

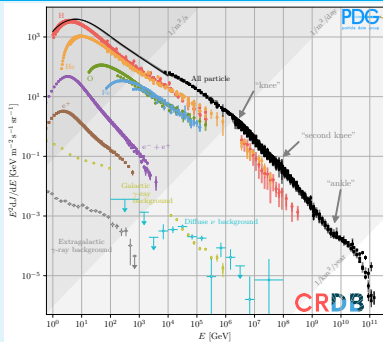
Galactic cosmic rays (III)

Philipp Mertsch

ECAP school
10 October 2023

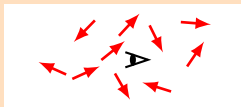
Recap (I): Fundamental observations

Spectrum



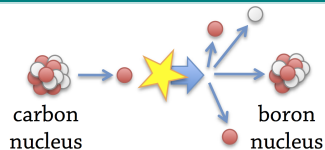
- Power law \rightarrow stochastic acceleration
- Features \rightarrow origin

Anisotropies



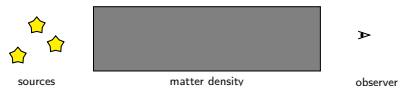
- Dipole $a = \mathcal{O}(10^{-3} \dots -2)$
- \rightarrow Requires isotropisation

Composition

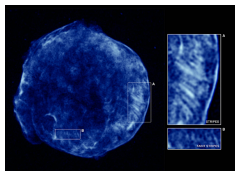


- Primaries: present in sources
- Secondaries: produced on route
- \rightarrow Requires isotropisation

Recap (II): Key insights



$$\frac{dN_1}{dt} = -\frac{N_1}{t_{\text{esc}}} - N_1\Gamma_1 + Q_1$$
$$\frac{dN_2}{dt} = -\frac{N_2}{t_{\text{esc}}} - N_2\Gamma_2 + \Gamma_{1\rightarrow 2}N_1$$



The slab model

- Definition of grammage
- From B/C: average grammage of \sim (a few) g cm^{-2}
- Requires crossing the disk thousands of times

The leaxy-box model

- $t_{\text{res}} = \mathcal{O}(10)$ Myr
- Cosmic-ray clocks
- Grammage rigidity-dependent

Supernova remnants

- 1 Presence of strong shocks
- 2 Observation of PeV particles
- 3 Energetics

Recap (III): The transport equation

$$\begin{aligned} \frac{\partial \psi_j}{\partial t} = & \nabla \cdot (\kappa \cdot \nabla \psi_j - \mathbf{U} \psi_j) && \text{spatial diffusion and advection} \\ & + \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi_j \right) && \text{momentum diffusion} \\ & + \frac{\partial}{\partial p} \left(-\frac{dp}{dt} \psi_j + \frac{p}{3} (\nabla \cdot \mathbf{U}) \psi_j \right) && \text{momentum change incl. adiabatic} \\ & - v n_{\text{gas}} \sigma_j \psi_j - \frac{\psi_j}{\tau_j} && \text{spallation and decay} \\ & + v n_{\text{gas}} \sum_{k>j} \sigma_{k \rightarrow j} \psi_k + \sum_{k>j} \frac{\psi_k}{\tau_{k \rightarrow j}} && \text{spallation and decay} \\ & + S_j && \text{primary sources} \end{aligned}$$

Addition (I)

- Charged particles interact resonantly with wave:

$$k_{\text{res}} = r_g^{-1} \propto p^{-1}$$

- Scattering rate Γ is gyrofrequency Ω reduced by density of resonant plasma waves:

$$\Gamma(k) \simeq \Omega \frac{\delta \tilde{B}^2(k)}{B_0^2} \simeq \Omega \frac{k P(k)}{B_0^2}$$

- The power spectrum of resonant plasma waves is:

