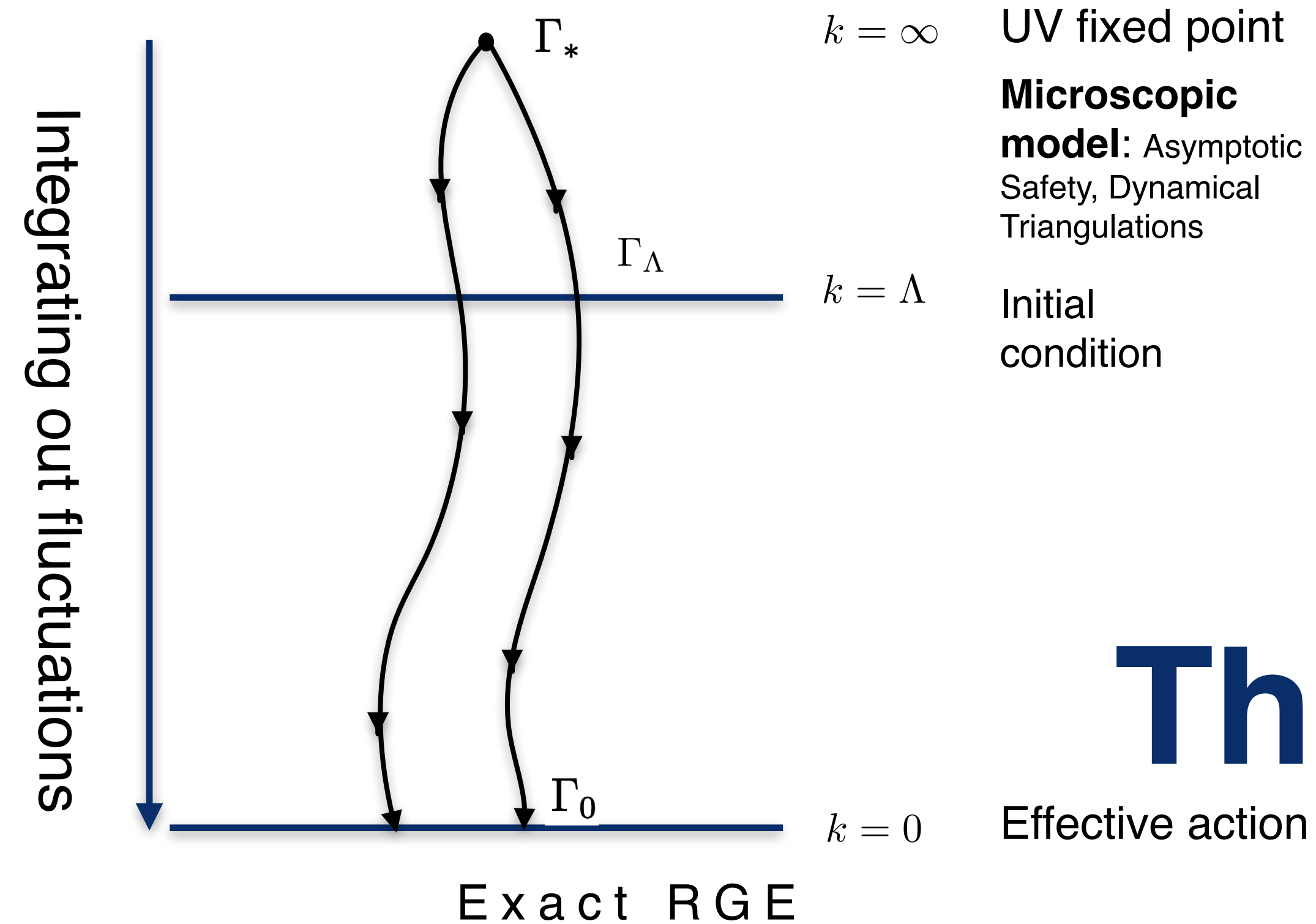
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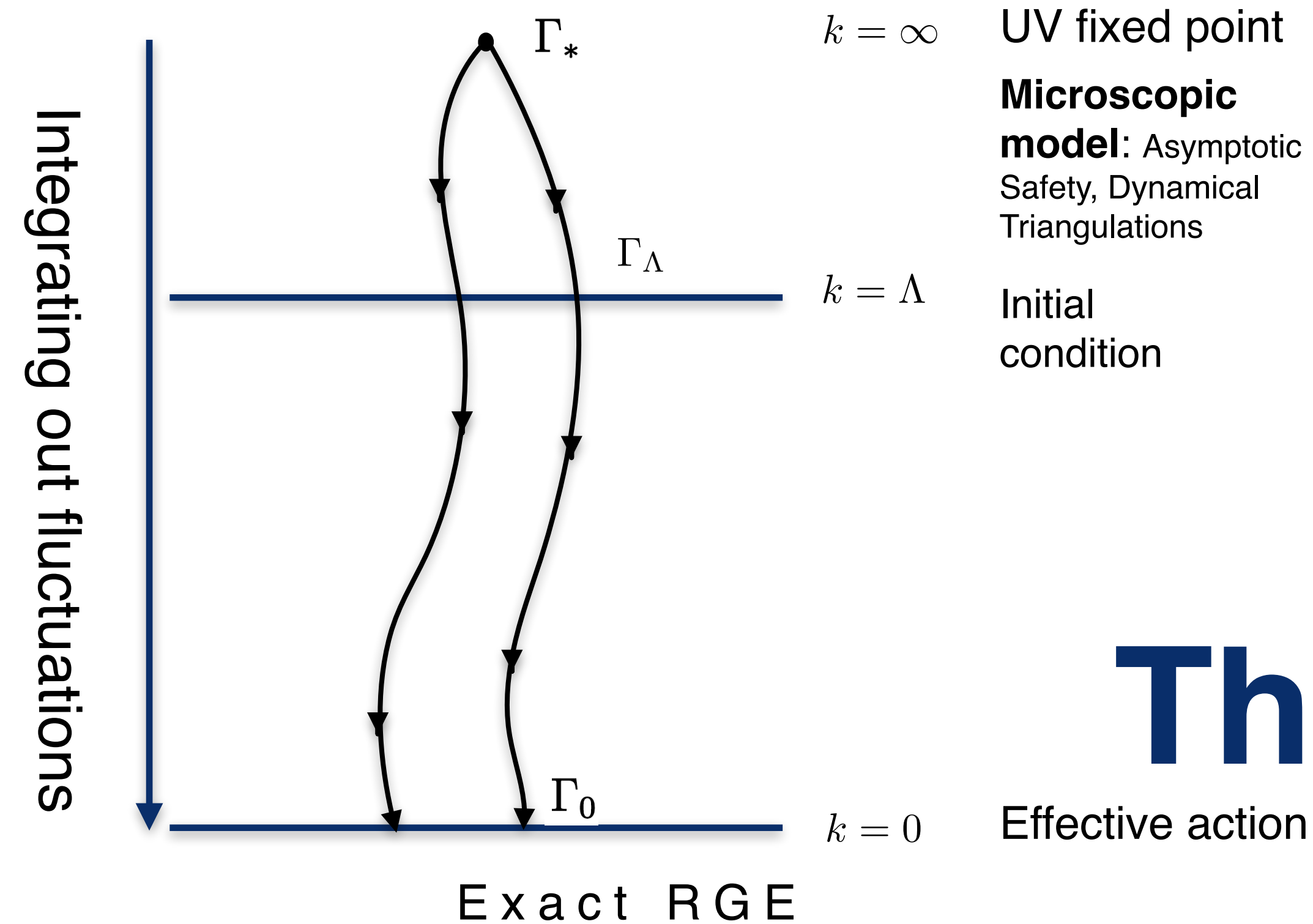
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Functional RGE

Functional RG

scale-dependent version of the effective action,
the Effective Average Action

Wetterich (1991)
Reuter and Wetterich (1994)

Functional RG in a nutshell

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**Alternative
manipulation of the
path integral**

- Implement the underlying RG idea already at the level of the EAA, fully independently of the bare action.

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Alternative manipulation of the path integral

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Generating functional

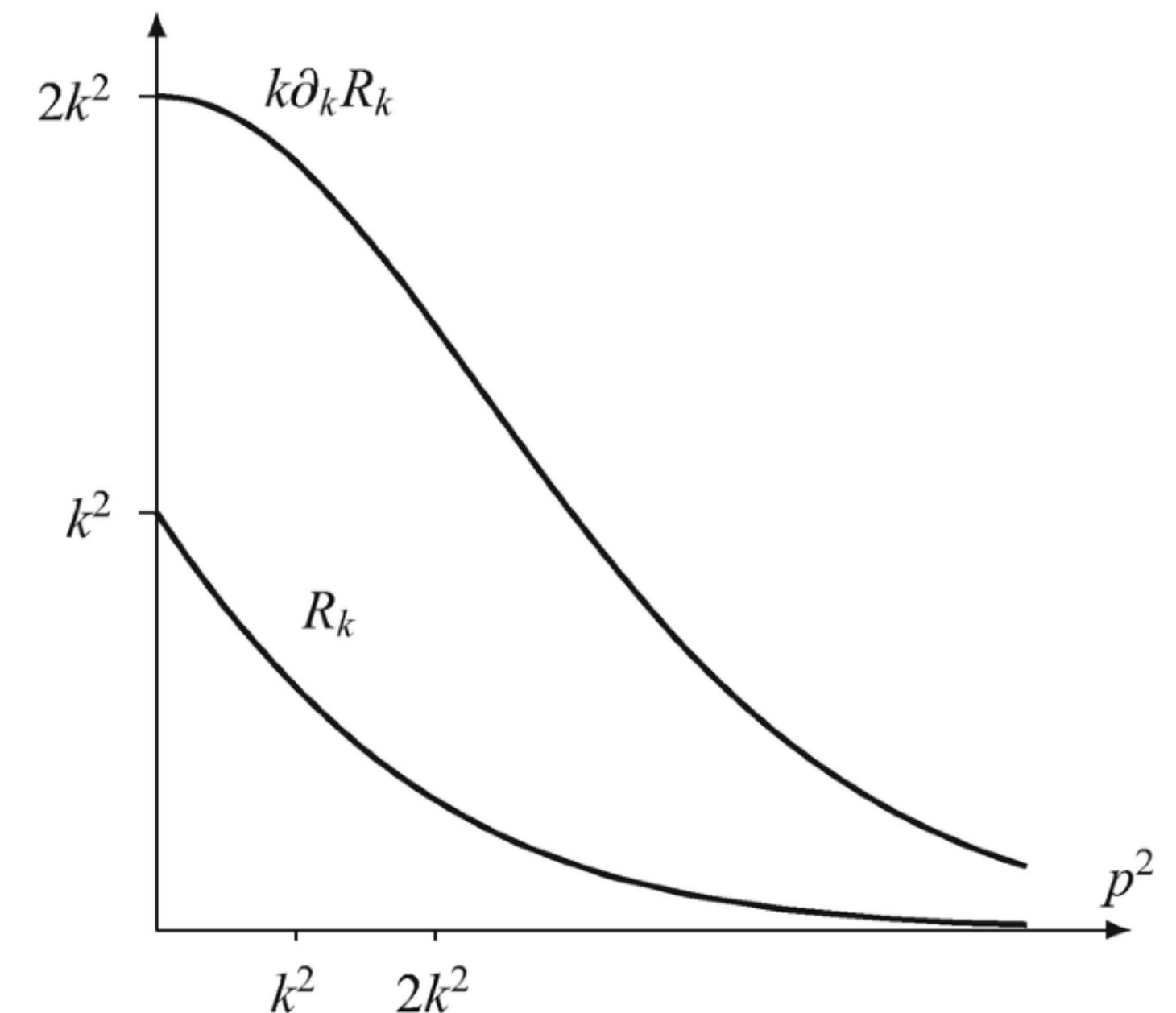
$$W_k[J] = \log \int \mathcal{D}\hat{\phi} \exp \left(-S[\hat{\phi}] - \Delta S_k[\hat{\phi}] + \int d^d x J(x) \hat{\phi}(x) \right)$$

Smooth cutoff

$$\Delta S_k[\hat{\phi}] = \frac{1}{2} \int d^d x \hat{\phi}(x) \mathcal{R}_k(-\square) \hat{\phi}(x)$$

RG kernel:
mass-like IR
regulator

$$\mathcal{R}_k(p^2) \approx \begin{cases} k^2 & \text{for } p^2 \ll k^2 \\ 0 & \text{for } p^2 \gg k^2 \end{cases}$$



