ELKO SPINOR AS DARK MATTER

specific signatures and constraints for the interacting model

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General remarks

Dark matter has strong evidence due to gravitational effects such as

- Galaxy rotation curve
- gravitational lensing effects

However,

- No interaction with standard matter particles

Therefore, some **NON-Luminous** matter must be throughout the universe.

According to cosmology $\sim 85\%$ **of the total mass of the universe is associated to dark matter.**

General remarks

- Standard gravitational theory implies in decaying velocity outside the region with the major quantity of matter (Luminous) at $R \sim 10,000$ light years.(Red line.)
- The observed curve must be explained by some non-luminous matter in the region $> R$

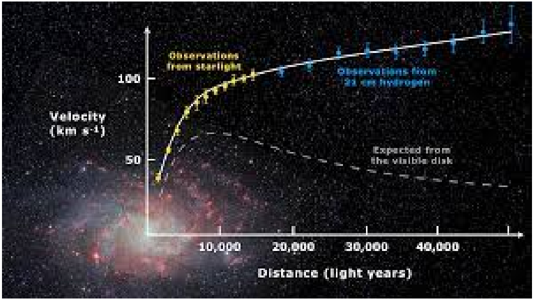

Figure: spiral galaxy Messier 33

General remarks

ELKO spinor is defined as eigenspinors of charge conjugation operator. It transforms differently than Dirac ones under discrete symmetries C, P, T. They are candidates for a non-standard Wigner class.

It has some important characteristics:

- Mass dimension one
- Just obey KG equation
- Then, **cannot** enter the standard model doublets in a **renormalizable** manner
- It is also important to highlight the fact that ELKO spinors enter as CLASS 5 spinors in LOUNESTO classification, DIFFERENTLY FROM MAJORANA fermions. Another characteristic feature is the fact that they DO NOT HAVE A DEFINITE HELICITY, each quiral component has a different one.

XENON-nT (noble gas)-Gran Sasso-Italy

- On the ground, low contamination, and $v = 232 Km/s$ with relation to DM approximately in rest in the center of the galaxy.
- Recognize scintillation due to nucleon recoil.
- Well known background. Searching for excess with statistical significance in terms of standard deviation.

Experimental searches

However, there are recent searches for Dark matter in nucleon recoil LAB experiments

- Xenon-nT, Lux, zeplin, CDMS, etc (scintillation)
- phonon mediated detections-2022
- Cross section $\sigma \leq 10^{-39} cm^2$
- LAB **WIMP Mass expected to be of order** GeV event rate peak 2206.06772
- Tritium contamination in XENON (2020): fake $m \sim 2,3 Kev$ WIMP
- Estimate from ELKO and Galaxy rotation curve $m \sim 0,1 KeV$

To be detected in LAB (ordinary matter devices.) there must be coupling involving an **INTERMEDIATE boson** and the ELKO and also one involving this boson and the nucleons

Perspectives

- **A coupling with photon would be fine**(It is possible to find an exotic RENORMALIZABLE one.)
- But how to keep the darkness property with such coupling?
- We show that it leads to ZERO non-polarized Compton-like and pair annihilation squared amplitudes!

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Expressing the CHARGE CONJUGATION involutive operator as

$$\mathrm{C} = \begin{pmatrix} \mathbb{O} & i\Theta \\ -i\Theta & \mathbb{O} \end{pmatrix} \mathrm{K}, \tag{1}$$

One can define the ELKO, its **EIGENSPINORS** given in a $\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)$ representation (INTRINSICALLY NEUTRAL)

$$\mathrm{C}\lambda^S_{\pm}(p^\mu) = +\lambda^S_{\pm}(p^\mu), \qquad \mathrm{C}\lambda^A_{\pm}(p^\mu) = -\lambda^A_{\pm}(p^\mu). \tag{2}$$

$$\lambda^S_{\pm} = \lim_{\mathbf{p}\to 0} \begin{pmatrix} +i\Theta\left[\phi_{\pm}(p^\mu)\right]^* \\ \phi_{\pm}(p^\mu) \end{pmatrix}, \qquad \lambda^A_{\pm} = \lim_{\mathbf{p}\to 0} \begin{pmatrix} -i\Theta\left[\phi_{\mp}(p^\mu)\right]^* \\ \phi_{\mp}(p^\mu) \end{pmatrix}. \tag{3}$$

with the right/left handed structures transforming as $\left(\frac{1}{2},0\right)$ / $\left(0,\frac{1}{2}\right)$

$$(\boldsymbol{\sigma}\cdot\widehat{\mathbf{p}})\phi_{\pm}(k^\mu) = \pm\phi_{\pm}(k^\mu). \tag{4}$$

$$\boldsymbol{\sigma}\cdot\widehat{\mathbf{p}}\left[\Theta\phi^*_{\pm}(p^\mu)\right] = \mp\left[\Theta\phi^*_{\pm}(p^\mu)\right]. \tag{5}$$