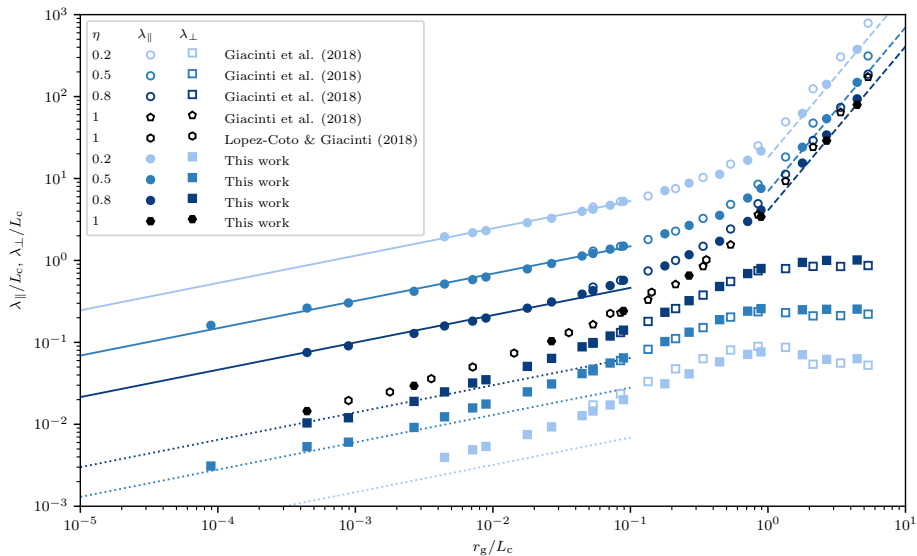
$$P(k) \sim \int d^3 r e^{i\mathbf{r}\cdot\mathbf{k}} \langle \delta \mathbf{B}(\mathbf{r}_0) \delta \mathbf{B}(\mathbf{r}_0 + \mathbf{r}) \rangle \propto k^{-5/3}$$

- Interaction with ensemble of random waves \rightarrow diffusion:

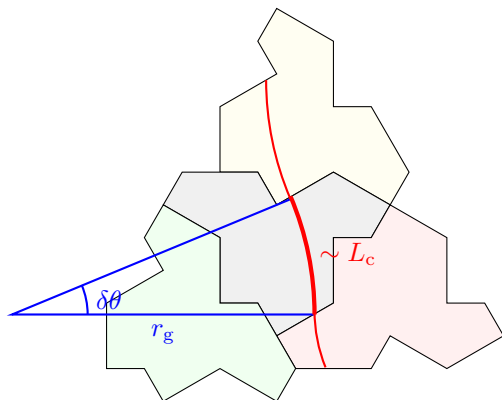
$$\kappa \simeq \frac{v^2}{3\Gamma} \propto \frac{1}{\Omega} \left(\frac{k P(k)}{B_0^2} \right)^{-1} \Big|_{k=k_{\text{res}}} \propto p^{1/3}$$

Addition (II)

Kuhlen, Mertsch, Phan (2022) [arXiv:2211.05881]



Addition (III)



- Mean-square deflection from crossing one domain:

$$\langle(\delta\theta)^2\rangle \simeq \left(\frac{L_c}{r_g}\right)^2$$

- Number of domains crossed in time Δt :

$$N \simeq \frac{c\Delta t}{L_c}$$

- Mean-square deflection from crossing N domains:

$$\langle(\Delta\theta)^2\rangle = N\langle(\delta\theta)^2\rangle \simeq \frac{c\Delta t}{L_c} \left(\frac{L_c}{r_g}\right)^2$$

- When $\langle(\Delta\theta)^2\rangle = 1$, $\Delta t \equiv t_{sc}$:

$$t_{sc} = \frac{L_c}{c} \left(\frac{r_g}{L_c}\right)^2$$

- Spatial diffusion coefficient:

$$\kappa = \frac{c^2}{3} t_{sc} = \frac{cL_c}{3} \left(\frac{r_g}{L_c}\right)^2$$