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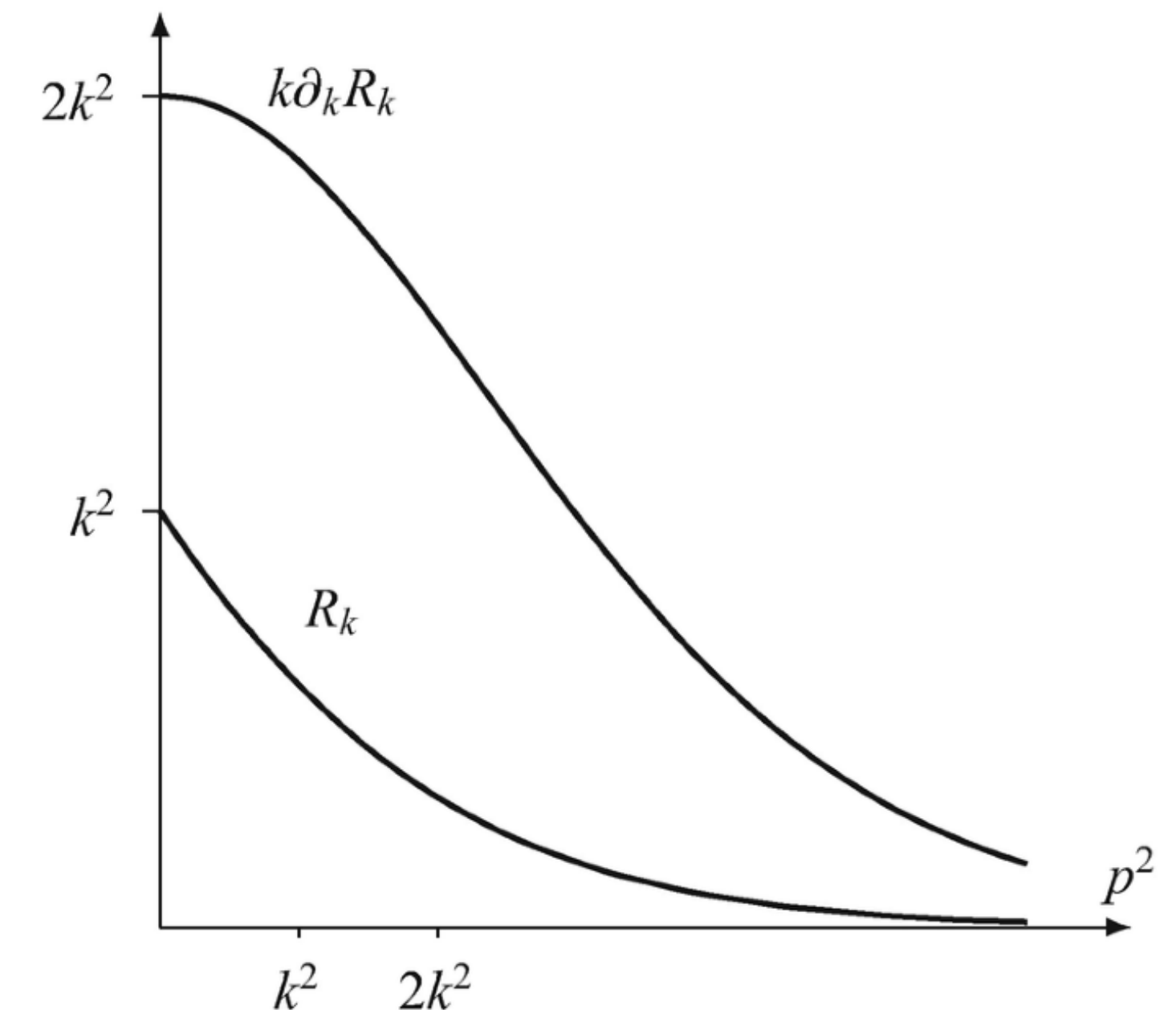
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Legendre transform

Effective Average Action

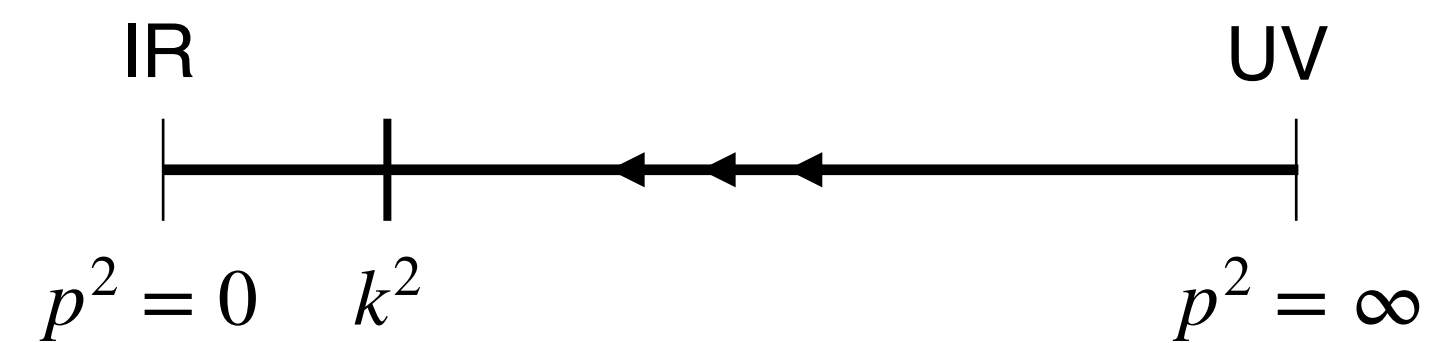
$$\tilde{\Gamma}_k[\phi] = \int d^d x J_k[\phi](x) \phi(x) + W_k[J_k[\phi]] \longrightarrow \Gamma_k[\phi] = \tilde{\Gamma}_k[\phi] - \Delta S_k[\phi] = \int d^d x J(x) \phi(x) + W_k[\phi] - \Delta S_k[\phi]$$



The EAA $\Gamma_k[\phi]$

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It represents the scale-dependent version of the standard effective action.

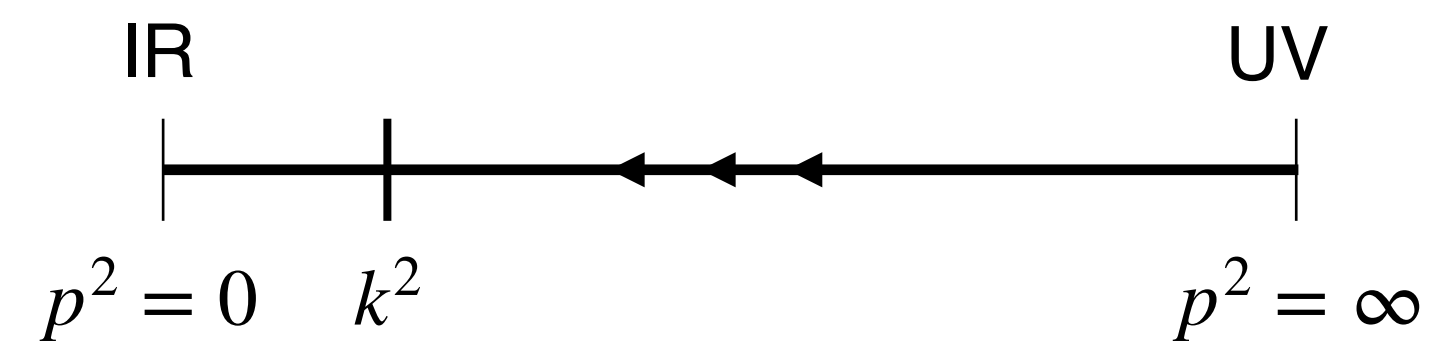


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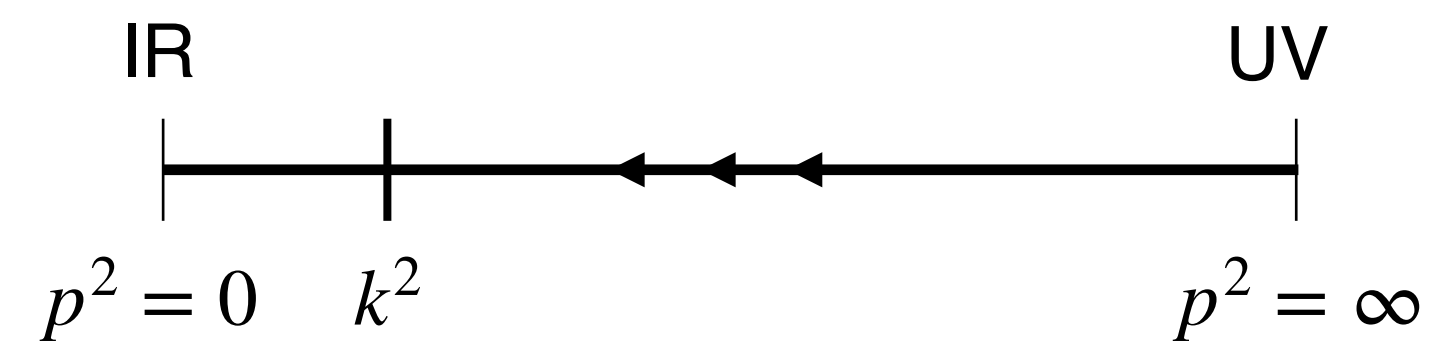
It satisfies the Functional Renormalization Group Equation:

$$k \partial_k \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[(\Gamma_k^{(2)} + \mathcal{R}_k)^{-1} k \partial_k \mathcal{R}_k \right]$$

- UV- and IR finite
- Fully nonperturbative or exact

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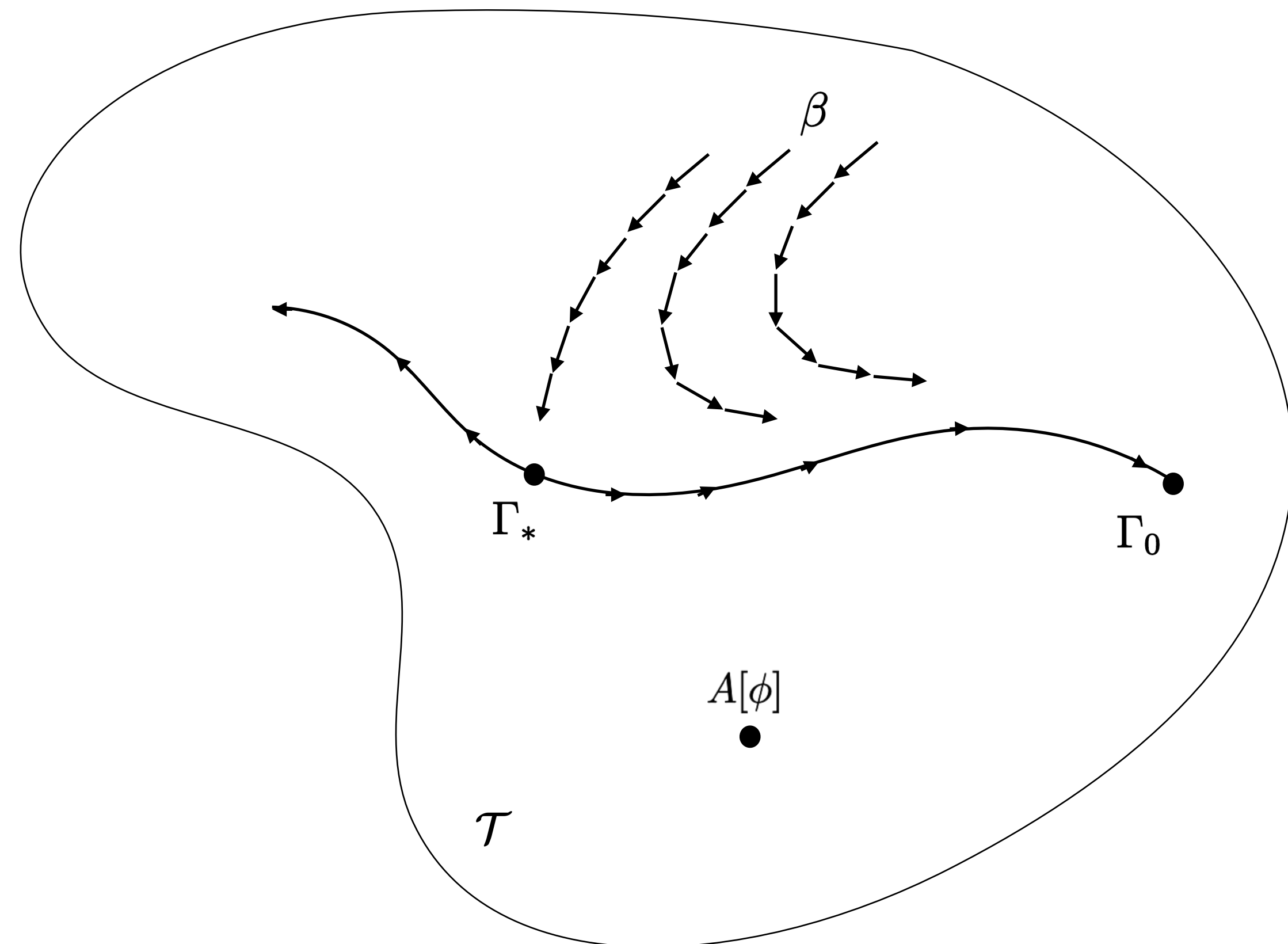
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Predictive solutions do exist in theories that are otherwise perturbatively non-renormalizable.