Defining ELKO

- Defining parity as $\mathrm{P} = m^{-1}\gamma^{\mu} p_{\mu}$ and the time-reversal operator $\mathrm{T} = -i\gamma_5 \mathrm{C}$ yields
$$(\mathrm{CPT})^2 = \mathbb{I}, \textit{and } \{\mathrm{C}, \mathrm{P}\} = 0 \tag{6}$$
- ELKO transform differently than Dirac: (Possible non-Standard Wigner Class)

$$\mathrm{P}\lambda_{\pm}^{S/A}(p^{\mu}) = \pm / \mp im\lambda_{\mp}^{S/A}(p^{\mu}) \tag{7}$$

$$\mathrm{T}\lambda_{\pm}^{S/A}(p^{\mu}) = \pm im\lambda_{\mp}^{S/A}(p^{\mu}) \tag{8}$$

- ELKO: general non-vanishing momenta by boost $\lambda_{\pm}^{S/A}(p^{\mu}) = \mathrm{D}(L(p)) \lim_{\mathbf{p}\to 0} \lambda_{\pm}^{S/A}(p^{\mu})$.

Field expansion prescription:

$$\mathfrak{f}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{(2mE(\mathbf{p}))^{1/2}} \sum_{\alpha} \left[a_{\alpha}(\mathbf{p})\varepsilon_{\alpha}(\mathbf{p}) e^{-ip^{\mu}x_{\mu}} + b_{\alpha}^{\dagger}(\mathbf{p})\chi_{\alpha}(\mathbf{p}) e^{ip^{\mu}x_{\mu}} \right]. \tag{9}$$

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Rotationally covariant basis, Dual, spin sums (Nuclear Physics B, Vol. 987 (2023) 116092)

This basis overcomes a recent criticism, successfully implementing FULL ROTATION COVARIANCE (in a Wigner anti-linear setting.)

$$\varepsilon_1(p) = \lambda^S_+(p)\ , \quad \varepsilon_2(p) = \lambda^S_-(p) \tag{10}$$

$$\varepsilon_3(p) = -i\lambda^A_+(p)\ , \quad \varepsilon_4(p) = -i\lambda^A_-(p), \tag{11}$$

and the anti-self-conjugate one has

$$\begin{aligned} \chi_1(p) = \lambda^A_+(p)\ , \quad \chi_2(p) = \lambda^A_-(p) \\ \chi_3(p) = -i\lambda^S_+(p)\ , \quad \chi_4(p) = -i\lambda^S_-(p). \end{aligned} \tag{12}$$

with both bases being defined in terms of the ELKO spinors.

The dual is defined as

$$\vec{\varepsilon}_\sigma(p) = \Big[\mathcal{P}\varepsilon_\sigma\Big]^\dagger(p)\gamma_0, \qquad \vec{\chi}_\sigma(p) = \Big[-\mathcal{P}\chi_\sigma\Big]^\dagger(p)\gamma_0, \tag{13}$$

with $\mathcal{P} = \frac{\not p}{m}$.

The Dual, spin sums

- Successfully implementing Well-defined expressions for products, spin sums, etc

$$\vec{\varepsilon}_\alpha(p^\mu)\varepsilon_{\alpha'}(p^\mu) = m\delta_{\alpha\alpha'} = -\vec{\chi}_\alpha(p^\mu)\chi_{\alpha'}(p^\mu), \tag{14}$$

with spin sums

$$\sum_\alpha \varepsilon_\alpha(p^\mu)\,\vec{\varepsilon}_\alpha(p^\mu) = 2m\mathbb{I} = -\sum_\alpha \chi_\alpha(p^\mu)\,\vec{\chi}_\alpha(p^\mu), \tag{15}$$

with completeness relation

$\sum_\alpha \left[\varepsilon_\alpha(p^\mu)\,\vec{\varepsilon}_\alpha(p^\mu) - \chi_\alpha(p^\mu)\,\vec{\chi}_\alpha(p^\mu)\right] = 4m\mathbb{I}.$

