# ELKO SPINOR AS DARK MATTER
## specific signatures and constraints for the interacting model

(Recently accepted in Nucl. Phys. B)

G.B. de Gracia[1] A.A. Nogueira[2] R. da Rocha[1]

[1]Federal University of ABC, Center of Mathematics, Santo André, 09210-580, Brazil.

[2]Federal University of Alfenas, Physics Department: Institute of Exact Sciences, Alfenas 37133-840, Brazil

FAPESP/BAYLAT Workshop.
ERLANGEN-FAU

# Table of Contents


1. General remarks
2. Definition of ELKO spinor
3. Amplitudes

# Table of Contents

1 General remarks

2 Definition of ELKO spinor

3 Amplitudes

# General remarks

**Dark matter** has strong evidence due to gravitational effects such as

- Galaxy rotation curve
- gravitational lensing effects

**However**,

- Low interaction with standard matter particles

Therefore, some **NON-Luminous** matter must be throughout the universe.

**According to cosmology** $\sim 85\%$ **of the total mass of the universe is associated to dark matter.**

# General remarks

- Standard gravitational theory implies in decaying velocity outside the region with the major quantity of matter (Luminous) at $R \sim 15$ (kpc) light years.
- TThe observed curve must be explained by some non-luminous matter in the region $> R$

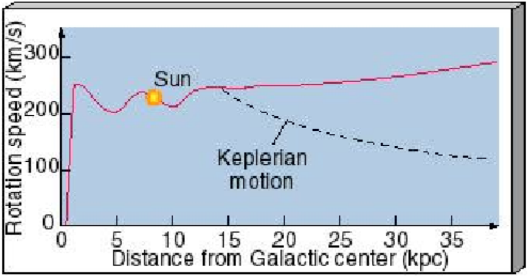



Figure: credit-astro.physics.uiowa.edu. (Milk way)

# General remarks

**ELKO** spinor is defined as eigenspinors of charge conjugation operator. It transforms differently than Dirac ones under discrete symmetries $C, P, T$. They are candidates for a non-standard Wigner class.

**It has some important characteristics:**

- Are the expansion coefficients of Mass dimension one fermion fields
- Each species Just obey KG equation
- Then, **cannot** enter the standard model doublets in a **renormalizable** manner
- It is also important to highlight the fact that ELKO spinors enter as CLASS 5 spinors in LOUNESTO classification, DIFFERENTLY FROM MAJORANA fermions. Another characteristic feature is the fact that they DO NOT HAVE A DEFINITE HELICITY, each quiral component has a different one.

# XENON-nT (noble gas)-Gran Sasso-Italy

- On the ground, low contamination, and $v = 232 Km/s$ with relation to DM approximately in rest in the center of the galaxy.
- Recognize scintillation due to nucleon recoil.
- Well known background. Searching for excess with statistical significance in terms of standard deviation.

# Experimental searches

However, there are recent searches for Dark matter in nucleon recoil LAB experiments

- Xenon-nT, Lux, zeplin, CDMS, etc (scintillation)
- phonon mediated detections-2022
- Cross section $\sigma \leq 10^{-39} cm^2$
- LAB **WIMP Mass expected to be of order** $GeV$ event rate peak 2206.06772
- Tritium contamination in XENON (2020): fake $m \sim 2,3Kev$ WIMP
- Previous estimate from ELKO and Galaxy rotation curve $m \sim 0,1KeV$

To be detected in LAB (ordinary matter devices.) there must be coupling involving an **INTERMEDIATE boson** and the ELKO and also one involving this boson and the nucleons

# Perspectives

- Considering an approximately flat universe.
- **A coupling with photon would be fine**(It is possible to find an exotic RENORMALIZABLE one.)
- But how to keep the darkness property with such coupling?
- We show that it leads to ZERO non-polarized Compton-like and pair annihilation squared amplitudes!