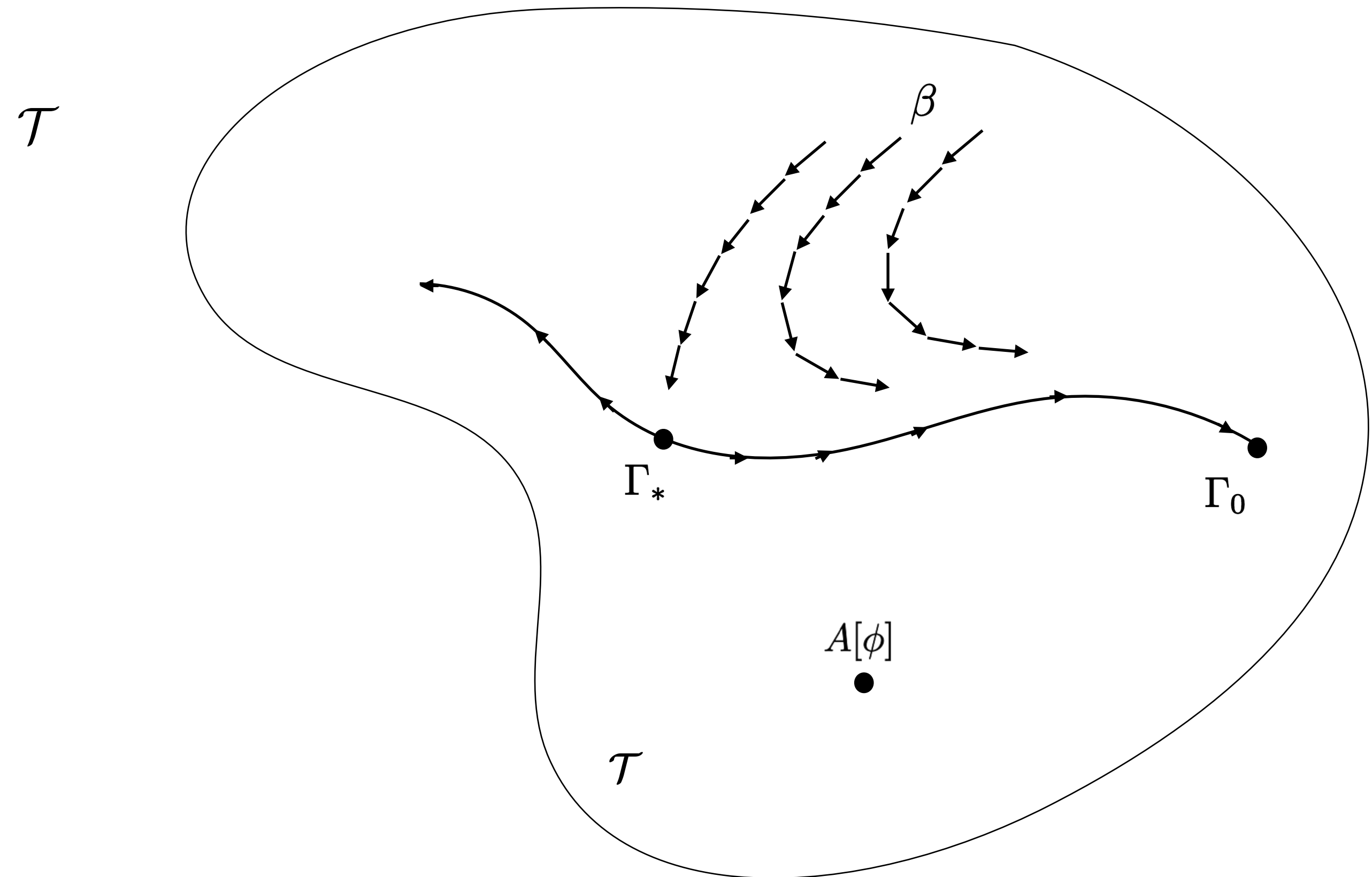
The geometrical setting



The geometrical setting

THEORY SPACE

space of functionals over which the EAA is supposed to be defined.



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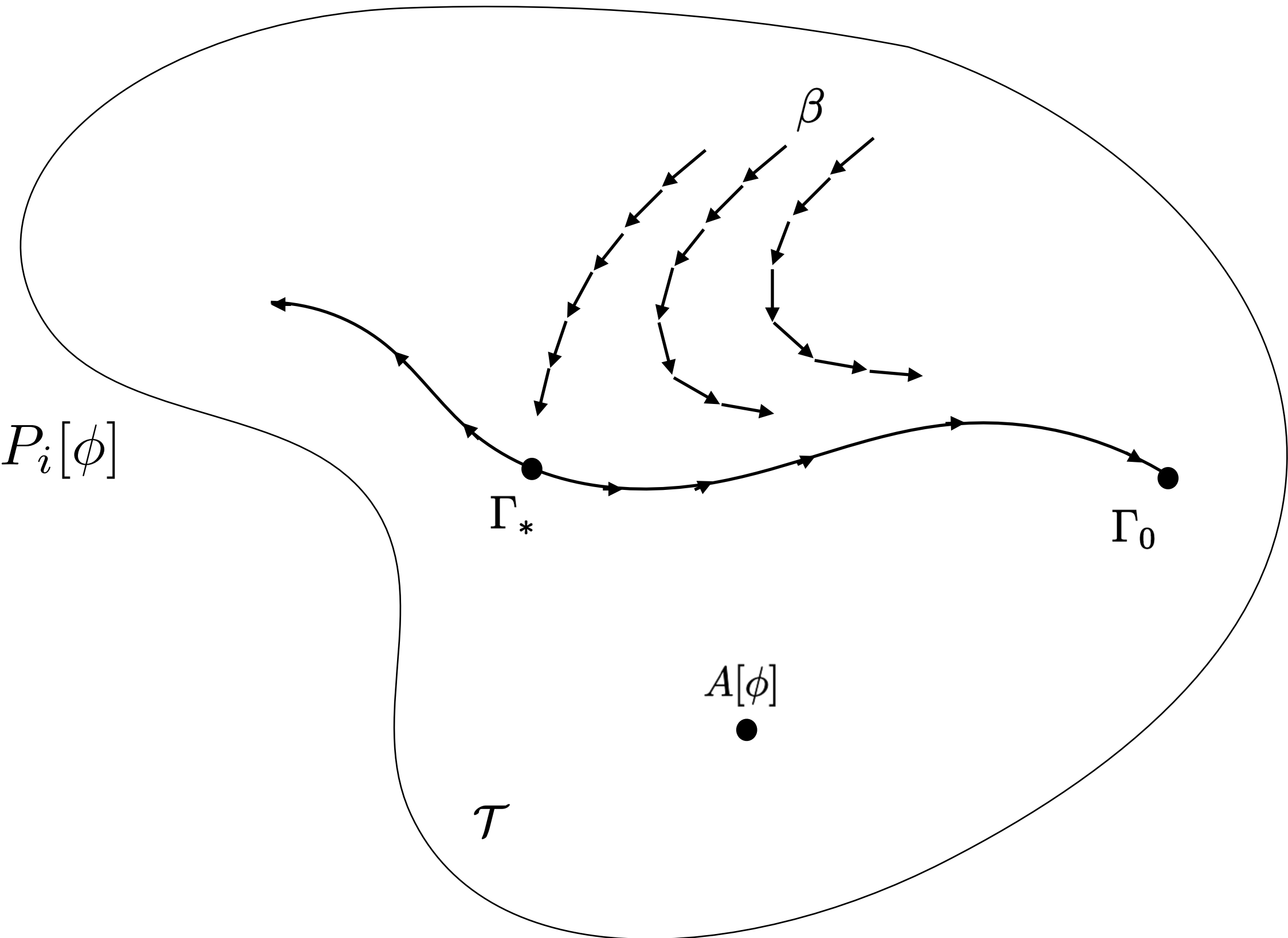
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BASIS ON THEORY SPACE

expansion of the elements of theory space in basis functionals and coupling constants.

$$A[\phi] = \sum_{i=1}^{\infty} U^i P_i[\phi]$$



The geometrical setting

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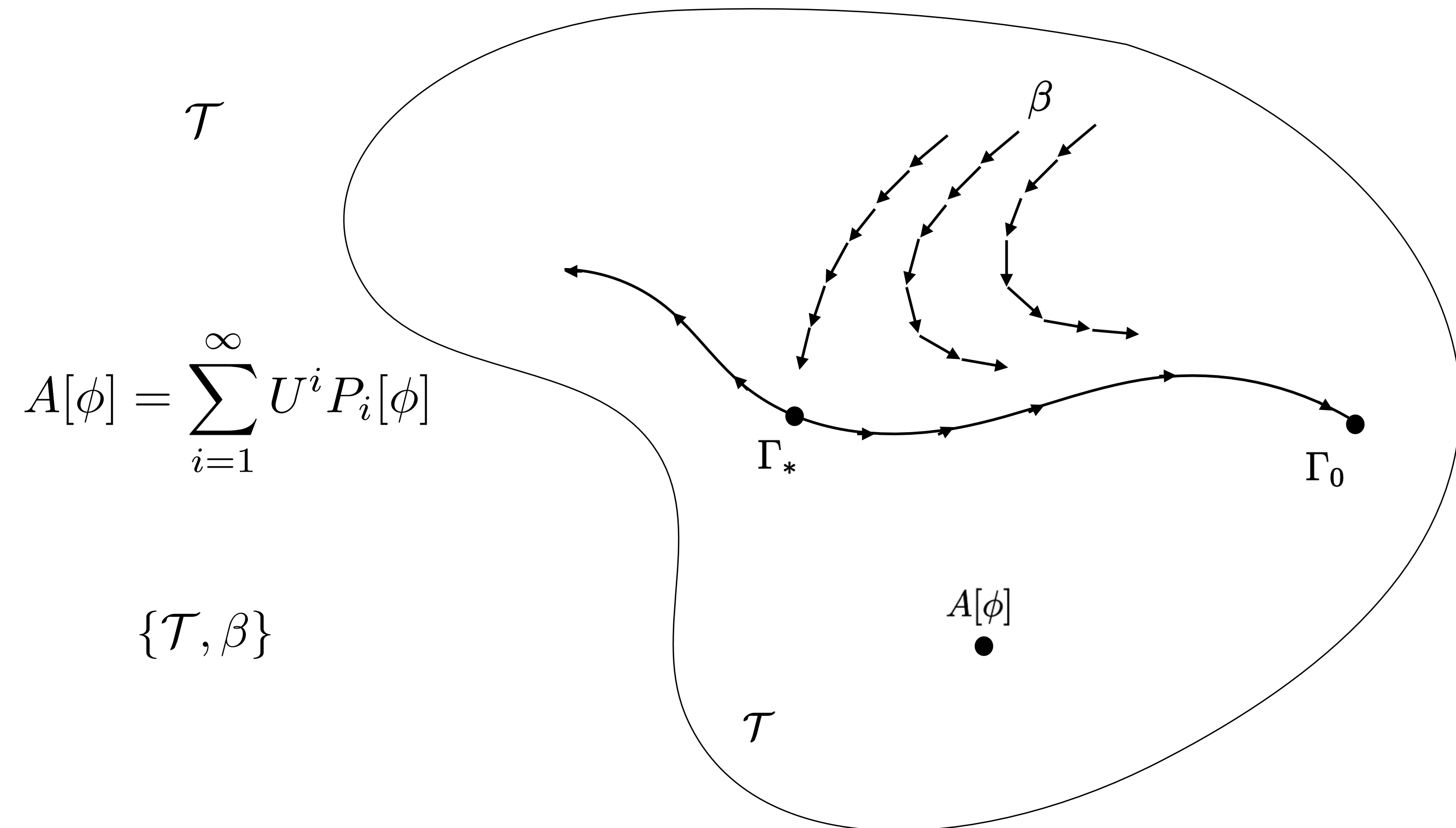
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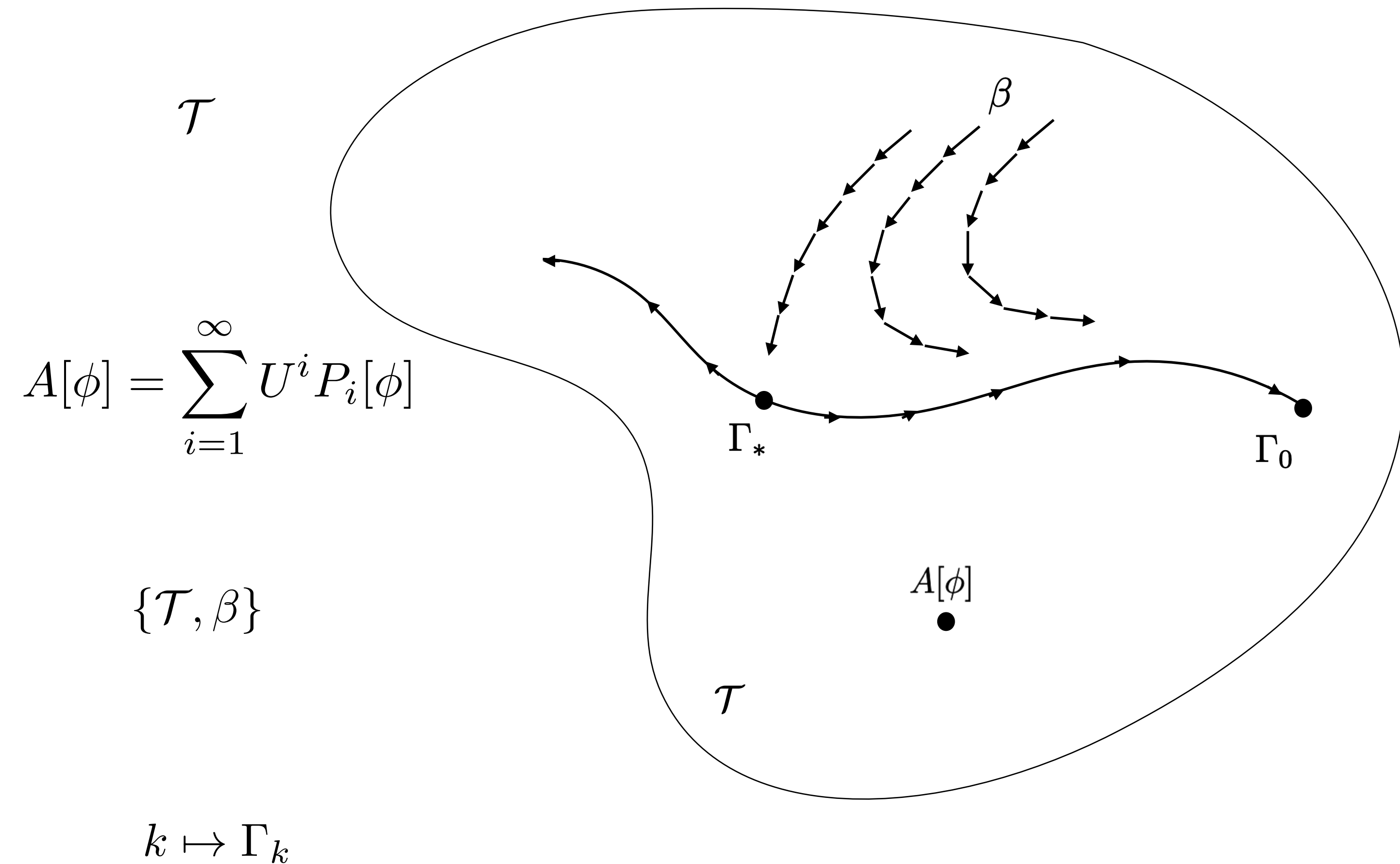
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pair of theory space and RG trajectories.