- ELKO do not obey Dirac, just Klein-Gordon equation

$$\gamma_\mu p^\mu \lambda^S_\pm(p^\mu) = \pm im\lambda^S_\mp(p^\mu), \qquad \gamma_\mu p^\mu \lambda^A_\pm(p^\mu) = \mp im\lambda^A_\mp(p^\mu), \tag{16}$$

- **The model has the following Lagrangian ensuring KG equation**

$$\mathcal{L}_{\text{FREE}}(x) = \partial_\mu \overset{\neg}{\mathfrak{f}}(x)\partial^\mu \mathfrak{f}(x) - m^2 \overset{\neg}{\mathfrak{f}}(x)\mathfrak{f}(x) \quad (17)$$

It is invariant under ‡ conjugation (generalized Hermitian conjugation) $\mathcal{L}_{\text{FREE}} = \mathcal{L}^{\ddagger}_{\text{FREE}}$ defined as

$$\left[\lambda^{S/A}_{\alpha}(\mathbf{p})\right]^{\ddagger} = -i\alpha \left[\lambda^{S/A}_{-\alpha}(\mathbf{p})\right]^{\dagger}. \quad (18)$$

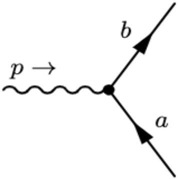
- The interaction lagrangian couples directly with physical fields being immediately gauge invariant

$$\mathcal{L} = \frac{g}{2} i \overset{\neg}{\mathfrak{f}} \sigma_{\mu\nu} F^{\mu\nu} \mathfrak{f} \quad (19)$$

With $\sigma_{\mu\nu} \equiv \frac{1}{2}[\gamma_\mu, \gamma_\nu]$

Interaction part

The interaction vertex



$$= igp_\mu \sigma^{\mu\nu}_{ba} \qquad (20)$$

The bosonic sector is composed of the Podolsky Lagrangian (No mixing gauge)

$$\mathcal{L}_{\text{POD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2M^2}\partial_\mu F^{\mu\nu}\partial^\gamma F_{\gamma\nu} + \frac{\lambda}{2}\left(a^2\Box + 1\right)\left(\partial_\mu A^\mu\right)^2 \qquad (21)$$

Electron-positron scattering with 12 **GeV** $\lesssim \sqrt{s} \lesssim 46.8$ **GeV yields the bound** $M \gtrsim 370$ **GeV.**

- **The amplitude for the Compton scattering reads** $\left(\Gamma_\mu(k) = \gamma_\mu \not{k} - k_\mu\right)$

$$\mathcal{M}_{\alpha,\alpha',\beta,\beta'} = \frac{g^2}{2m\left(s - m^2\right)} i\,\vec{\varepsilon}_{\beta}(p')\Gamma^{\nu}(k')\Gamma^{\sigma}(k)\epsilon^{*\alpha}_{\nu}(k')\epsilon_{\sigma}^{\alpha'}(k)\varepsilon_{\beta'}(p) + \frac{g^2}{2m\left(u - m^2\right)} i\,\vec{\varepsilon}_{\beta}(p')\Gamma^{\sigma}(k)\Gamma^{\nu}(k')\epsilon^{*\alpha}_{\nu}(k')\epsilon_{\sigma}^{\alpha'}(k)\varepsilon_{\beta'}(p). \quad (22)$$

Replacing the s channel by t, $\vec{\varepsilon}_{\beta}(p')$ by $\vec{\chi}_{\beta}(p')$ and with all external photons being conjugated, we obtain the amplitude for the DARK MATTER pair annihilation

Compton and pair annihilation non-polarized process

- **In both $\dagger$ and $\ddagger$ prescription, the probability is proportional to**

$$\sum_{\alpha,\alpha',\beta,\beta'} \mathcal{M}_{\alpha,\alpha',\beta,\beta'} \mathcal{M}^{\ddagger}_{\alpha,\alpha',\beta,\beta'} \tag{23}$$

is proportional to a linear combination of the following terms

$$Tr[\Gamma^{\nu}(k)\Gamma^{\sigma}(k')\Gamma_{\sigma}(k')\Gamma_{\nu}(k)], \qquad Tr[\Gamma^{\sigma}(k')\Gamma^{\nu}(k)\Gamma_{\sigma}(k')\Gamma_{\nu}(k)] \tag{24}$$

with both being proportional to $(k^{\mu}k_{\mu})(k'_{\mu}k'^{\mu}) = 0$ In interacting case, the massive excitation becomes a Merlin mode, just contributing to internal lines.