# Table of Contents


1. General remarks
2. Definition of ELKO spinor
3. Amplitudes

# Defining ELKO

Expressing the CHARGE CONJUGATION involutive operator as

$$\mathrm{C} = \begin{pmatrix} \mathbb{O} & i\Theta \\ -i\Theta & \mathbb{O} \end{pmatrix} \mathrm{K}, \tag{1}$$

One can define the ELKO, its **EIGENSPINORS** given in a $\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)$ representation (INTRINSICALLY NEUTRAL)

$$\mathrm{C}\lambda^S_\pm(p^\mu) = +\lambda^S_\pm(p^\mu), \qquad \mathrm{C}\lambda^A_\pm(p^\mu) = -\lambda^A_\pm(p^\mu). \tag{2}$$

$$\lambda^S_\pm = \lim_{\mathbf{p}\to 0} \begin{pmatrix} +i\Theta\left[\phi_\pm(p^\mu)\right]^* \\ \phi_\pm(p^\mu) \end{pmatrix}, \qquad \lambda^A_\pm = \lim_{\mathbf{p}\to 0} \begin{pmatrix} -i\Theta\left[\phi_\mp(p^\mu)\right]^* \\ \phi_\mp(p^\mu) \end{pmatrix}. \tag{3}$$

with the right/left handed structures transforming as $\left(\frac{1}{2},0\right)$ / $\left(0,\frac{1}{2}\right)$

$$(\boldsymbol{\sigma}\cdot\widehat{\mathbf{p}})\phi_\pm(k^\mu) = \pm\phi_\pm(k^\mu). \tag{4}$$

$$\boldsymbol{\sigma}\cdot\widehat{\mathbf{p}}\left[\Theta\phi^*_\pm(p^\mu)\right] = \mp\left[\Theta\phi^*_\pm(p^\mu)\right]. \tag{5}$$

# Defining ELKO

- Defining parity as $\mathrm{P} = m^{-1}\gamma^{\mu} p_{\mu}$ and the time-reversal operator $\mathrm{T} = -i\gamma_5 \mathrm{C}$ yields [Phys. Reports 2205.04754v1 (2022)]
$$(\mathrm{CPT})^2 = \mathbb{I}, \textit{and } \{\mathrm{C}, \mathrm{P}\} = 0 \tag{6}$$
- ELKO transform differently than Dirac: (Possible non-Standard Wigner Class)

$$\mathrm{P}\lambda^{S/A}_{\pm}(p^{\mu}) = \pm / \mp im\lambda^{S/A}_{\mp}(p^{\mu}) \tag{7}$$

$$\mathrm{T}\lambda^{S/A}_{\pm}(p^{\mu}) = \pm im\lambda^{S/A}_{\mp}(p^{\mu}) \tag{8}$$

- ELKO: general non-vanishing momenta by boost $\lambda^{S/A}_{\pm}(p^{\mu}) = \mathrm{D}(L(p)) \, \lim_{\mathbf{p}\to 0} \lambda^{S/A}_{\pm}(p^{\mu})$.

Field expansion prescription:

$$\mathfrak{f}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{(2mE(\mathbf{p}))^{1/2}} \sum_{\alpha} \left[ a_{\alpha}(\mathbf{p}) \varepsilon_{\alpha}(\mathbf{p}) e^{-ip^{\mu}x_{\mu}} + b^{\dagger}_{\alpha}(\mathbf{p}) \chi_{\alpha}(\mathbf{p}) e^{ip^{\mu}x_{\mu}} \right]. \tag{9}$$

# Table of Contents


1. General remarks
2. Definition of ELKO spinor
3. Amplitudes

# Rotationally covariant basis, Dual, spin sums (Nuclear Physics B, Vol. 987 (2023) 116092)

This basis overcomes a recent criticism, successfully implementing FULL ROTATION COVARIANCE (in a Wigner anti-linear setting.)