Outline

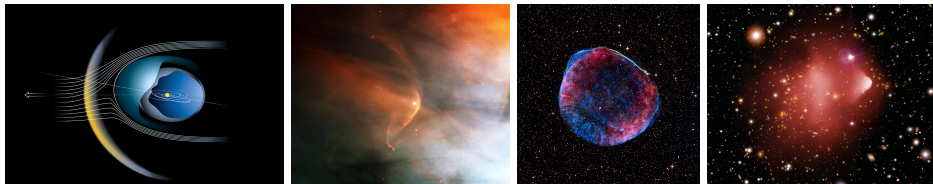
- ① Motivation
- ② Fundamental observations
 - Spectrum
 - Anisotropy
 - Composition
- ③ Key insights
 - Grammage
 - Cosmic ray clocks
 - Rigidity-dependence
 - Source candidates
- ④ The transport equation
- ⑤ Exercises
- ⑥ Shock acceleration
 - Shocks
 - Macroscopic approach
 - Microscopic approach
 - Additional effects
- ⑦ Galactic transport
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 - Green's function
 - Numerical codes
- ⑧ Open question 1: The positron excess
 - Sources of positrons
 - Acceleration of secondaries
- ⑨ Open question 2: Self-confinement
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- ⑫ Open question 5: Small-scale anisotropies
 - Data
 - Test particle simulations
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 - 3D gas maps
- ⑭ Summary & Conclusions

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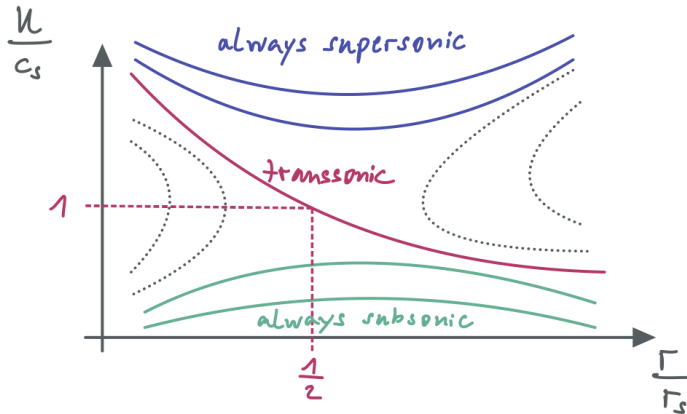
Shocks are ubiquitous

- A flow is characterised by ρ , \mathbf{U} , T
- In general, functions of position
- If they change abruptly, this is called a shock
- Shocks are very common in astrophysics:
 - Blast wave explosion
 - Accretion (onto star, black hole, galaxy cluster)
 - Mergers
 - Supersonic winds / outflows
 - Bow shocks (=supersonic flow around obstacle)



Shocks are inevitable

- SN is expelling material at $U_{\text{sh}} > c_s$
- Supersonic flow close to SN, subsonic flow at infinity
- Nature of supersonic and subsonic flows very different



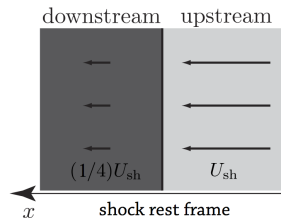
→ Connection must be discontinuous: formation of shock

Parallel shock

- Consider non-relativistic shock in its rest frame
- Discontinuity in gas density and velocity:

$$\rho_2 = r\rho_1 \quad \text{and} \quad U_2 = \frac{1}{r}U_1$$

→ Gas is compressed and slowed down



Compression ratio

Depends on the ratio of specific heats γ :

$$r \simeq \frac{\gamma + 1}{\gamma - 1}$$

For ideal mono-atomic gas: $\gamma = 5/3 \Rightarrow r = 4$

The transport equation

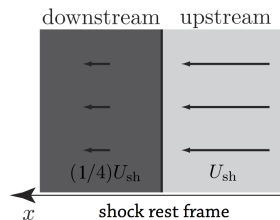
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Macroscopic approach

Seminal papers in 1977/78 by Krymsky; Axford, Leer, Skaldron; Blandford, Ostriker



- Consider steady-state transport equation for phase-space density f :

$$U \frac{\partial f}{\partial x} - \frac{\partial}{\partial x} \kappa \frac{\partial f}{\partial x} - \frac{p}{3} \frac{dU}{dx} \frac{\partial f}{\partial p} = 0$$

- For $x \neq 0$,

$$f(x, p) = \begin{cases} g_1(p) \exp \left[\frac{x}{\kappa(p)/U} \right] + f_1(p) & \text{for } x < 0 \\ f_2(p) & \text{for } x > 0 \end{cases}$$

Macroscopic approach

Seminal papers in 1977/78 by Krymsky; Axford, Leer, Skaldron; Blandford, Ostriker