- **This is our definition of darkness**

From the path integral U(1) invariance $\delta A^{\mu}(x) = \partial_{\mu}\Lambda$, $\delta\mathfrak{f}(x) = 0$ (Neutral.)

$$
\begin{aligned}
Z &= N\int\prod_{\mu=0}^{3}\mathcal{D}A^{\mu}\,\mathcal{D}\,\vec{\mathfrak{f}}\,\mathcal{D}\mathfrak{f} \\
&\times\exp\Big[i\int d^4x - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2M^2}\partial_{\mu}F^{\mu\nu}\partial^{\gamma}F_{\gamma\nu} \\
&\qquad + (\partial_{\mu}\,\vec{\mathfrak{f}}\,\partial^{\mu}\mathfrak{f} - m^2\,\vec{\mathfrak{f}}\,\mathfrak{f}) + \frac{ig}{2}\,\vec{\mathfrak{f}}\,\sigma_{\mu\nu}\mathfrak{f}\,(F^{\mu\nu}) \\
&\qquad + \lambda(\frac{\Box}{M^2}+1)(\partial_{\mu}A^{\mu})^2 + J_{\mu}A^{\mu} + \bar{T}\mathfrak{f} + \vec{\mathfrak{f}}\,T\Big)\Big],
\end{aligned}
\tag{25}
$$

one can obtain the Ward identity

$$
K^{\alpha}\frac{\delta^3\Gamma}{\delta\,\vec{\mathfrak{f}}\,(p)\delta\mathfrak{f}(\tilde{p})\delta A_{\alpha}(k)} = 0 \tag{26}
$$

The existence of the basis $\mathfrak{G}_a = \{\mathbb{I}_{4\times4}, \gamma_\mu, \gamma_\mu\gamma_5, \gamma_5, \sigma_{\mu\nu}\}$ allows us to conclude (until 2-loops)

$$g\Gamma_\alpha^{2-LOOP}(p, \tilde{p}, k = p - \tilde{p}) = g\mathfrak{F}(p, \tilde{p}, k = p - \tilde{p})\sigma_{\alpha\nu}k^\nu. \qquad (27)$$

This, together with Schwinger-Dyson equations ensuring the ELKO propagator structure, may prove Darkness prevalence radiative corrections until g^6 order in cross-section.

- It is valid until g^6 even when considering 4-point 1PI contributions and complete vertexes to Compton. However, box-box products violate the property. That is why it is valid just until g^6 and not g^8.

- One loop contributions to Compton scattering.

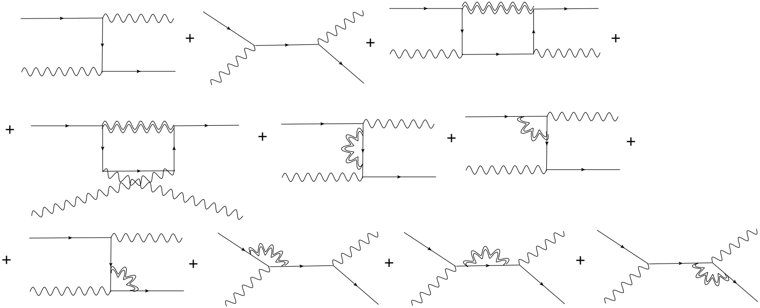


Figure: Compton radiative corrections.

Nucleon Recoil Bounds

CONSIDERING...

- **The effective low energy vertex for photon-proton interaction is**

$$\mathcal{V}_\mu \equiv \frac{1}{m'}\sigma_{\mu\nu}p^\nu + \gamma_\mu \quad (28)$$

  with $m' \sim 1GeV$ denoting the proton mass.
- Using our machinery and the fact that the velocity $232Km/s$ of the DM approaching the target made of protons
- The cross-section bound from LUX (for spin-dependent WIMP-proton interaction) $\sigma \leq 10^{-39}cm^2$
- considering the very stringent limits, a WIMP with mass $\sim GeV$ is our focus. Namely, (2206.06772) leads to an event rate peak at a possible WIMP with

$$M \approx 5GeV \quad (29)$$

WE CAN CALCULATE THE CROSS-SECTION $\mathfrak{f} + p \rightarrow \mathfrak{f} + p$ (natural units.)

$$g \leq 10^{-5} \quad (30)$$