Constraining the VSR origin of graviton mass [1] through binary pulsars [2]

Alessandro Santoni, Jorge Alfaro, Alex Soto

Pontificia Universidad Católica de Chile
Vienna University of Technology
asantoni@uc.cl

Astroparticle School 2023, Obertrubach, Germany

October 3, 2023

[1] J. Alfaro and A. Santoni. "Very special linear gravity: A gauge-invariant graviton mass", Phys. Lett. B 829:137080 (2022)

[2] A. Santoni, J. Alfaro and A. Soto, "Graviton mass bounds in very special relativity from binary pulsar's gravitational waves", Phys. Rev. D 108, 044072

Very Special Relativity[3] (VSR)

- New Spacetime symmetry: **T(4) + Lorentz Subgroups***
- VSR $+$ P, T or CP $\rightarrow$ Full Lorentz Group

Small **CP VIOLATION** $\Longleftrightarrow$ Small **VSR EFFECTS** .

Preferred light-like spacetime direction $n_\mu = (1,0,0,1)$

$$n_\mu \underset{SIM(2)}{\longrightarrow} e^{\phi} n_\mu \, .$$

Ratios of scalar products with n^μ are invariants under VSR

[3] Andrew G Cohen and Sheldon L Glashow. "Very special relativity", Physical review letters, 97(2):021601 (2006)

Quadratic VSR Lagrangian of the $h_{\mu\nu}$ spin-2 field ($|h_{\mu\nu}| << \eta_{\mu\nu}$):

$$L = \frac{1}{2}\, h^{\mu\nu} O_{\mu\nu\alpha\beta} h^{\alpha\beta} \qquad (1)$$

Ingredients:

- Metric $\eta_{\mu\nu}$
- Momentum p^{μ}
- VSR vector $N^{\mu} = n^{\mu}/(n \cdot p)$

$$O = 3\,\eta\eta + 9\,pp\eta + 12\,pN\eta + 12\,ppNN\ , \qquad (2)$$

Rules:

- Index Symmetries: $\mu \Longleftrightarrow \nu$, $\alpha \Longleftrightarrow \beta$, $\mu\nu \Longleftrightarrow \alpha\beta$
- Gauge Invariance: $O_{\mu\nu\alpha\beta}\, p^{\alpha} = 0$

Gauge Choices $\partial_\mu h^{\mu\nu} = 0; \; N_\mu h^{\mu\nu} = 0; \; h = h^\mu_\mu = 0$

$\rightarrow$ New E.o.M. is Klein-Gordon

$$(p^2 - m_g^2) h_{\mu\nu} = 0 \tag{3}$$

m_g is a graviton mass! Interesting for:

- Universe's Accelerated Expansion
- Dark Matter[4]

Gauge Invariance $\Longleftrightarrow$ only **2 Physical Degrees of Freedom**.

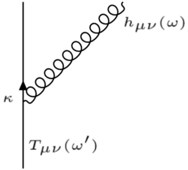
[4] K. Aoki and S. Mukohyama. "Massive gravitons as darkmatter and gravitational waves", Physical Review D94:024001 (2016)

Binary stars are great astronomical experimental tool since their discovery back in 1974.

Why Binaries for VSR?

1. Small effects integrated over long times
2. Most common source of GW

"Effective Field theory" (EFT) calculation with classical source[5]



The vertex should be gauge invariant and linear in $h_{\mu\nu}$

$$\to \frac{\kappa}{2} T_{\mu\nu} h^{\mu\nu} \text{ with } \kappa = \sqrt{32\pi G} \quad (4)$$

Then, the total energy loss rate will be simply

$$\frac{dE}{dt} = \int \frac{\omega}{T} dP = \int \frac{d^3k}{2T(2\pi)^3} \sum_\lambda |A_\lambda|^2 \quad (5)$$

with the **unpolarized squared amplitude**

$$\sum_\lambda |A_\lambda|^2 = \frac{\kappa^2}{4} \tilde{T}^*_{\mu\nu} \tilde{T}_{\alpha\beta} \sum_\lambda \epsilon^{*\mu\nu}_\lambda \epsilon^{\alpha\beta}_\lambda$$

[5]Goldberger, Walter D. "Effective field theories and gravitational radiation." Les Houches. Vol. 86. Elsevier, 351-396 (2007)

