Galactic cosmic rays (II)

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Outline

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Definitions

- We will consider the differential (in momentum p) particle density ψ,

$$\psi(\mathbf{r}, p, t) = 4\pi p^2 f_0(\mathbf{r}, p, t) = \frac{\mathrm{d}n}{\mathrm{d}p}$$

- Can also express this as differential in energy E, $\tilde{\psi}$,

$$\tilde{\psi}(\mathbf{r}, p, t) = \frac{\mathrm{d}n}{\mathrm{d}E} = \frac{\mathrm{d}p}{\mathrm{d}E}\psi(\mathbf{r}, p, t) = \frac{E}{pc^2}\psi(\mathbf{r}, p, t)$$

- What is measured is the intensity

$$J(\mathbf{r}, p, t) = \frac{\mathrm{d}^4 N}{\mathrm{d}A\,\mathrm{d}t\,\mathrm{d}\Omega\,\mathrm{d}E} = \frac{v}{4\pi}\tilde{\psi}(\mathbf{r}, p, t) = p^2 f_0(\mathbf{r}, p, t)$$

The transport equation

$$
\begin{aligned}
\frac{\partial \psi_j}{\partial t} = & \nabla \cdot (\kappa \cdot \nabla \psi_j - \mathbf{U} \psi_j) && \text{spatial diffusion and advection} \\
& + \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi_j \right) && \text{momentum diffusion} \\
& + \frac{\partial}{\partial p} \left(- \frac{\mathrm{d}p}{\mathrm{d}t} \psi_j + \frac{p}{3} (\nabla \cdot \mathbf{U}) \psi_j \right) && \text{momentum change incl. adiabatic} \\
& - v n_{\text{gas}} \sigma_j \psi_j - \frac{\psi_j}{\tau_j} && \text{spallation and decay} \\
& + v n_{\text{gas}} \sum_{k>j} \sigma_{k \to j} \psi_k + \sum_{k>j} \frac{\psi_k}{\tau_{k \to j}} && \text{spallation and decay} \\
& + S_j && \text{primary sources}
\end{aligned}
$$

Mathematical description

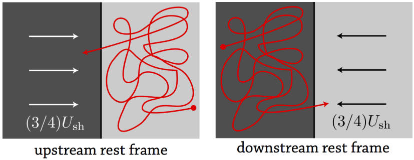
Ginzburg & Syrovatskii (1964)

Transport equation

$$\frac{\partial \psi_j}{\partial t} - \nabla \cdot (\kappa \cdot \nabla \psi_j - \mathbf{u} \psi_j) + \frac{\partial}{\partial p} \left(\frac{p}{3} (\nabla \cdot \mathbf{u}) \psi_j \right) = q_j$$

↓

Application to blast wave:



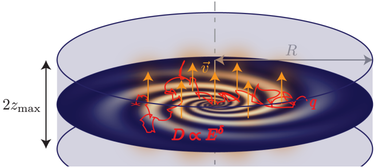
Shock acceleration

Source spectrum: $q(\mathcal{R}) \propto \mathcal{R}^{-2.4\ldots-1.9}$

Axford, Leer, Skadron (1977); Krymskii (1977); Bell (1978); Blandford, Ostriker (1978)

↓

Application to galactic halo:



Diffusive escape

Observed spectrum: $\psi(\mathcal{R}) \propto \frac{q(\mathcal{R})}{\kappa(\mathcal{R})}$ e.g. $\frac{\mathcal{R}^{-2.2}}{\mathcal{R}^{0.6}} \propto \mathcal{R}^{-2.8}$

The transport equation

$$
\begin{aligned}
\frac{\partial \psi_j}{\partial t} =& \nabla \cdot (\kappa \cdot \nabla \psi_j - \mathbf{U} \psi_j) \\
&+ \frac{\partial}{\partial p}\left(p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi_j \right) \\
&+ \frac{\partial}{\partial p}\left(-\frac{\mathrm{d}p}{\mathrm{d}t} \psi_j + \frac{p}{3} (\nabla \cdot \mathbf{U}) \psi_j \right) \\
&- v n_{\text{gas}} \sigma_j \psi_j - \frac{\psi_j}{\tau_j} \\
&+ v n_{\text{gas}} \sum_{k>j} \sigma_{k \to j} \psi_k + \sum_{k>j} \frac{\psi_k}{\tau_{k \to j}} \\
&+ S_j
\end{aligned}
$$