- Can derive matching conditions and find for the spectrum at shock,

$$f_2(p) = \Gamma p^{-\Gamma} \int_0^p dp' p'^{\Gamma-1} f_1(p') + \text{const.} \times p^{-\Gamma}$$

with spectral index $\Gamma \equiv \frac{3r}{r-1}$

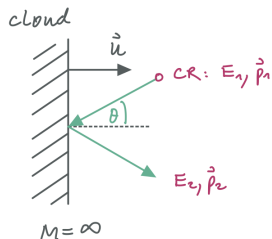
- With $r \simeq \frac{\gamma+1}{\gamma-1} = 4$, $f(0, p) \propto p^{-4} \Rightarrow \psi(0, p) = 4\pi p^2 f(0, p) \propto p^{-2}$

Strong ($r = 4$) shock accelerates CRs to p^{-2} spectrum!

Microscopic approach

Fermi (1949)

- Collision of particle with ∞ massive cloud



- Two types of collisions:
 - $\theta \in [0, \frac{\pi}{2}]$ "head-on"
 - $\theta \in [\frac{\pi}{2}, \pi]$ "trailing"

1 Transform E and p_x to cloud frame (primed):

$$E'_1 = \gamma(E_1 + Up_1 \cos \theta)$$
$$p'_{1x} = p'_1 \cos \theta' = \gamma \left(p_1 \cos \theta + \frac{U}{c^2} E_1 \right)$$

Microscopic approach

- 1 Transform E and p_x to cloud frame (primed):

$$E'_1 = \gamma(E_1 + Up_1 \cos \theta)$$
$$p'_{1x} = p'_1 \cos \theta' = \gamma \left(p_1 \cos \theta + \frac{U}{c^2} E_1 \right)$$

- 2 Collision in cloud frame:

- Energy conserved: $E'_2 = E'_1$
- Momentum flipped: $p'_{2x} = -p'_{1x}$

- 3 Transform back to observer frame:

$$E_2 = \gamma (E'_2 - Up'_{2x})$$
$$= \gamma \left(\gamma (E_1 + Up_1 \cos \theta) + U \gamma \left(p_1 \cos \theta + \frac{U}{c^2} E_1 \right) \right)$$
$$= \gamma^2 \left(E_1 + 2Up_1 \cos \theta + E_1 \left(\frac{U}{c} \right)^2 \right)$$
$$= \gamma^2 E_1 \left(1 + 2U \frac{v_1}{c^2} \cos \theta + \left(\frac{U}{c} \right)^2 \right)$$

- 3 Transform back to observer frame:

$$E_2 = \gamma^2 E_1 \left(1 + 2U \frac{v_1}{c^2} \cos \theta + \left(\frac{U}{c} \right)^2 \right)$$

- 4 Expand γ^2 in U/c

$$\gamma^2 = \left(1 - (U/c)^2 \right)^{-1} \simeq 1 + (U/c)^2$$

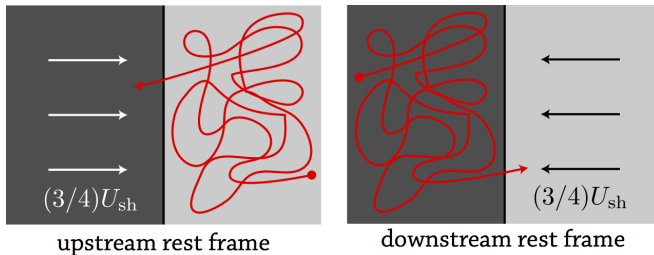
and let $v_1 \simeq c$:

$$\begin{aligned} \Rightarrow E_2 &\simeq E_1 \left(1 + 2 \frac{U}{c} \cos \theta + 2 \left(\frac{U}{c} \right)^2 \right) \\ \Rightarrow \Delta E \equiv E_2 - E_1 &= E_1 \left(2 \frac{U}{c} \cos \theta + 2 \left(\frac{U}{c} \right)^2 \right) \end{aligned}$$

$\cos \theta > 1 \Rightarrow$ energy gain
 $\cos \theta < 1 \Rightarrow$ energy loss