Classically, the period loss rate is proportional to the energy one

$$\dot{P}_b = \frac{dP_b}{dt} \propto \frac{dE}{dt} \tag{6}$$

Thus, defining $\delta \equiv m_g/\Omega$, we have

$$\dot{P}_{VSR} = -\frac{192\pi\, T_{\odot}^{\frac{5}{3}}}{5} \frac{\tilde{m}_1 \tilde{m}_2}{\tilde{M}^{\frac{1}{3}}} \left(\frac{P_b}{2\pi}\right)^{-\frac{5}{3}} \sum_{N_{min}} f(N, e, \delta, \hat{n}), \tag{7}$$

with f made up of combinations of Bessel functions.

$$\textit{GR Limit} \quad \rightarrow \quad \lim_{\delta \to 0} \sum_{N} f(N, e, \delta, \hat{n}) = \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1-e^2)^{\frac{7}{2}}} \tag{8}$$

Bounds on VSR graviton mass

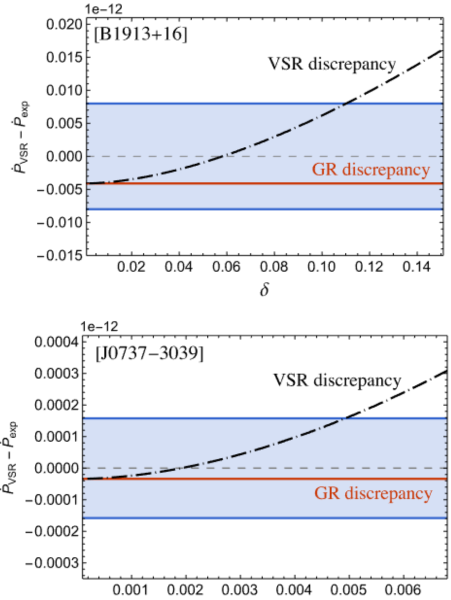
Using data from

- Hulse-Taylor binary PSR B1913+16
- Double Pulsar PSR J0737-3039

We constrain VSR mass

$$m_g \lesssim 10^{-21} eV$$

Still worse than the kinematical bound from GW170817 ($\lesssim 10^{-22} eV$)



1. VSLG as a gauge invariant theory of massive gravitons
2. Massive graviton emission in VSR from EFT's perspective
3. Bounds on the VSR graviton mass: $m_g \lesssim 10^{-21} eV$

Upcoming Work

- More complete statistical analysis
- Pulsar Timing Arrays
- Ultra Light Dark Matter candidate (?)

Thanks for your attention!

Backup: VSR Subgroups Properties

*Subgroups of interest ($T_1 \equiv K_x + J_y$, $T_2 \equiv K_y - J_x$)

- $T(2) = T_1 + T_2$
- $E(2) = T(2) + J_z$ and $HOM(2) = T(2) + K_z$
- $SIM(2) = T(2) + J_z + K_z$

T(2), E(2) - Have an invariant 4−Vector

Possibility to build local terms that violate Lorentz invariance
→ Free Particle's propagation gets affected

HOM(2) - No new Invariants

- Implies Special Relativity's kinematics
- CPT not granted

SIM(2) - No new Invariants

- Implies Special Relativity's kinematics
- CPT granted

Equations of Motion (E.o.M.): $O_{\mu\nu\alpha\beta}h^{\alpha\beta} = 0$, where[6]