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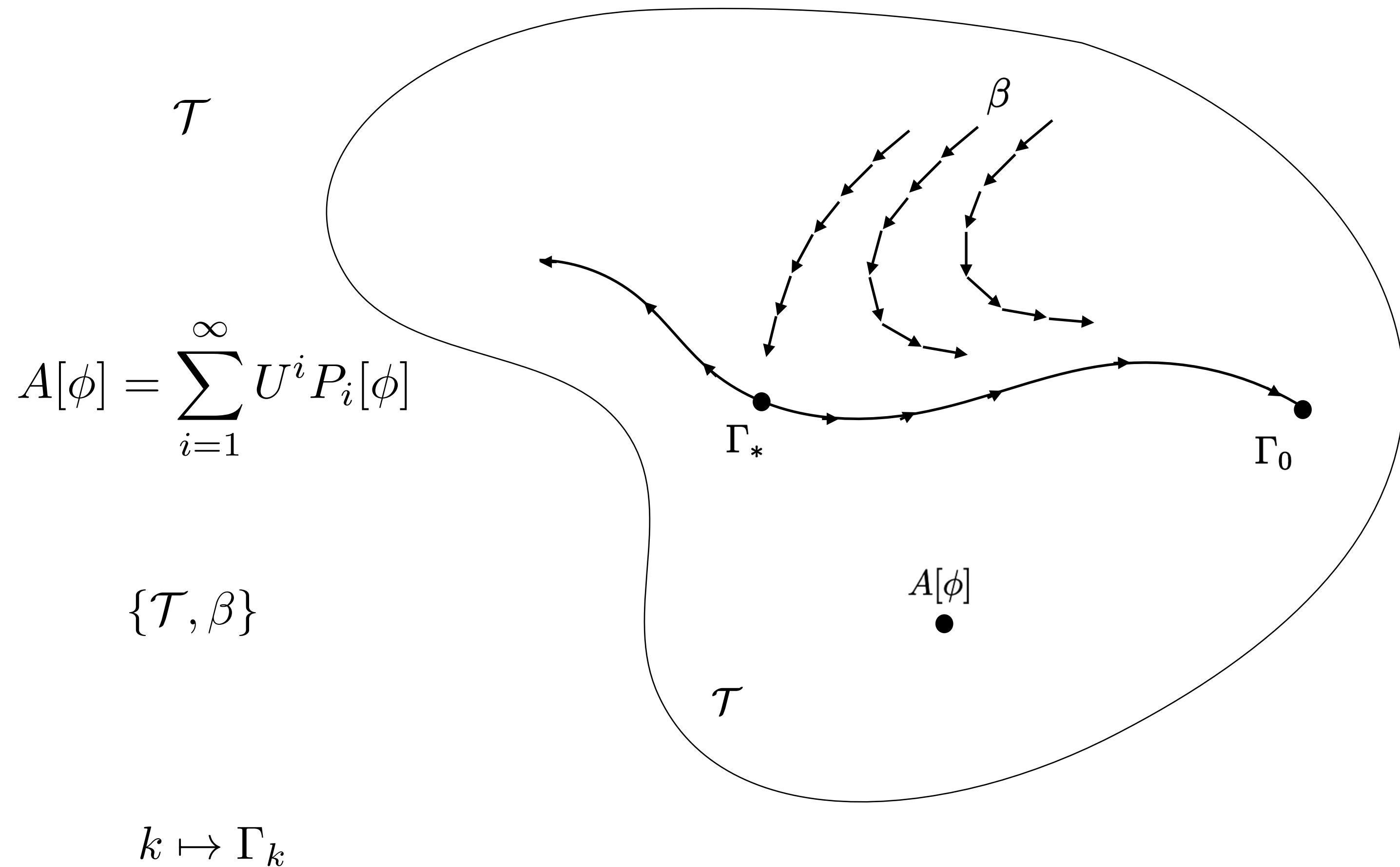
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Fixed points & Scaling exponents

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Gaussian FP: the scaling exponent agrees with the canonical mass dimension (generally $u_*^i = 0$)

Non-Gaussian FP (interacting, UV FP): at least one of the scaling exponents differs from the canonical mass dimension ($u_*^i \neq 0$)

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STABILITY MATRIX

$$k\partial_k u^i(k) = \sum_j \mathcal{B}^i_j(u^i(k) - u_*^i), \quad \mathcal{B}^i_j(u_*) \equiv \partial_j \beta^i(u_*)$$

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SCALING EXPONENTS

$$u^i(k) = u_*^i + \sum_I C_I V_I^i \left(\frac{k_0}{k} \right)^{\theta_I}$$

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They encode **physical information** about the **universality class** of the system and its **scaling (observable) behavior**.

À LA WEINBERG

Asymptotic Safety

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Asymptotic Safety

a mechanism which renders physical scattering amplitudes finite (but non-vanishing) at energy scales exceeding the Planck scale.

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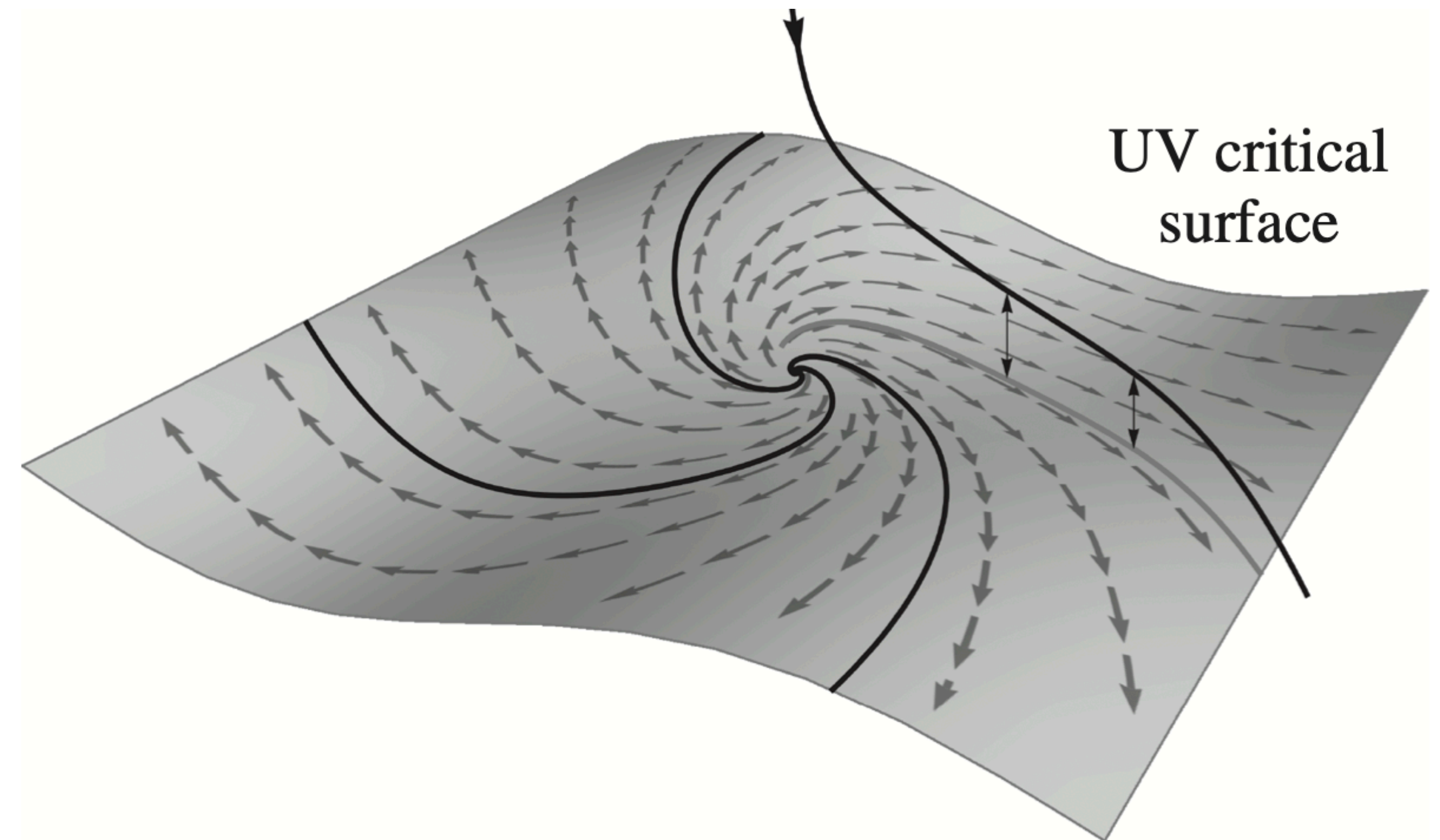
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À LA EFFECTIVE AVERAGE ACTION

nonperturbative renormalization & predictivity

Renormalization consists in constructing a complete trajectory, a trajectory which lies entirely within theory space.



À LA WEINBERG

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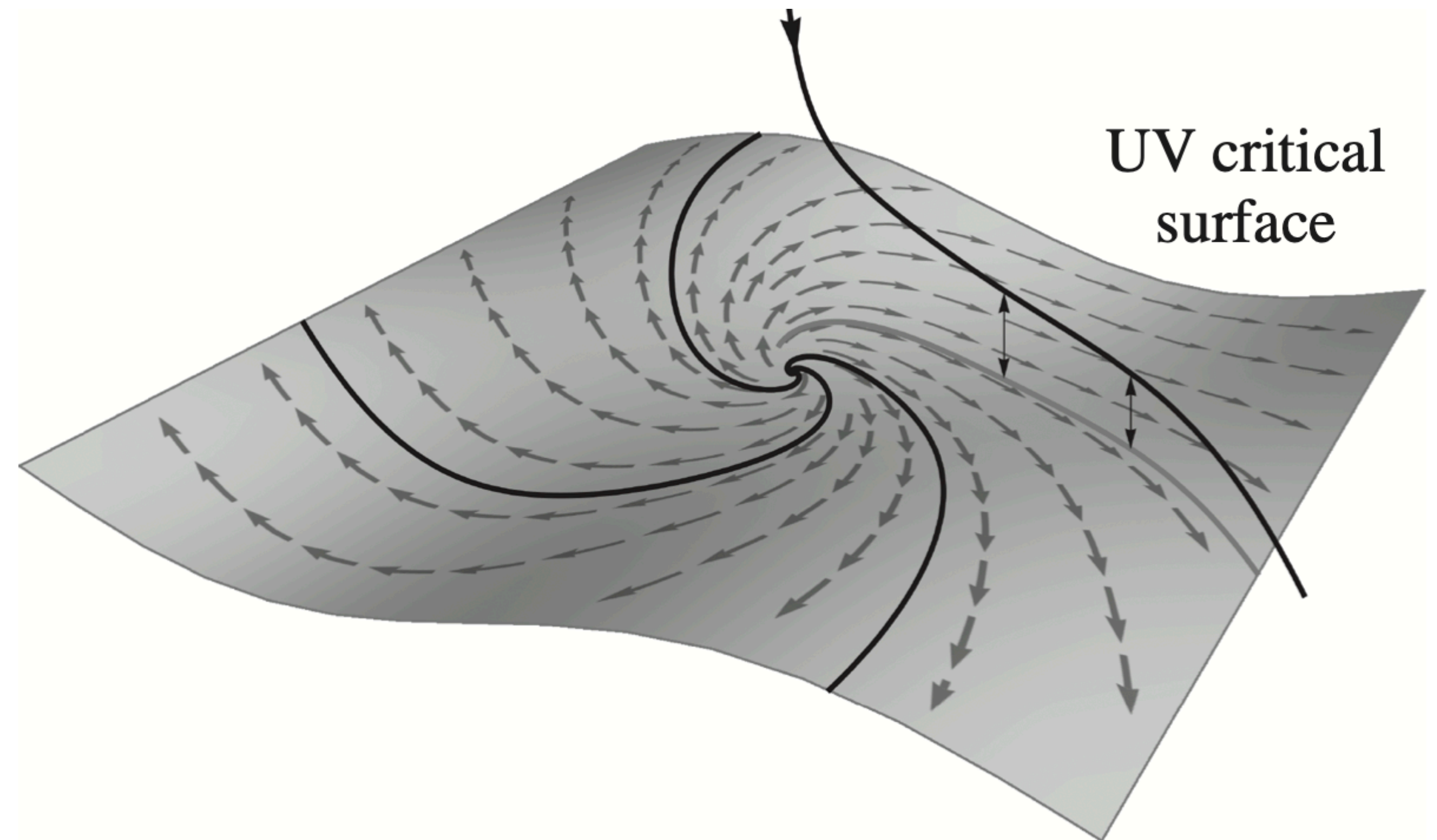
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UV critical surface: surface spanned by the eigenvectors of relevant couplings.