$$\varepsilon_1(p) = \lambda^S_+(p)\ , \quad \varepsilon_2(p) = \lambda^S_-(p) \tag{10}$$

$$\varepsilon_3(p) = -i\lambda^A_+(p)\ , \quad \varepsilon_4(p) = -i\lambda^A_-(p), \tag{11}$$

and the anti-self-conjugate one has

$$\begin{aligned} \chi_1(p) &= \lambda^A_+(p)\ , \quad \chi_2(p) = \lambda^A_-(p) \\ \chi_3(p) &= -i\lambda^S_+(p)\ , \quad \chi_4(p) = -i\lambda^S_-(p). \end{aligned} \tag{12}$$

with both bases being defined in terms of the ELKO spinors.

The dual is defined as

$$\bar{\varepsilon}_\sigma(p) = \Big[\mathcal{P}\varepsilon_\sigma\Big]^\dagger(p)\gamma_0, \qquad \bar{\chi}_\sigma(p) = \Big[-\mathcal{P}\chi_\sigma\Big]^\dagger(p)\gamma_0, \tag{13}$$

with $\mathcal{P} = \frac{\not{p}}{m}$.

# The Dual, spin sums

- Successfully implementing Well-defined expressions for products, spin sums, etc

$$\overset{\neg}{\varepsilon}_{\alpha}(p^{\mu})\varepsilon_{\alpha'}(p^{\mu}) = m\delta_{\alpha\alpha'} = -\overset{\neg}{\chi}_{\alpha}(p^{\mu})\chi_{\alpha'}(p^{\mu}), \tag{14}$$

with spin sums

$$\sum_{\alpha} \varepsilon_{\alpha}(p^{\mu})\overset{\neg}{\varepsilon}_{\alpha}(p^{\mu}) = 2m\mathbb{I} = -\sum_{\alpha} \chi_{\alpha}(p^{\mu})\overset{\neg}{\chi}_{\alpha}(p^{\mu}), \tag{15}$$

**with completeness relation**

$\sum_{\alpha}\left[\varepsilon_{\alpha}(p^{\mu})\overset{\neg}{\varepsilon}_{\alpha}(p^{\mu}) - \chi_{\alpha}(p^{\mu})\overset{\neg}{\chi}_{\alpha}(p^{\mu})\right] = 4m\mathbb{I}.$

- ELKO do not obey Dirac, just Klein-Gordon equation

$$\gamma_{\mu}p^{\mu}\lambda^{S}_{\pm}(p^{\mu}) = \pm im\lambda^{S}_{\mp}(p^{\mu}), \qquad \gamma_{\mu}p^{\mu}\lambda^{A}_{\pm}(p^{\mu}) = \mp im\lambda^{A}_{\mp}(p^{\mu}), \tag{16}$$

- **The model has the following Lagrangian ensuring KG equation**

$$\mathcal{L}_{\text{FREE}}(x) = \partial_\mu \, \vec{\mathfrak{f}}\,(x)\partial^\mu \mathfrak{f}(x) - m^2 \, \vec{\mathfrak{f}}\,(x)\mathfrak{f}(x) \tag{17}$$

It is invariant under ‡ conjugation (generalized Hermitian conjugation) $\mathcal{L}_{\text{FREE}} = \mathcal{L}^{\ddagger}_{\text{FREE}}$ defined as

$$\left[\lambda^{S/A}_{\alpha}(\mathbf{p})\right]^{\ddagger} = -i\alpha \left[\lambda^{S/A}_{-\alpha}(\mathbf{p})\right]^{\dagger} . \tag{18}$$

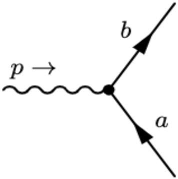
- The interaction lagrangian couples directly with physical fields being immediately gauge invariant (photon interaction with neutral fermions!)