$$
\begin{aligned}
O_{\mu\nu\alpha\beta} = \chi \Big(& p_\mu p_\nu \eta_{\alpha\beta} - \frac{1}{2}p_\mu p_\alpha \eta_{\nu\beta} - \frac{1}{2}p_\mu p_\beta \eta_{\nu\alpha} + p_\alpha p_\beta \eta_{\mu\nu} - \frac{1}{2}p_\nu p_\beta \eta_{\mu\alpha} - \frac{1}{2}p_\nu p_\alpha \eta_{\mu\beta} \\
& - p^2 \eta_{\mu\nu}\eta_{\alpha\beta} + \frac{1}{2}p^2 \eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}p^2 \eta_{\mu\beta}\eta_{\nu\alpha} + m_g^2 \eta_{\mu\nu}\eta_{\alpha\beta} - \frac{m_g^2}{2}\eta_{\mu\alpha}\eta_{\nu\beta} - \frac{m_g^2}{2}\eta_{\mu\beta}\eta_{\nu\alpha} \\
& - m_g^2 N_\mu N_\nu p_\alpha p_\beta + \frac{m_g^2}{2}N_\mu N_\alpha p_\nu p_\beta + \frac{m_g^2}{2}N_\mu N_\beta p_\nu p_\alpha + \frac{m_g^2}{2}N_\nu N_\alpha p_\mu p_\beta + \frac{m_g^2}{2}N_\nu N_\beta p_\mu p_\alpha - m_g^2 N_\alpha N_\beta p_\mu p_\nu \\
& + m_g^2 p^2 N_\mu N_\nu g_{\alpha\beta} - \frac{m_g^2}{2}p^2 N_\mu N_\alpha g_{\nu\beta} - \frac{m_g^2}{2}p^2 N_\mu N_\beta g_{\nu\alpha} - \frac{m_g^2}{2}p^2 N_\nu N_\beta \eta_{\mu\alpha} - \frac{m_g^2}{2}p^2 \eta_{\mu\beta} N_\nu N_\alpha + m_g^2 p^2 N_\alpha N_\beta \eta_{\mu\nu} \\
& - m_g^2 \eta_{\mu\nu} N_\alpha p_\beta - m_g^2 \eta_{\mu\nu} p_\alpha N_\beta + \frac{m_g^2}{2}\eta_{\mu\alpha} N_\nu p_\beta + \frac{m_g^2}{2}\eta_{\mu\alpha} p_\nu N_\beta + \frac{m_g^2}{2}\eta_{\mu\beta} N_\nu p_\alpha + \frac{m_g^2}{2}\eta_{\mu\beta} p_\nu N_\alpha \\
& + \frac{m_g^2}{2}\eta_{\nu\alpha} N_\mu p_\beta + \frac{m_g^2}{2}\eta_{\nu\alpha} p_\mu N_\beta + \frac{m_g^2}{2}\eta_{\nu\beta} N_\mu p_\alpha + \frac{m_g^2}{2}\eta_{\nu\beta} p_\mu N_\alpha - m_g^2 \eta_{\alpha\beta} N_\mu p_\nu - m_g^2 \eta_{\alpha\beta} p_\mu N_\nu \Big).
\end{aligned}
$$

Quite messy!

[6] Jorge Alfaro and Alessandro Santoni. "Very special linear gravity: A gauge-invariant graviton mass", Physics Letters B 829:137080 (2022)

Backup: $T_{\mu\nu}$ of a Binary

The energy momentum tensor of a Binary system is

$$T^{\mu\nu}(t,\vec{x}) = \mu\, U^\mu U^\nu \delta^3(\vec{x}-\vec{r}(t))\,, \tag{9}$$

where $\mu = m_1 m_2/M$ is the reduced mass, $\vec{r}(t)$ is the reduced mass trajectory and $U^\mu = (1, \dot{r}_x, \dot{r}_y, 0)$ its non-relativistic four-velocity.

The **keplerian orbit** is parametrized as

$$\vec{r}(t) = b\left(\cos\phi - e, \sqrt{1-e^2}\sin\phi, 0\right)\,,$$
$$\Omega t = \phi - e\sin\phi \;\text{ with }\; \Omega = \sqrt{\frac{GM}{b^3}}\,. \tag{10}$$

Where b and e are respectively the semi-major axis and the eccentricity of the orbit.

Backup: Main steps of the EFT Calculation

$$\frac{dE}{dt} = \frac{\kappa^2}{8(2\pi)^3 T} \int \rho(\omega)\, \omega^2\, \tilde{T}^*_{\mu\nu}\, \tilde{T}_{\alpha\beta}\, S^{\mu\nu\alpha\beta}\, d\omega\, d\Omega_k \qquad (11)$$

1. Calculating $S^{\mu\nu\alpha\beta}$ through symmetry arguments
2. Working in the "far zone" or "radiation zone" approximation $b << \lambda << d$ so that

$$\tilde{T}^{ij}(\omega, \vec{k}) \simeq \int d^3x\, \tilde{T}^{ij}(\omega, \vec{x}) \equiv \tilde{T}^{ij}(\omega) \qquad (12)$$

3. Exploiting the (quasi-)periodic motion of binaries, to switch the $\omega-$integral with a sum over the keplerian modes $N \rightarrow \omega_N = N\Omega$
4. Realizing the $\int d\Omega_k$ integral using the tensorial structure of its components