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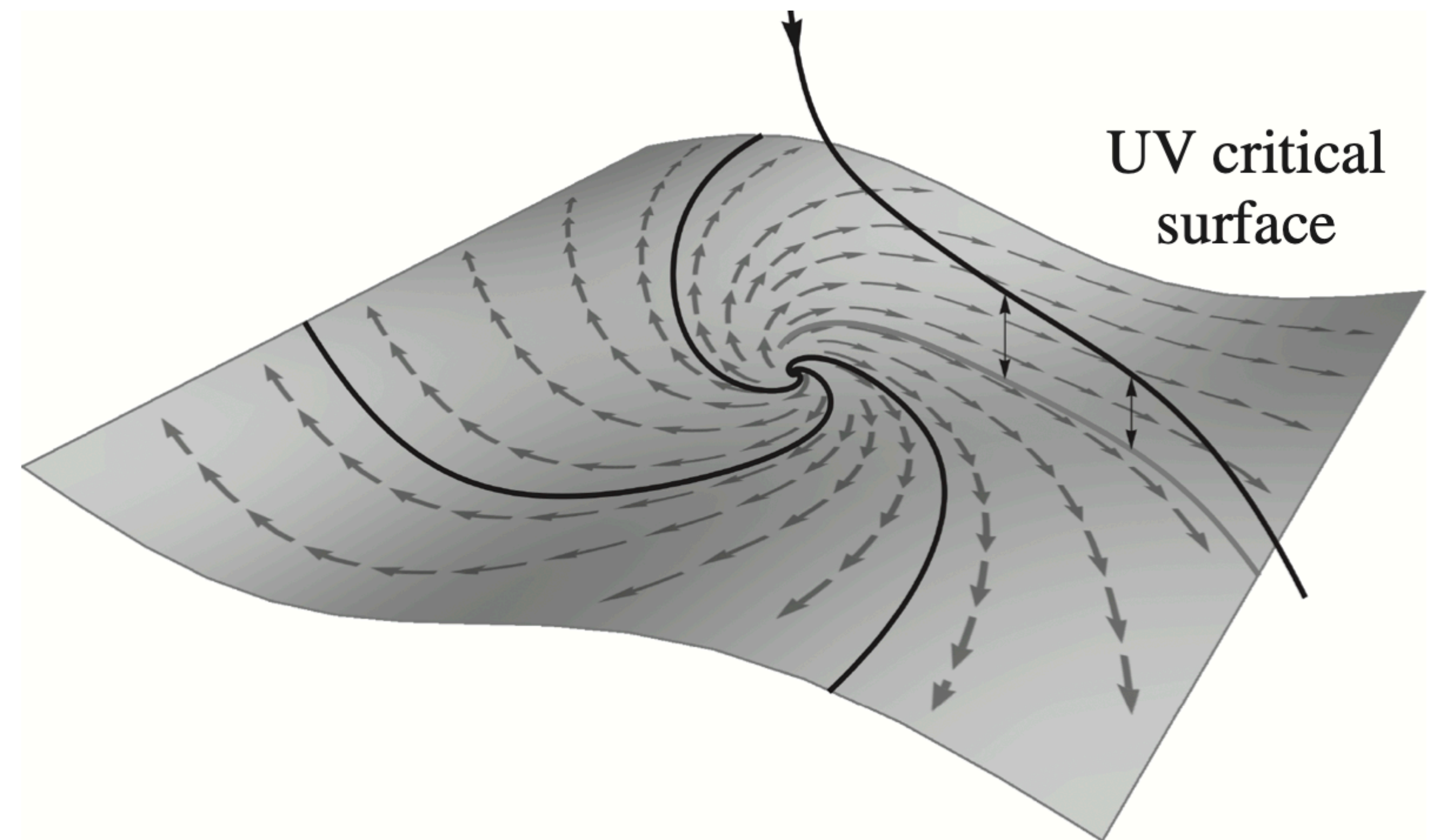
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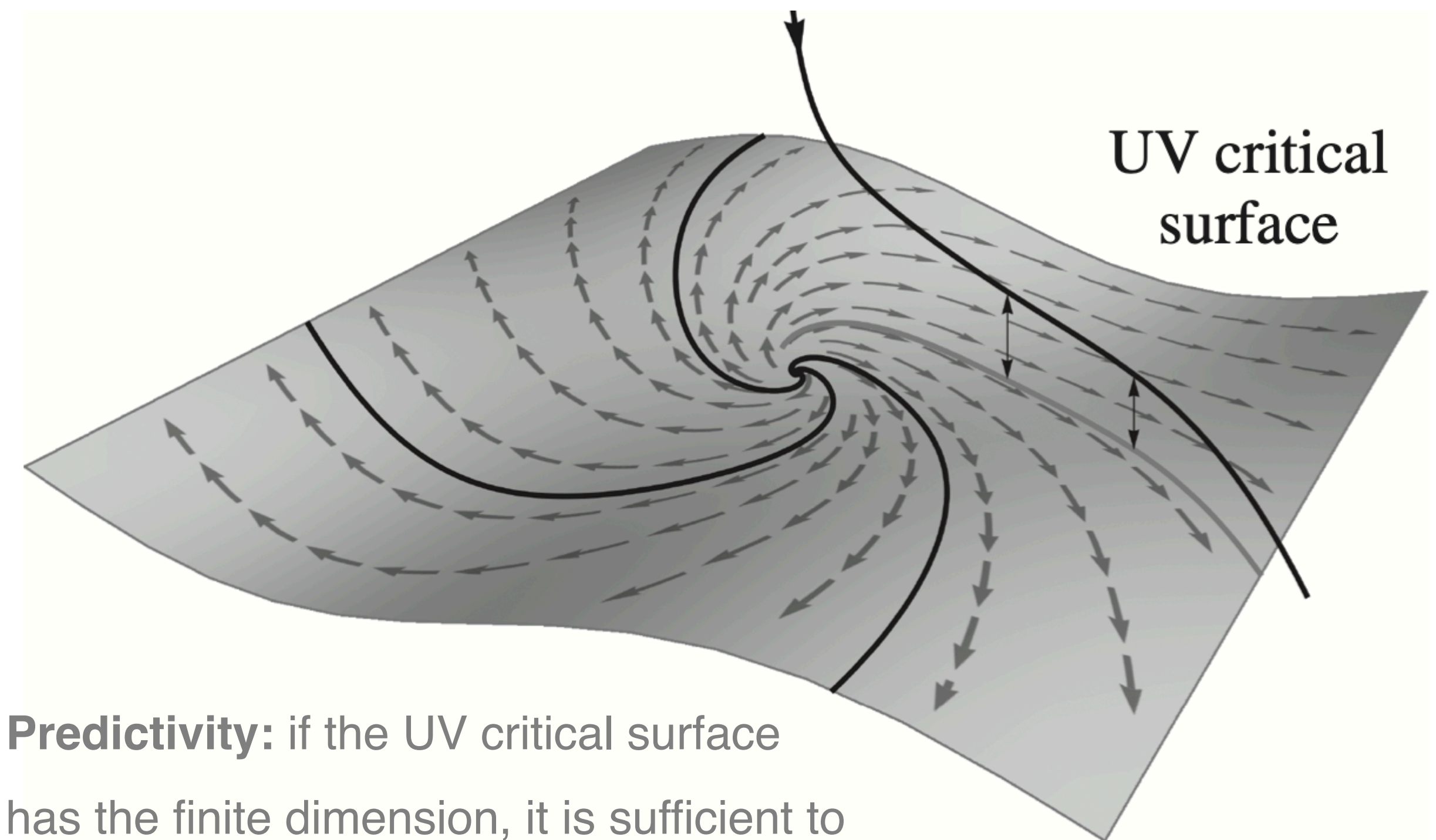
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Predictivity: if the UV critical surface has the finite dimension, it is sufficient to perform only a **finite number of measurements** in order to uniquely identify Nature's RG trajectory.

Truncations

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**Nonperturbative
approximate
solution**

$$\Gamma_k[\phi] = \sum_{i=1}^N U^i(k) P_i[\phi]$$

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RHS: Evaluate the trace using heat kernel techniques

- Compare term by term the coefficients multiplying the same operator
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What has been done?

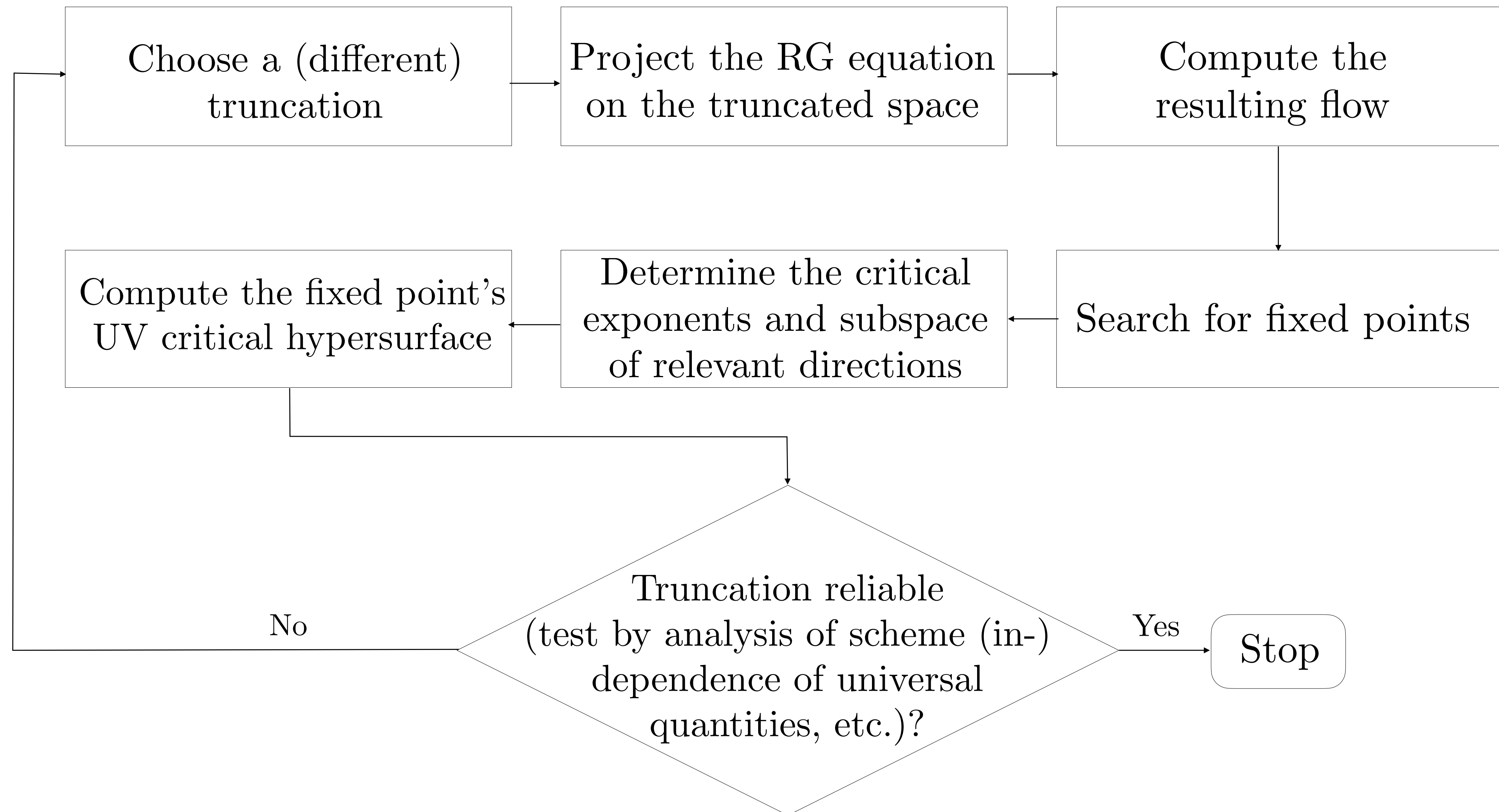
up to order R^7

with matter

different carrier fields

Reuter, Saueressig, Percacci, Falls, Litim, Eichhorn,...