spatial diffusion and advection

momentum diffusion

momentum change incl. adiabatic

spallation and decay

spallation and decay

primary sources

CRs are a collisionless plasma

- From local measurement $\varepsilon_{\mathrm{CR}} \sim 1\,\mathrm{eV\,cm}^{-3} \Rightarrow n_{\mathrm{CR}} \sim 10^{-9}\mathrm{cm}^{-3}$
- Inelastic collisions, typically $\sigma \sim 100\,\mathrm{mb} = 10^{-25}\,\mathrm{cm}^2$

$$\begin{aligned} \Rightarrow \lambda &= (n_{\mathrm{CR}}\sigma)^{-1} \\ &= 10^{34}\,\mathrm{cm} \simeq 3 \times 10^{12}\,\mathrm{kpc} \end{aligned}$$

$\rightarrow$ (mean free path) $\gg$ (size of the system)

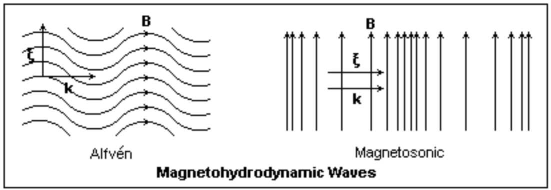
$\rightarrow$ CRs described by the collisionless Boltzmann equation:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \dot{\mathbf{x}} \cdot \nabla f + \dot{\mathbf{p}} \cdot \nabla_{\mathbf{p}} f = 0$$

Diffusion

- But CRs interact with the magnetic field in the background plasma
- Homogeneous field $\mathbf{B}_0 \rightarrow$ gyration
- Magneto-hydrodynamic (MHD) equations allow for small-amplitude solutions $\delta\mathbf{B}$: plasma waves

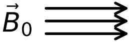
Examples



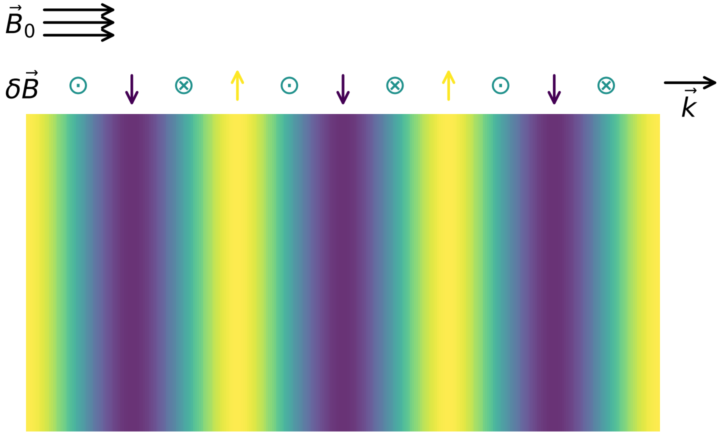
- Can perturbatively derive a diffusion equation:

$$\frac{\partial \psi}{\partial t} - \nabla \cdot (\kappa \cdot \nabla - \mathbf{U})\,\psi = 0$$

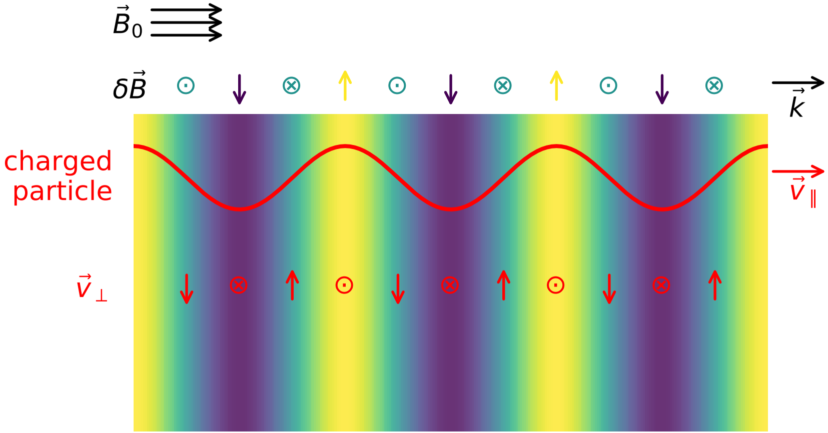
Momentum-dependence of diffusion coefficient