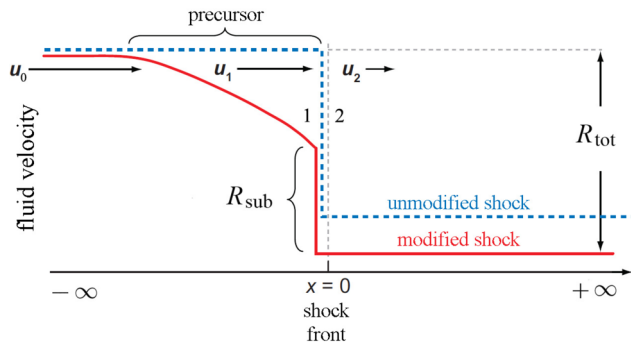
Microscopic approach

Seminal papers in 78 by Bell



- Particle crossing shock always suffers “head-on” collisions
- Systematic energy gain, $\Delta E/E \sim (U_{sh}/c)$

Non-linear shock acceleration



Shock modification

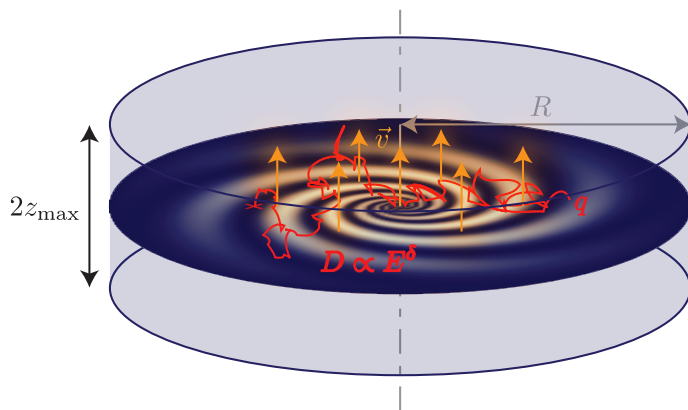
- CRs contribute to the pressure of the system
 - Slow-down of flow \rightarrow precursor
 - Compression depends on position
- \rightarrow Spectral curvature

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The transport equation

$$\begin{aligned} & \frac{\partial \psi_j}{\partial t} - \nabla \cdot (\kappa \cdot \nabla \psi_j - \mathbf{U} \psi_j) - \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial}{\partial p} \frac{1}{p^2} \psi_j \right) + \frac{\partial}{\partial p} \left(\frac{dp}{dt} \psi_j - \frac{p}{3} (\nabla \cdot \mathbf{U}) \psi_j \right) \\ = & S_j + \sum_{k>j} \left(v n_{\text{gas}} \sigma_{k \rightarrow j} + \frac{1}{\tau_{k \rightarrow j}} \right) \psi_k - \left(v n_{\text{gas}} \sigma_j + \frac{1}{\tau_j} \right) \psi_j \end{aligned}$$



Boundary conditions: $\psi_j(r, z) = 0$ for $z = \pm z_{\text{max}}$ or $r = R$

- Can derive leaky box model from transport equation

$$\frac{\partial \psi_j}{\partial t} - \nabla \cdot (\kappa \cdot \nabla \psi_j - \mathbf{U} \psi_j) = q_j + \sum_{j < k} (v n_{\text{gas}} \sigma_{k \rightarrow j}) \psi_k - (v n_{\text{gas}} \sigma_i) \psi_j$$

- Integrate over CR halo, $N_j \equiv \int dV \psi_j$:

$$\frac{\partial N_j}{\partial t} - \int dV \left[\nabla \cdot \underbrace{(\kappa \cdot \nabla \psi_j - \mathbf{U} \psi_j)}_{= -\mathbf{F}_j} \right] = \frac{\partial N_j}{\partial t} + \underbrace{\int d\mathbf{S} \cdot \mathbf{F}_j}_{\equiv -\frac{N_j}{\tau_{\text{esc}}}} = Q_j + \sum_{j < k} (v n_{\text{gas}} \sigma_{k \rightarrow j}) N_k - (v n_{\text{gas}} \sigma_i) N_j$$