The Program



Gravity

Asymptotic Safety

Background independence

Background
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Background field method

$$g_{\mu\nu} = \langle \hat{g}_{\mu\nu} \rangle = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

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$$g_{\mu\nu} = \langle \hat{g}_{\mu\nu} \rangle = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

The dynamical degrees of freedom are quantized on this background.

$$\langle \hat{h}_{\mu\nu} \rangle_{\bar{g}} = 0 \iff \langle \hat{g}_{\mu\nu} \rangle_{\bar{g}} = \bar{g}_{\mu\nu} \quad \text{for} \quad \bar{g} = \bar{g}(k)^{\text{sc}}$$

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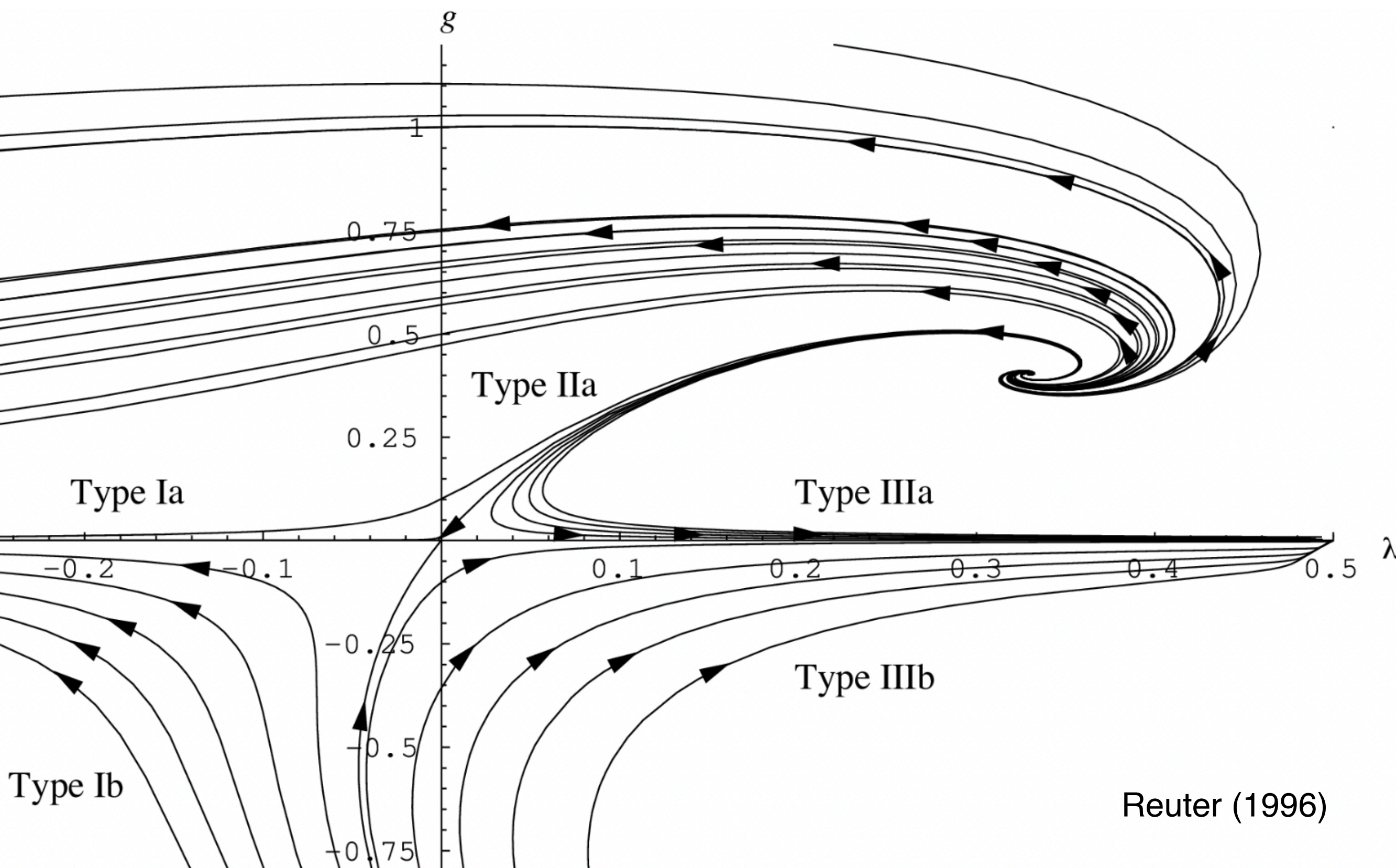
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The background is re-adjusted in such a way that it becomes self-consistent, meaning that the expectation value of the fluctuation vanishes.

$$\frac{\delta}{\delta h_{\mu\nu}(x)} \Gamma_k [h; \bar{g}] \Big|_{h=0, \bar{g}=\bar{g}(k)^{\text{sc}}} = 0$$

Quantum Einstein Gravity

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^d x \sqrt{g} (R - 2\Lambda(k)) + \text{gauge fixing} + \text{ghosts}$$

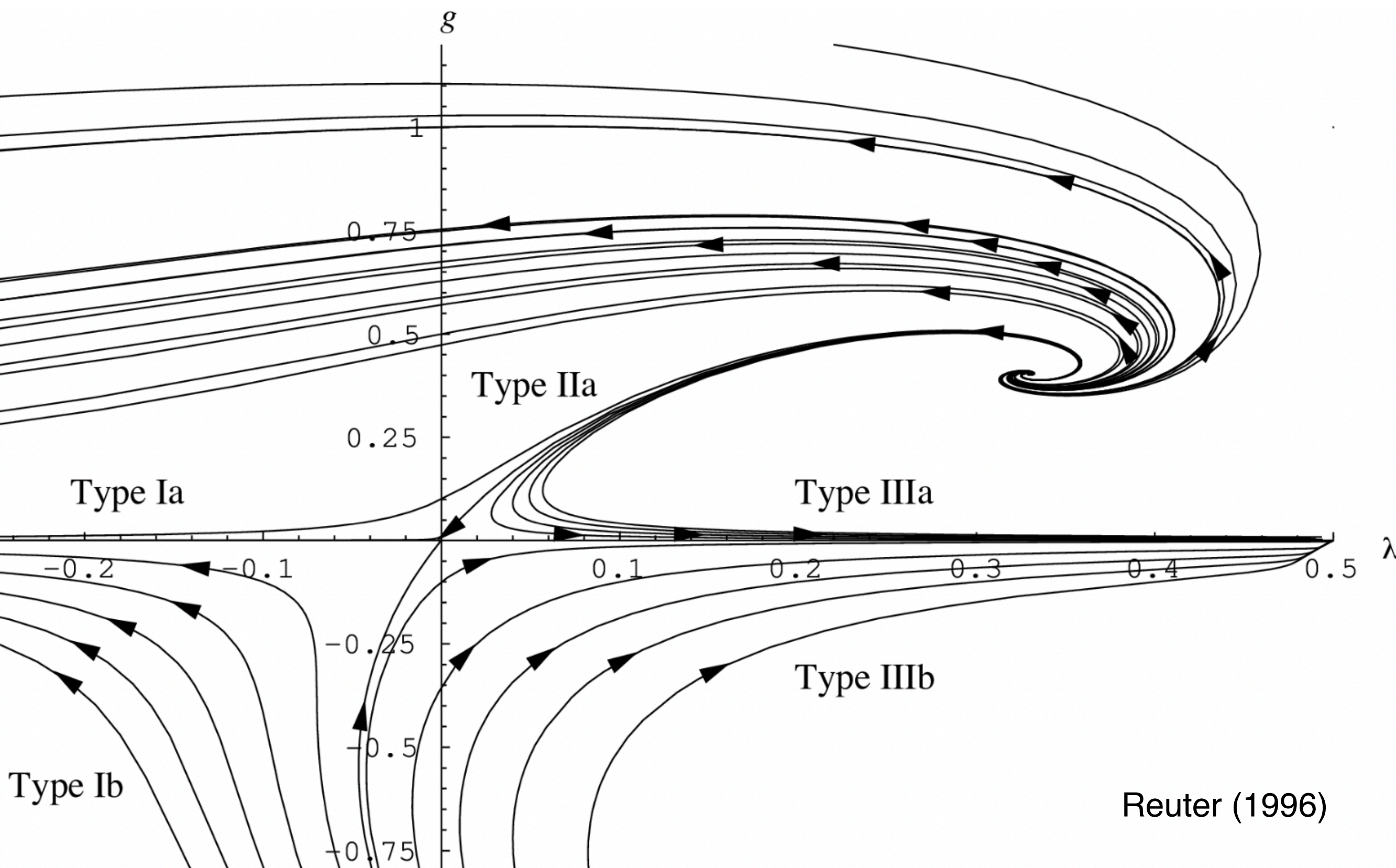


Reuter (1996)

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$$g(k) = G(k)k^2$$
$$\lambda(k) = \Lambda(k)k^{-2}$$



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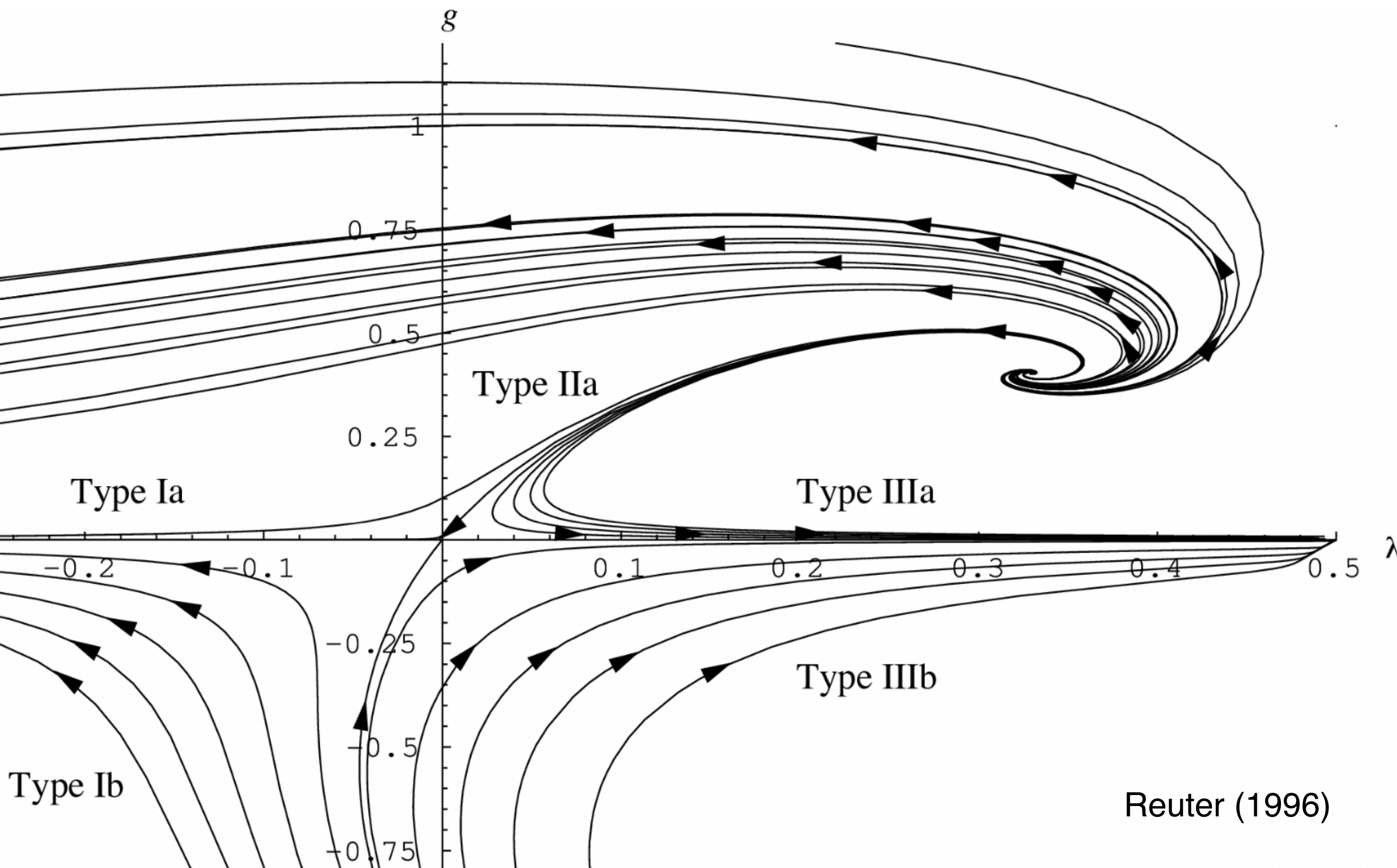
$$\lambda(k) = \Lambda(k)k^{-2}$$