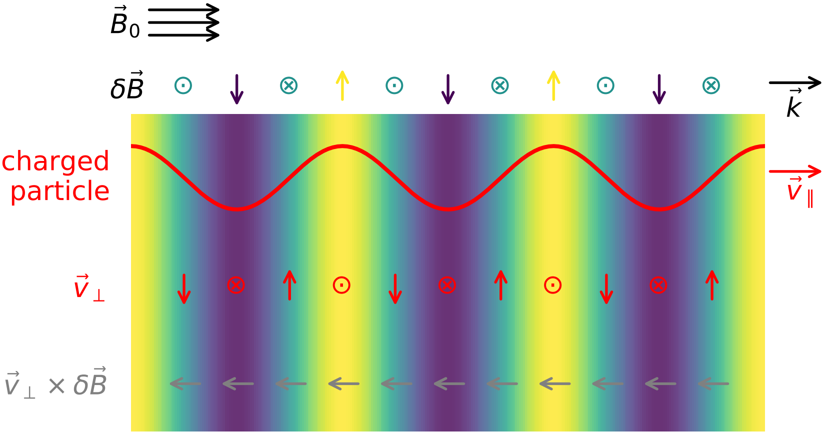
Momentum-dependence of diffusion coefficient



Momentum-dependence of diffusion coefficient

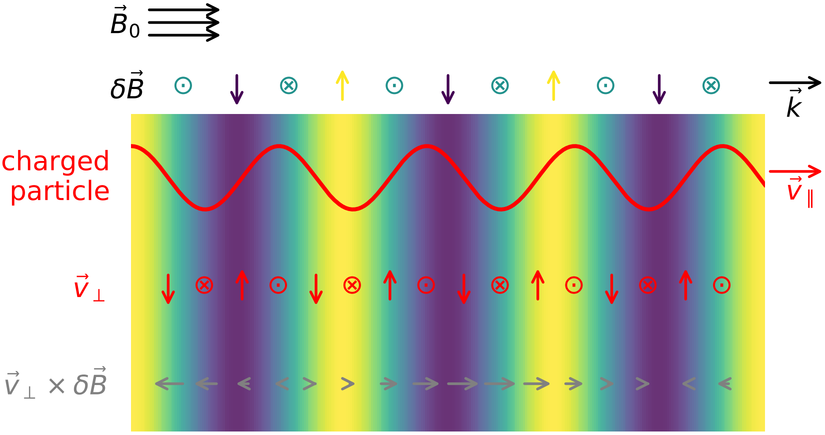


Momentum-dependence of diffusion coefficient



If $kr_g = 1 \quad \Rightarrow \quad$ coherent force over many gyrations

Momentum-dependence of diffusion coefficient



If $kr_g \neq 1$ $\Rightarrow$ **NO** coherent force

Diffusion

- Charged particles interact resonantly with wave:

$$k_{\mathrm{res}} = r_g^{-1} \propto p^{-1}$$

- Scattering rate Γ is gyrofrequency Ω reduced by density of resonant plasma waves:

$$\Gamma(k) \simeq \Omega \frac{\delta \tilde{B}^2(k)}{B_0^2} \simeq \Omega \frac{k\, P(k)}{B_0^2}$$

- The power spectrum of resonant plasma waves is:

$$P(k) \sim \int \mathrm{d}^3 r\, \mathrm{e}^{i\mathbf{r}\cdot\mathbf{k}} \langle \delta\mathbf{B}(\mathbf{r}_0)\delta\mathbf{B}(\mathbf{r}_0 + \mathbf{r})\rangle \propto k^{-5/3}$$

- Interaction with ensemble of random waves $\to$ diffusion:

$$\kappa \simeq \frac{v^2}{3\Gamma} \propto \frac{1}{\Omega} \left(\frac{k\, P(k)}{B_0^2} \right)^{-1} \bigg|_{k=k_{\mathrm{res}}} \propto p^{1/3}$$