Simplified 1D model

Simplify transport equation:

$$\frac{\partial \psi_j}{\partial t} - \kappa \frac{\partial^2 \psi_j}{\partial z^2} - U \frac{\partial \psi_j}{\partial z} + \left(v n_{\text{gas}}(z) \sigma_j + \frac{1}{\tau_j} \right) \psi_j = q_j + \sum_{j < k} \left(v n_{\text{gas}}(z) \sigma_{k \rightarrow j} + \frac{1}{\tau_{k \rightarrow j}} \right) \psi_k$$
$$-\kappa \frac{\partial^2 \psi_j}{\partial z^2} - U \frac{\partial \psi_j}{\partial z} + \left(2h\delta(z) v n_{\text{gas}} \sigma_j + \frac{1}{\tau_j} \right) \psi_j = 2h\delta(z) q_j + \sum_{j < k} \left(2h\delta(z) v n_{\text{gas}} \sigma_{k \rightarrow j} + \frac{1}{\tau_{k \rightarrow j}} \right) \psi_k$$

Solution

$$\psi_j(z, p) = \frac{2z_{\text{max}} \left(q_j + \sum_{k > j} v n_{\text{gas}} \sigma_{k \rightarrow j} \psi_k \right) e^{\frac{zU}{2\kappa}} \sinh \left[\frac{z_{\text{max}} - z}{z_j} \right]}{\left(2h v n_{\text{gas}} \sigma_j + U + \frac{\kappa}{z_{\text{max}}} \frac{z_{\text{max}}}{z_j} \coth \left[\frac{z_{\text{max}}}{z_j} \right] \right) \sinh \left[\frac{z_{\text{max}}}{z_j} \right]}$$

$$\text{with } z_j \equiv \left(\left(\frac{U}{2\kappa} \right)^2 - \frac{1}{\tau_j \kappa^2} \right)^{-1/2}$$

Simplified 1D model

Simplify transport equation:

$$\frac{\partial \psi_j}{\partial t} - \kappa \frac{\partial^2 \psi_j}{\partial z^2} - U \frac{\partial \psi_j}{\partial z} + \left(v n_{\text{gas}}(z) \sigma_j + \frac{1}{\tau_j} \right) \psi_j = q_j + \sum_{j < k} \left(v n_{\text{gas}}(z) \sigma_{k \rightarrow j} + \frac{1}{\tau_{k \rightarrow j}} \right) \psi_k$$
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Solution

$$\psi_j(0, p) = \frac{2z_{\text{max}} \left(q_j + \sum_{k > j} v n_{\text{gas}} \sigma_{k \rightarrow j} \psi_k \right) e^{\frac{zU}{2\kappa}} \sinh \left[\frac{z_{\text{max}} - z}{z_j} \right]}{\left(2h v n_{\text{gas}} \sigma_j + U + \frac{\kappa}{z_{\text{max}}} \frac{z_{\text{max}}}{z_j} \coth \left[\frac{z_{\text{max}}}{z_j} \right] \right) \sinh \left[\frac{z_{\text{max}}}{z_j} \right]}$$

with $z_j \equiv \left(\left(\frac{U}{2\kappa} \right)^2 - \frac{1}{\tau_j \kappa^2} \right)^{-1/2} \rightarrow \infty$; evaluate at $z = 0$

$$\text{ambient CR spectrum} = \frac{\text{source spectrum}}{\text{diffusion coefficient}}$$

Green's function approach

- Solve simplified transport equation for e^\pm

$$\frac{\partial \psi_\pm}{\partial t} - \nabla \cdot \kappa \cdot \nabla \psi_\pm - \frac{\partial}{\partial E} \left(\frac{dE}{dt} \psi_\pm \right) = \delta(\mathbf{r} - \mathbf{r}_0) \delta(t) Q(E)$$

- For homogeneous energy loss rate dE/dt , energy becomes pseudo time
- Heat equation