$$\mathcal{L} = \frac{g}{2} i \, \vec{\mathfrak{f}} \, \sigma_{\mu\nu} F^{\mu\nu} \mathfrak{f} \tag{19}$$

**With** $\sigma_{\mu\nu} \equiv \frac{1}{2}[\gamma_\mu, \gamma_\nu]$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

# Interaction part

## The interaction vertex




$$= igp_{\mu}\sigma^{\mu\nu}_{ba} \tag{20}$$

The BOSONIC sector is composed of the Podolsky Lagrangian (No mixing gauge)

$$\mathcal{L}_{\text{POD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2M^2}\partial_{\mu}F^{\mu\nu}\partial^{\gamma}F_{\gamma\nu} + \frac{\lambda}{2}\left(a^2\Box + 1\right)\left(\partial_{\mu}A^{\mu}\right)^2 \tag{21}$$

**Electron-positron scattering with** $12$ **GeV** $\lesssim \sqrt{s} \lesssim 46.8$ **GeV yields the bound** $M \gtrsim 370$ **GeV.**

- **The amplitude for the Compton scattering reads** $\left(\Gamma_\mu(k) = \gamma_\mu \not{k} - k_\mu\right)$

$$\mathcal{M}_{\alpha,\alpha',\beta,\beta'} = \frac{g^2}{2m\left(s-m^2\right)} i \, \vec{\varepsilon}_\beta(p')\Gamma^\nu(k')\Gamma^\sigma(k)\epsilon^{*\alpha}_\nu(k')\epsilon^{\alpha'}_\sigma(k)\varepsilon_{\beta'}(p) + \frac{g^2}{2m\left(u-m^2\right)} i \, \vec{\varepsilon}_\beta(p')\Gamma^\sigma(k)\Gamma^\nu(k')\epsilon^{*\alpha}_\nu(k')\epsilon^{\alpha'}_\sigma(k)\varepsilon_{\beta'}(p). \quad (22)$$

Replacing the $s$ channel by $t$, $\vec{\varepsilon}_\beta(p')$ by $\vec{\chi}_\beta(p')$ and with all external photons being conjugated, we obtain the amplitude for the DARK MATTER pair annihilation

# Compton and pair annihilation non-polarized process

- **In both † and ‡ prescription, the probability is proportional to**

$$\sum_{\alpha,\alpha',\beta,\beta'} \mathcal{M}_{\alpha,\alpha',\beta,\beta'}\mathcal{M}^{\ddagger}_{\alpha,\alpha',\beta,\beta'} \tag{23}$$

is proportional to a linear combination of the following terms

$$Tr[\Gamma^{\nu}(k)\Gamma^{\sigma}(k')\Gamma_{\sigma}(k')\Gamma_{\nu}(k)], \qquad Tr[\Gamma^{\sigma}(k')\Gamma^{\nu}(k)\Gamma_{\sigma}(k')\Gamma_{\nu}(k)] \tag{24}$$

with both being proportional to $(k^{\mu}k_{\mu})(k'_{\mu}k'^{\mu}) = 0$ In interacting case, the massive excitation becomes UNSTABLE just contributing to internal lines. The asymptotic spectrum is free of massive photons.

- **This is our definition of darkness**

# Darkness survive full quantum corrections?

**From the path integral U(1) invariance** $\delta A^\mu(x) = \partial_\mu \Lambda$, $\delta \mathfrak{f}(x) = 0$ (Neutral.)