Momentum diffusion

- Waves carry a momentum
- When scattering particles, can transfer momentum to CRs
- If angular distribution of waves is not isotropic, CRs get accelerated
- Stochastic process, can be described as diffusion in momentum space:

$$\frac{\partial f}{\partial t} - \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial f}{\partial p} \right) = 0$$

$$\frac{\partial \psi}{\partial t} - \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi \right) = 0$$

Momentum change

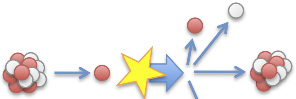
- CRs lose energy by interactions with radiation fields:
  - Synchrotron emission
  - Inverse Compton scattering
- ... or gas:
  - Bremsstrahlung
  - Ionisation losses
  - Coulomb scattering
- Adiabatic losses or gains:

$$\left(\frac{\mathrm{d}p}{\mathrm{d}t} \right)_{\mathrm{ad}} = -\frac{p}{3} \left(\nabla \cdot \mathbf{U} \right)$$

- In transport equation:

$$\frac{\partial \psi}{\partial t} - \frac{\partial}{\partial p} \left(-\frac{\mathrm{d}p}{\mathrm{d}t} \psi + \frac{p}{3} \left(\nabla \cdot \mathbf{U} \right) \psi \right) = 0$$

Spallation and decay



- In leaky box model, considered a global rate, e.g.

$$\frac{\mathrm{d}N_2}{\mathrm{d}t} = -\frac{N_2}{t_{\mathrm{esc}}} - N_2\Gamma_2 + \Gamma_{1\to 2}N_1$$

- In a diffusion model

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} \supset -v n_{\mathrm{gas}}\sigma_j\psi_j + v n_{\mathrm{gas}}\sum_{k>j}\sigma_{k\to j}\psi_k$$

- Additionally, decay:

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} \supset -\frac{\psi_j}{\tau_j} + \sum_{k>j}\frac{\psi_k}{\tau_{k\to j}}$$

The transport equation

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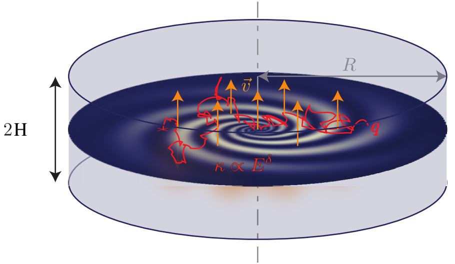
Outline

Exercise 1: Boron-to-carbon ratio

Boron-to-carbon ratio (I)



- Solve a 1D, simplified version of the transport equation with homogeneous diffusion, advection, spallation, but no energy losses, radioactive decays or second-order Fermi acceleration, assuming steady-state,
$$-\kappa \frac{\mathrm{d}^2\psi}{\mathrm{d}z^2} + V \frac{\mathrm{d}\psi}{\mathrm{d}z} = 2h\delta(z)(q - \Gamma\psi) \, .$$
- $\psi \mathrm{d}E$ is the number density of CRs with kinetic energy in $[E, E + \mathrm{d}E]$
- $\kappa = \kappa(E)$ is the diffusion coefficient
- V is the advection speed away from the disk
- h is the disk height
- $q = q(E)$ is the source spectrum
- $\Gamma = c\sigma_{\mathrm{tot}} n_{\mathrm{gas}}$ is the total destruction rate by inelastic collisions

Boron-to-carbon ratio (II)

1. Show that for $z \neq 0$, the solution is of the form

$$\psi = A + B \mathrm{e}^{\frac{Vz}{\kappa}} \, ,$$

with A, B functions of E.