$$k\partial_k g_k = \beta_g(g, \lambda)$$

$$\beta_g(g_*, \lambda_*) = 0$$

$$k\partial_k \lambda_k = \beta_\lambda(g, \lambda)$$

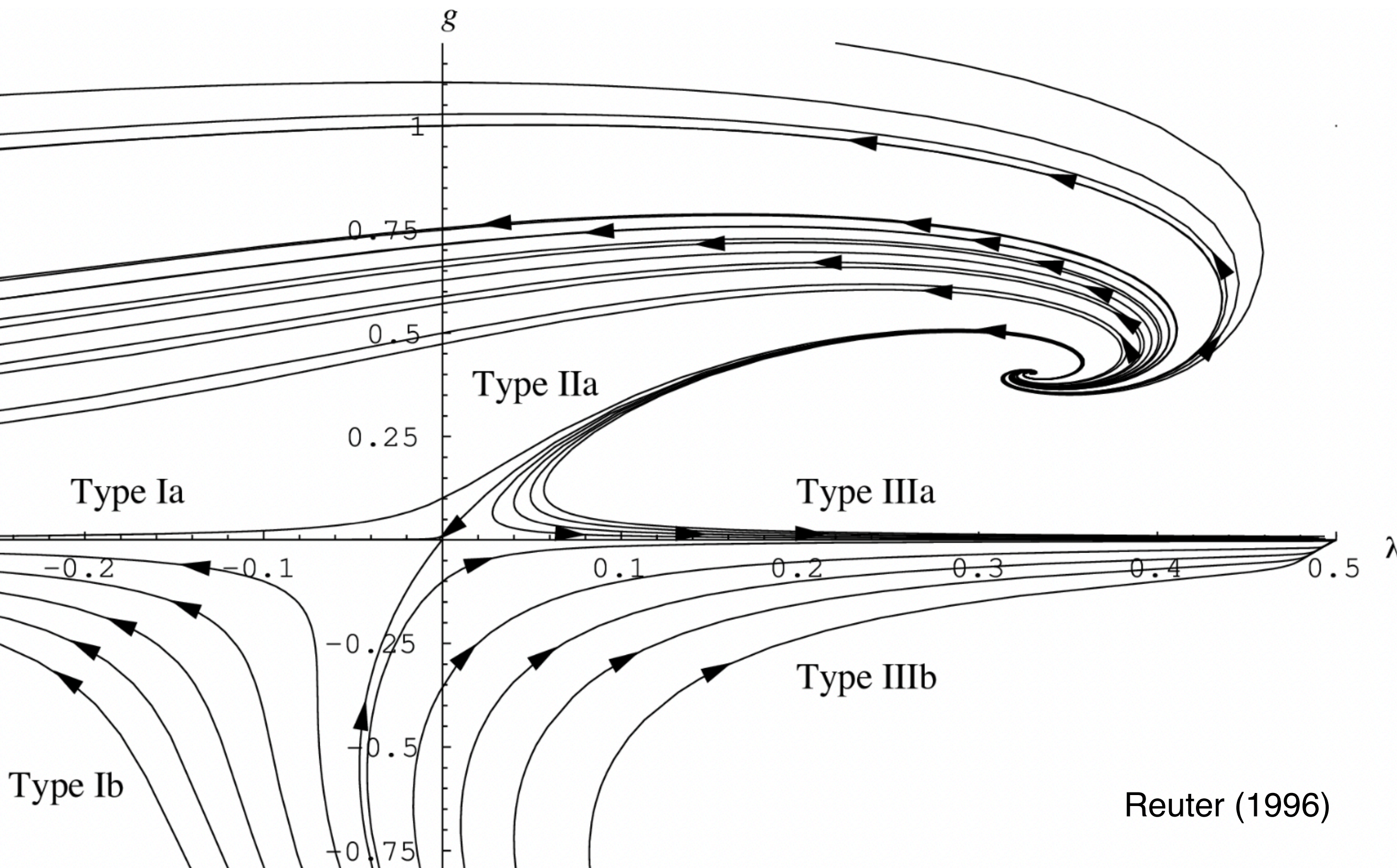
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Non-trivial fixed point.

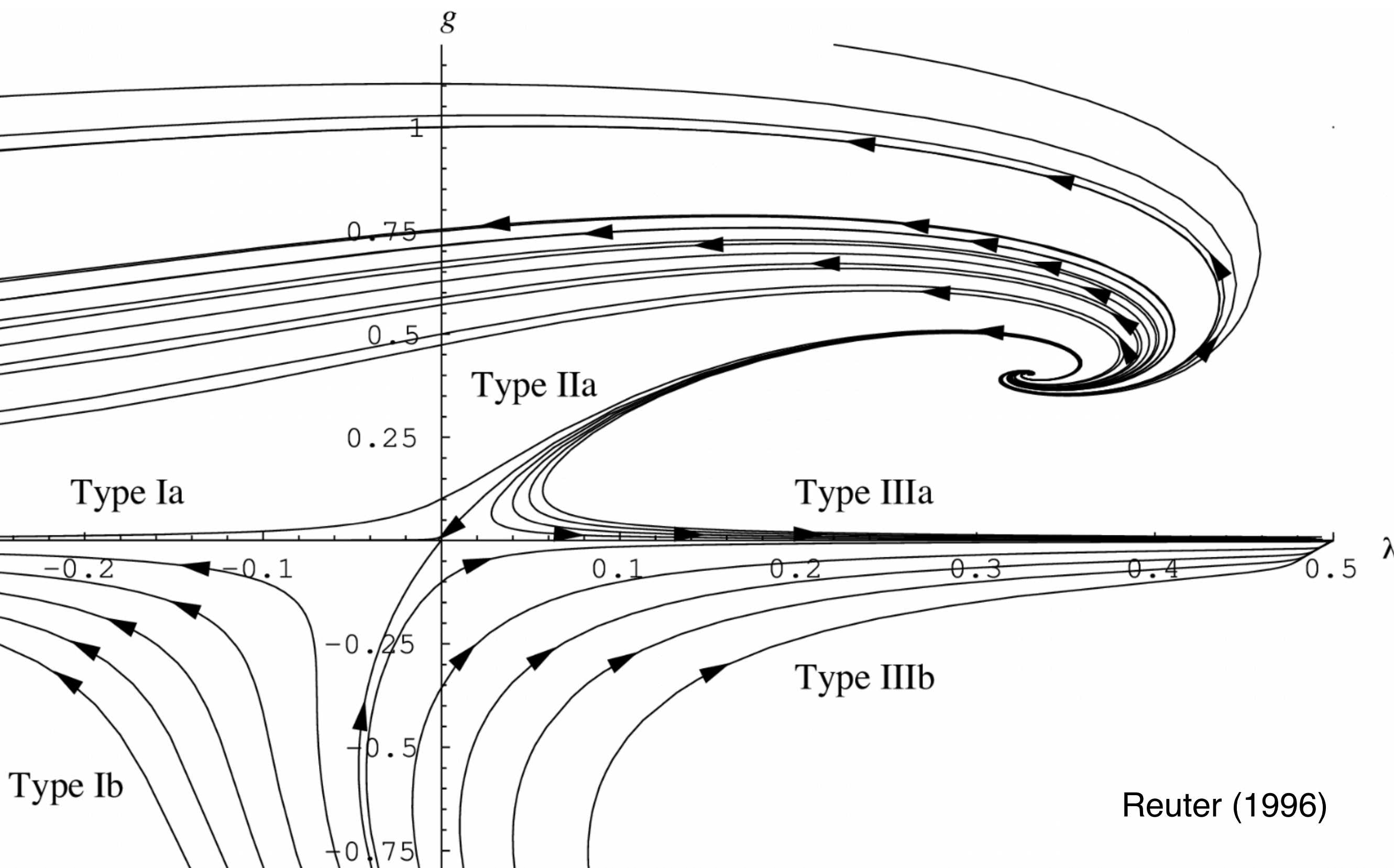
Complete trajectories.

**Finite number of
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Quantum Einstein Gravity

$$\Gamma_k = \frac{1}{16\pi G(k)} \int d^d x \sqrt{g} (R - 2\Lambda(k)) + \text{gauge fixing} + \text{ghosts}$$



$$g(k) = G(k)k^2$$

$$\lambda(k) = \Lambda(k)k^{-2}$$

$$k\partial_k g_k = \beta_g(g, \lambda) \quad \beta_g(g_*, \lambda_*) = 0$$

$$k\partial_k \lambda_k = \beta_\lambda(g, \lambda) \quad \beta_\lambda(g_*, \lambda_*) = 0$$

Non-trivial fixed point.

Complete trajectories.

**Finite number of
relevant directions.**

**ASYMPTOTIC
SAFE**

Reuter (1996)

Euclidean vs. Lorentzian

Euclidean vs. Lorentzian

Integrating out dofs **Timelike-spacelike:** no distinguished ordering of the modes with a standard canonical status is given.

State dependence - Observer dependence

Euclidean vs. Lorentzian

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Problem set 1

Problem set 2

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One can work out a Lorentzian heat kernel proper time regularization.

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Problem set 1 Obtain RG trajectories on a theory space which is constituted of functionals that are constructed on Lorentzian metrics.

The FRG as a machinery One can work out a Lorentzian heat kernel proper time regularization.

Problem set 2 Analyze the flows of hyperbolic kinetic operators, typically of the d'Alembertian in the background of the running self-consistent metrics.

RF, Reuter (2022)

Which physical information can
we extract from an asymptotic
safe theory of gravity?

Observables?

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Observables?

Relational observables.

Two options.

“Physical gauge fixing”

via material reference frame

~ reduced phase space quantisation

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Construct relational observables as additional operators

after gauge-fixing include matter (physical reference frame) in the EAA

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Flow of the composite operator

Pagani, Reuter (2016)

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$$\lim_{k \rightarrow \infty} \mathcal{O}_k = \hat{\mathcal{O}}|_{\hat{\phi} \rightarrow \phi}$$

$$\mathcal{O}_{k=0} = \langle \hat{\mathcal{O}} \rangle$$

Covariant formulation

Running relational observables

Covariant formulation

Construct a physical coordinate frame
e.g. by adding matter fields

s.t.

perform a diffeomorphism
transformation

transform the tensor

transform the physical frame

composed transformation leaves
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$$R(x) \mapsto \varphi * R(x)$$

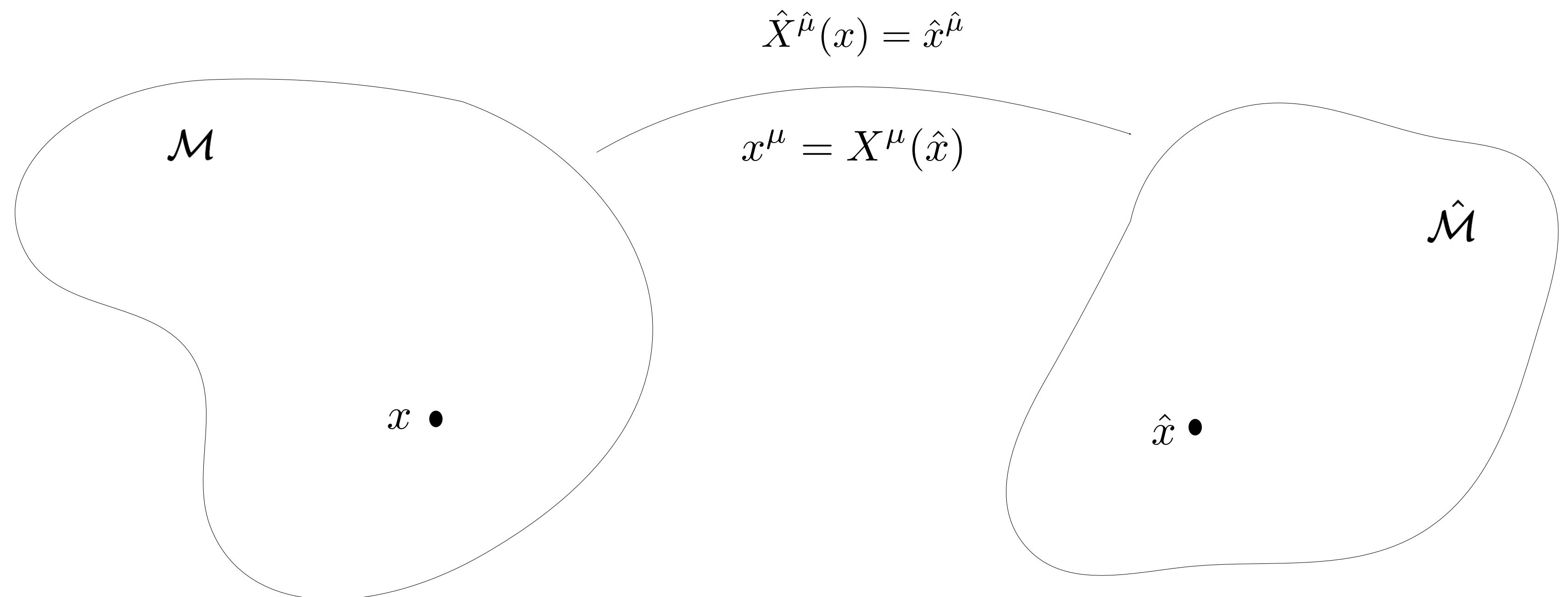
transform the physical frame

$$X \mapsto \varphi^{-1}(X)$$

composed transformation leaves
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$$R(X) \mapsto \varphi * R(\varphi^{-1}(X)) = R(X)$$