$$
\begin{aligned}
Z &= N \int \prod_{\mu=0}^{3} \mathcal{D}A^\mu \, \mathcal{D}\,\vec{\mathfrak{f}}\, \mathcal{D}\mathfrak{f} \\
&\times \exp\Bigg[ i \int d^4x - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2M^2} \partial_\mu F^{\mu\nu} \partial^\gamma F_{\gamma\nu} \\
&\qquad + \left(\partial_\mu \,\vec{\mathfrak{f}}\, \partial^\mu \mathfrak{f} - m^2 \,\vec{\mathfrak{f}}\, \mathfrak{f}\right) + \frac{ig}{2} \,\vec{\mathfrak{f}}\, \sigma_{\mu\nu} \mathfrak{f} \left(F^{\mu\nu}\right) \\
&\qquad + \lambda\left(\frac{\Box}{M^2} + 1\right)\left(\partial_\mu A^\mu\right)^2 + J_\mu A^\mu + \bar{T}\mathfrak{f} + \vec{\mathfrak{f}}\, T \Bigg)\Bigg],
\end{aligned} \tag{25}
$$

one can obtain the Ward-like identity

$$
K^\alpha \frac{\delta^3 \Gamma}{\delta \,\vec{\mathfrak{f}}\,(p) \delta \mathfrak{f}(\tilde{p}) \delta A_\alpha(k)} = 0 \tag{26}
$$

# Darkness survive full quantum corrections?

The existence of the basis $\mathfrak{G}_a = \{\mathbb{I}_{4\times4}, \gamma_\mu, \gamma_\mu\gamma_5, \gamma_5, \sigma_{\mu\nu}\}$ allows us to conclude (until 2-loops)

$$g\Gamma_\alpha^{2-LOOP}(p, \tilde{p}, k = p - \tilde{p}) = \; g\mathfrak{F}(p, \tilde{p}, k = p - \tilde{p})\sigma_{\alpha\nu}k^\nu. \qquad (27)$$

This, together with Schwinger-Dyson equations ensuring the ELKO propagator structure, may prove Darkness prevalence radiative corrections until $g^6$ order in cross-section.

- It is valid until $g^6$ even when considering 4-point 1PI contributions and complete vertexes to Compton. However, box-box products violate the property. That is why it is valid just until $g^6$ and not $g^8$.

# Darkness survives full quantum corrections?

- One loop contributions to Compton scattering. (The associated amplitude respects darkness for order $g^6$!)
- This is enough for phenomenological darkness (i.e. Standard QED measures 70 BOX events in hundred billion tentatives in LHC!... our coupling is $10^5$ times smaller!)

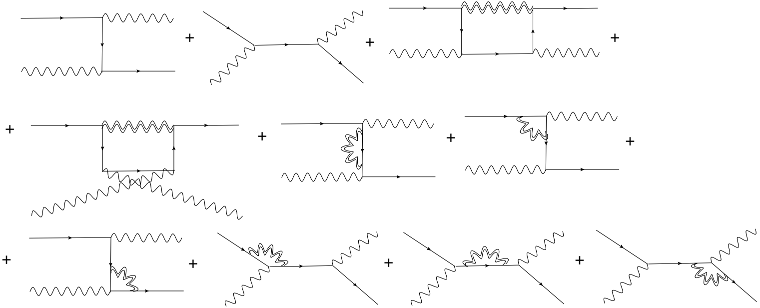



Figure: Compton radiative corrections.

# Nucleon Recoil Bounds

CONSIDERING...

- **The effective LOW ENERGY vertex for photon-proton interaction is**

$$\mathcal{V}_\mu \equiv \frac{1}{m'}\sigma_{\mu\nu}p^\nu + \gamma_\mu \tag{28}$$

with $m' \sim 1GeV$ denoting the proton mass.

- Using our machinery and the fact that the velocity $232Km/s$ of the DM approaching the target made of protons
- The cross-section bound from LUX (for spin-dependent WIMP-proton interaction) $\sigma \leq 10^{-39}cm^2$
- considering the very stringent limits, a WIMP with mass $\sim GeV$ is our focus. Namely, (2206.06772) leads to an event rate peak at a possible WIMP with

$$M \approx 5GeV \tag{29}$$

WE CAN CALCULATE THE CROSS-SECTION $\mathfrak{f} + p \to \mathfrak{f} + p$ (natural units.)

$$g \leq 10^{-5} \tag{30}$$