2. Integrate the transport equation across the Galactic disk and use the symmetry of the solution with respect to $z = 0$ to find the following relation between A and B,

$$-VB = qh - \Gamma h (A + B) \, .$$

3. Let H be the halo half-height. Employ the boundary condition at $|z| = H$, that is $\psi(\pm H) \equiv 0$, in order to express A through B,

$$A = -B \mathrm{e}^{\frac{VH}{\kappa}} \, .$$

4. Putting everything together, show that

$$\psi = \frac{-qh \left(\mathrm{e}^{\frac{Vz}{\kappa}} - \mathrm{e}^{\frac{VH}{\kappa}} \right)}{V - \Gamma h \left(1 - \mathrm{e}^{\frac{VH}{\kappa}} \right)} = \ldots = \underbrace{\frac{2qh}{V \left(1 + \coth \left(\frac{VH}{2\kappa} \right) \right) + 2 \Gamma h}}_{\psi(z=0)} \mathrm{e}^{\frac{Vz}{2\kappa}} \frac{\sinh \left(\frac{V(H-z)}{2\kappa} \right)}{\sinh \left(\frac{VH}{2\kappa} \right)} \, .$$

Boron-to-carbon ratio (III)

5. Assume one primary species, C, and one stable secondary species, B, and find the analytic expression for the boron-to-carbon ratio (B/C) in the disk,

$$\frac{\psi_2}{\psi_1} = \frac{2hc\sigma_{1\to 2} n_{\text{gas}}}{V\left(1 + \coth\left(\frac{VH}{2\kappa}\right)\right) + 2\Gamma_2 h} \, .$$

6. Adopting the following fixed paramters,

Quantity	Symbol	Value
Half-height of CR halo	H	5 kpc
Half-height of disk	h	0.1 kpc
Gas density in disk	n_{gas}	$1\,\text{cm}^{-3}$
Boron inelastic cross-section	$\sigma_{\text{tot,B}}$	230 mbarn
Production cross-section for boron	$\sigma_{\text{C}\to\text{B}}$	150 mbarn[a]

write a python script to plot the predicted B/C as a function of energy-per-nucleon E_n. For the diffusion coefficient, assume the following behaviour

$$\kappa(E) = \kappa_0 \left(\frac{E_n}{1\,\text{GeV}}\right)^{\delta} \, .$$

Typical values are $\kappa_0 = 0.1\,\text{kpc}^2\,\text{Myr}^{-1}$, $\delta = 1/3$ and $V = 10^{-2}\,\text{kpc}\,\text{Myr}^{-1}$.

[a] Really more like 30 mb, but here with fudge factor of 5 to take into account other channels.

Boron-to-carbon ratio (IV)

7. In `python`, write a code to plot AMS-02 data on B/C. To this end, `pip`-install the `CRDB` package,

```
$ pip install crdb
```

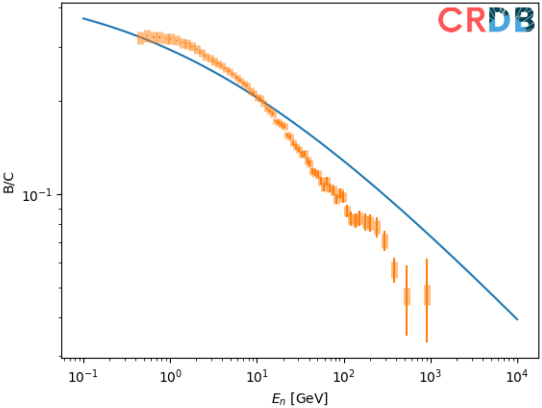
and use the following python fragment:

```python
import crdb
from crdb.mpl import draw_table, draw_logo
import matplotlib.pyplot as plt
import numpy as np

tab = crdb.query("B/C", energy_type="EKN")
## crdb query returns a structured Numpy array:
## the energies can be accessed as ‘t.e‘,
## the B/C values as ‘t.value‘
## and the statistical errors as ‘t.err_sta‘
plt.figure()
mask = crdb.experiment_masks(tab)['AMS02']
t = tab[mask]
draw_table(t, label="AMS02")
plt.xlabel(r"$E_n$ [GeV]")
plt.ylabel("B/C")
plt.loglog()
draw_logo(0.78, 1)
plt.show()
```

Boron-to-carbon ratio (V)

8. Overplotting the AMS-02 data with your model prediction for the typical parameter values, you should find



Now find the best-fit κ_0, δ and V by minimising the χ^2.

Fitting the boron-to-carbon ratio (VI)

- A couple of caveats:
  - Solar modulation
  - Isotopes
  - Actual cross-sections
  - Carbon is not a pure primary
  - Height of CR halo is also a free parameter

Any questions?