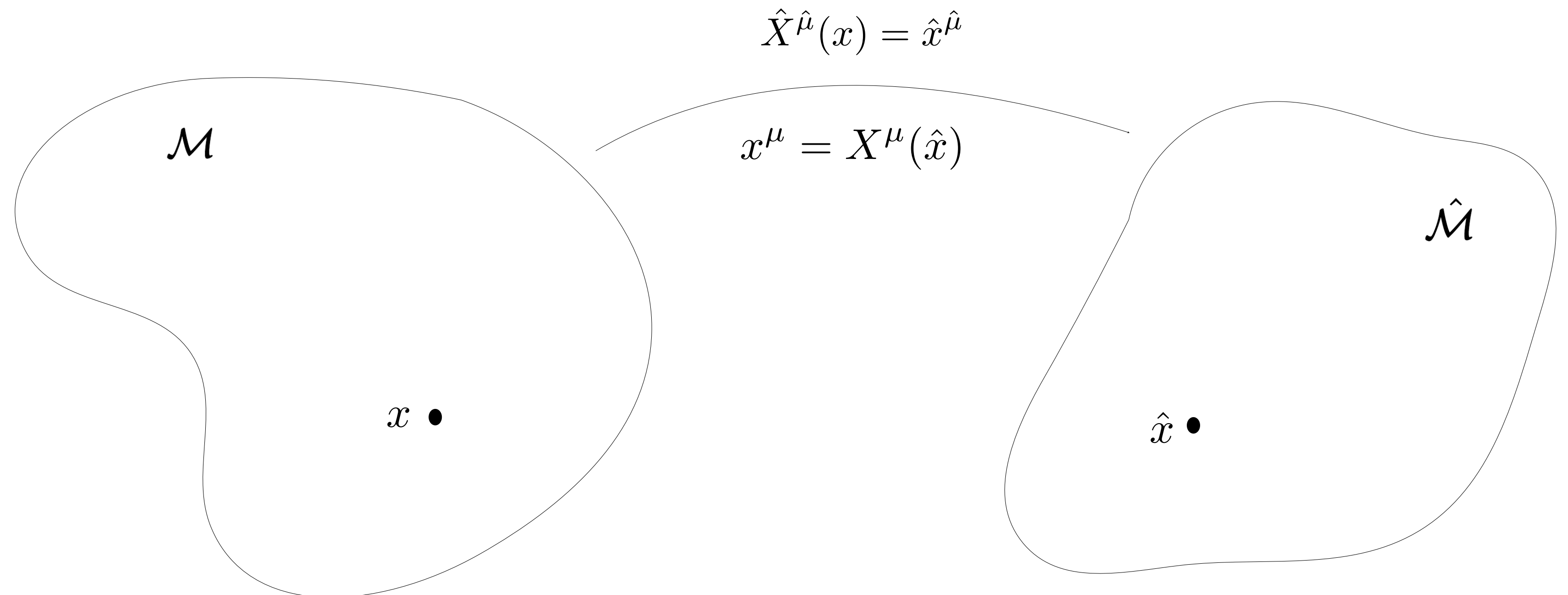
Physical coordinate system



Physical coordinate system

Introduce a
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$$\phi^a(x) = \{g_{\mu\nu}, \text{matter fields}\}$$



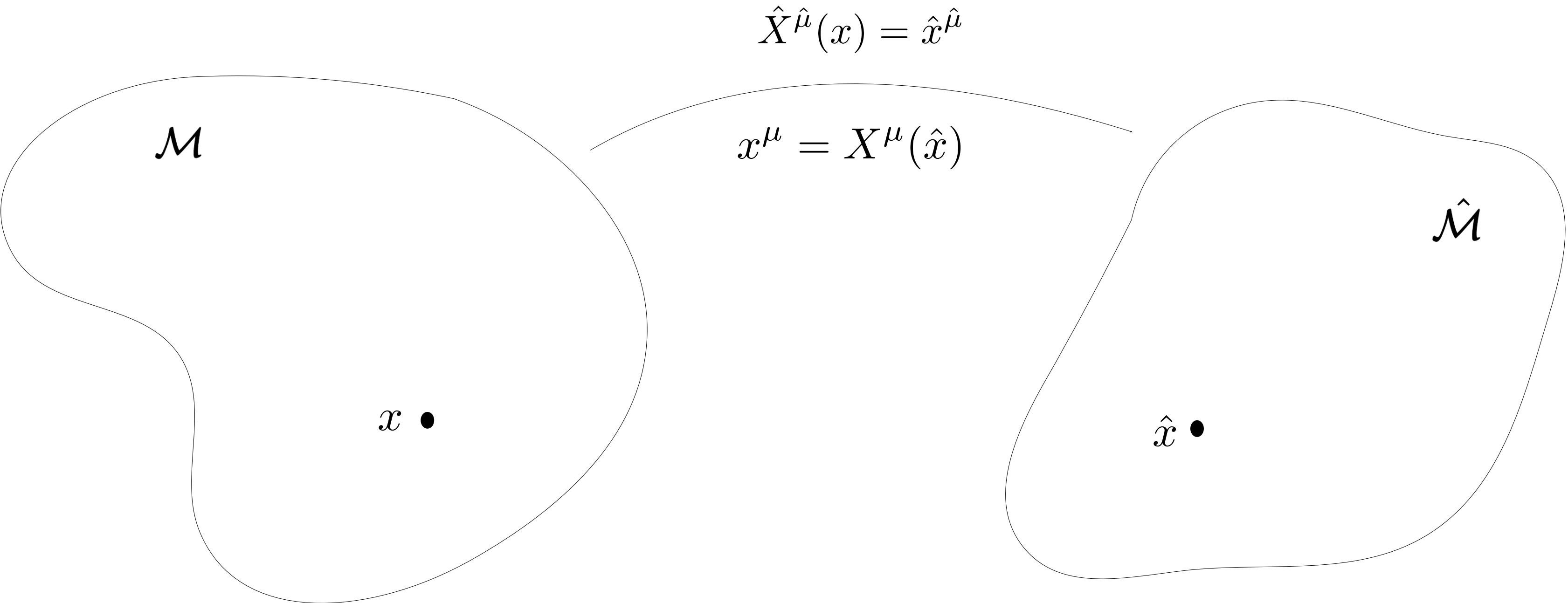
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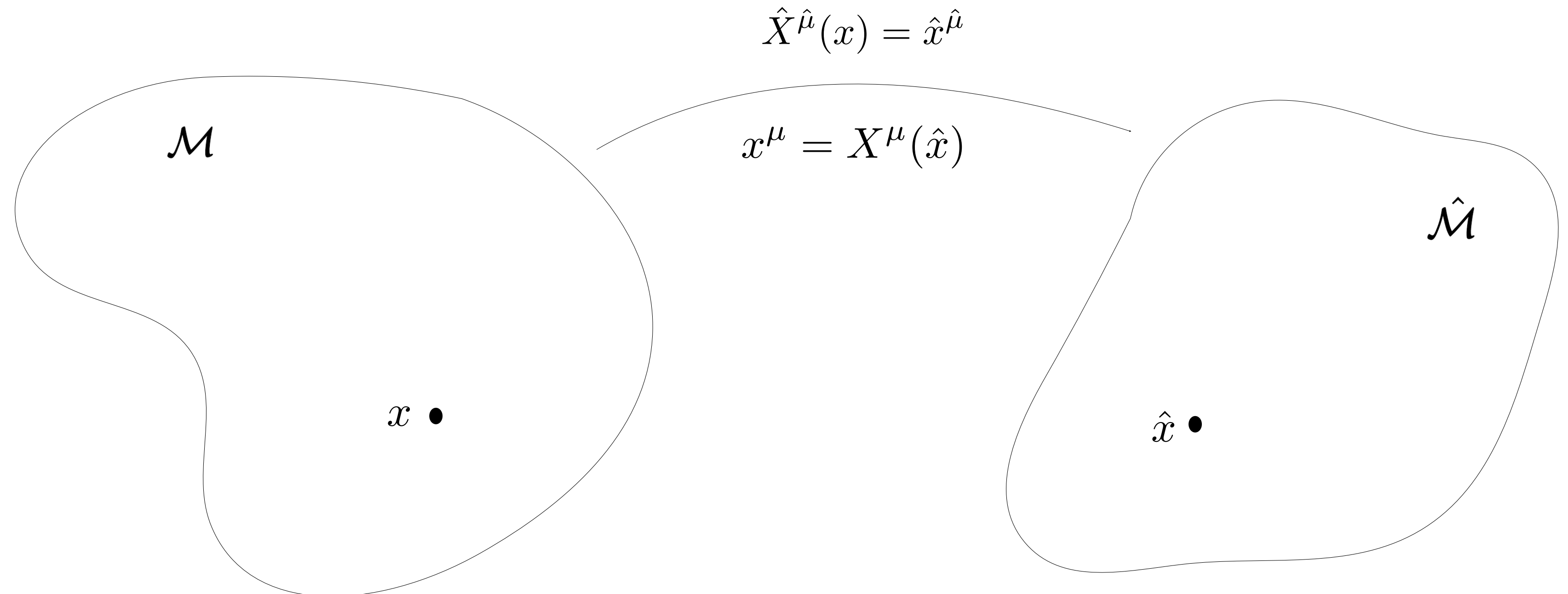
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Physical coordinate system

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Frame fields:

$$e_{\mu}^{\hat{\mu}}(x) = \partial_{\mu} \hat{X}^{\hat{\mu}}(x)$$

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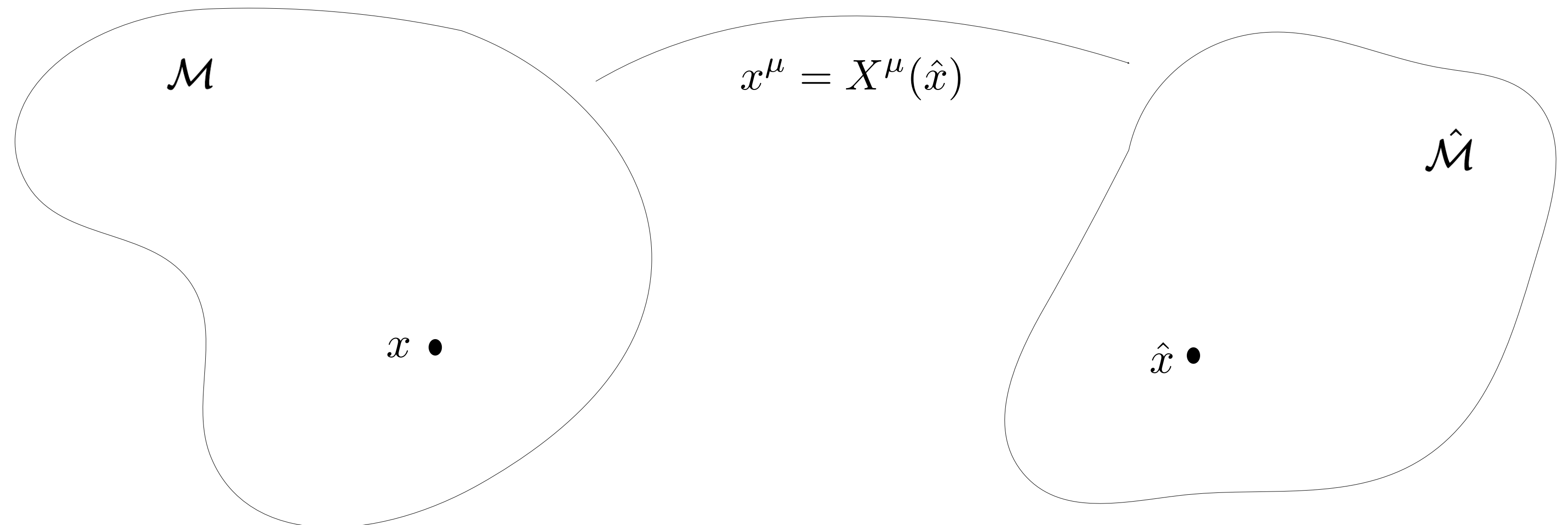
Invariant volume element: $\tilde{e} = \det e_{\mu}^{\hat{\mu}}$

$$\delta(X(\hat{x}), x) = \tilde{e}(x) \delta(\hat{x}, \hat{X}(x))$$

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$$\hat{X}^{\hat{\mu}}(x) = \hat{x}^{\hat{\mu}}$$

$$x^{\mu} = X^{\mu}(\hat{x})$$



Flow of the relational observables

**Relational
Effective
Average Action:**

$$\Gamma_k^{\text{rel.}} \equiv \int \mathrm{d}^4x \, \tilde{e}(x) \mathcal{L}_k^{\text{rel.}}(x) := \int \mathrm{d}^4x \, \tilde{e}(x) \sum_i a_{\hat{I}_i}(k) E_{i\hat{I}_i}^{\hat{I}_i}(x) O_i^{I_i}(x)$$

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**relational relational relational
volume Ricci scalar inverse metric**

Example:

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$$\begin{aligned} \Gamma_k^{\text{rel.}} &= \int \text{d}^4\hat{x} \left(a_0(k) + a_R(k) \hat{R}(\hat{x}) + a_1(k) \delta_{\hat{\mu}\hat{\nu}} \hat{g}^{\hat{\mu}\hat{\nu}}(\hat{x}) \right) \\ &= \int \text{d}^4x \, \tilde{e} \left(a_0(k) + a_R(k) R + a_1(k) \delta_{\hat{\mu}\hat{\nu}} g^{\mu\nu} (\partial_\mu \hat{X}^{\hat{\mu}}) (\partial_\nu \hat{X}^{\hat{\nu}}) \right) \end{aligned}$$

Recipe

**Find the fixed
points**

EAA

**Identify the
relational
observables**

PHYSICAL REFERENCE SYSTEM

**Compute the
flow of the
observables**

RELATIONAL EAA

**Scaling
dimension at
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RESULT

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E A A

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RELATIONAL E A A

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Matter content
SM (type II)
SM (type I)
SM + SF (type II)
SM + 3 ν (type II)

θ ₀	θ _R	θ ₁
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-4	-5.97467	-7.8177
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Conclusions & outlook

- AS seeks for a consistent and **predictive theory of the gravitational interactions within the framework of QFT** by invoking a **non-perturbative high-energy completion**.
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at the level of the scaling exponents, e.g. scaling of the correlation functions

at the level of the RG computations