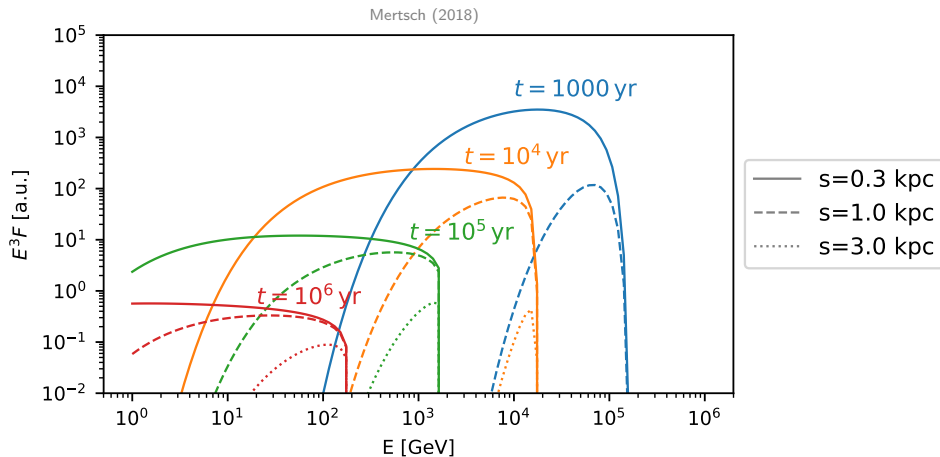
$$\Rightarrow \psi_\pm(\mathbf{r}, E, t) = \left(\pi \ell^2(E, t) \right)^{-3/2} e^{-|\mathbf{r} - \mathbf{r}_0|^2 / \ell^2(E, t)} \frac{\frac{dE}{dt}(E)}{\frac{dE}{dt}(E_0)} Q(E_0)$$

where $E_0 = E_0(E, t)$ and

$$\ell^2(E, t) = 4 \int_{E_0}^E dE' \frac{\kappa(E')}{dE/dt}$$

Green's function

$$\psi_{\pm}(\mathbf{r}, E, t) = \left(\pi \ell^2(E, t) \right)^{-3/2} e^{-|\mathbf{r}-\mathbf{r}_0|^2/\ell^2(E,t)} \frac{\frac{dE}{dt}(E)}{\frac{dE}{dt}(E_0)} Q(E_0)$$



with $\kappa_{xx} = \kappa_0 E^\delta$, $\frac{dE}{dt} = b_0 E^2$, $Q(E_0) \propto E_0^{-\Gamma} \exp[-E_0/E_c]$

Numerical codes

- In less restricted setups, e.g.
 - 3D
 - Inhomogeneous inputs, e.g. gas densities
 - Anisotropic diffusion
 - Self-generated turbulence

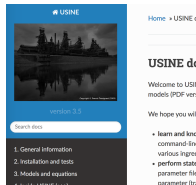
transport equation cannot be solved analytically

→ Solve numerically

- Many codes available:



GALPROP



USINE



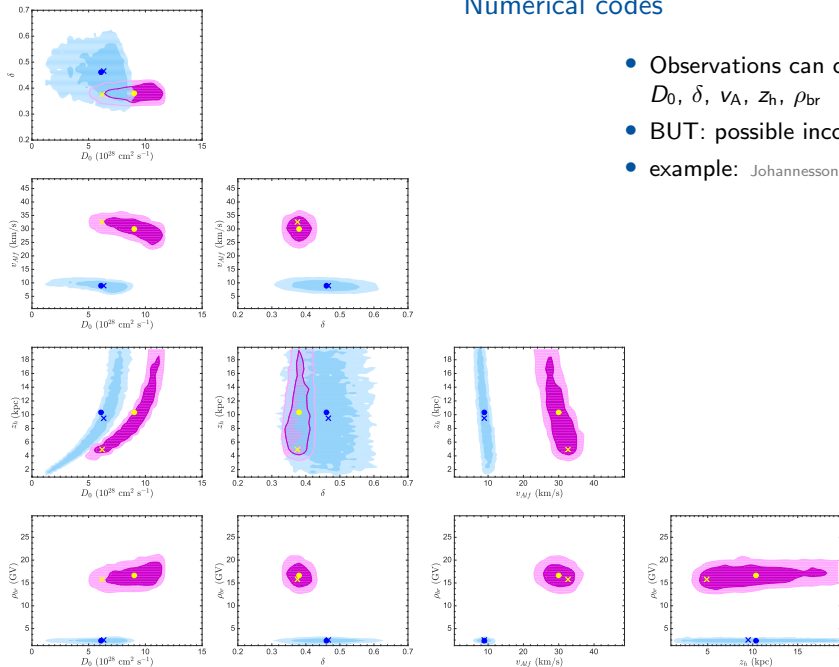
DRAGON



PICARD

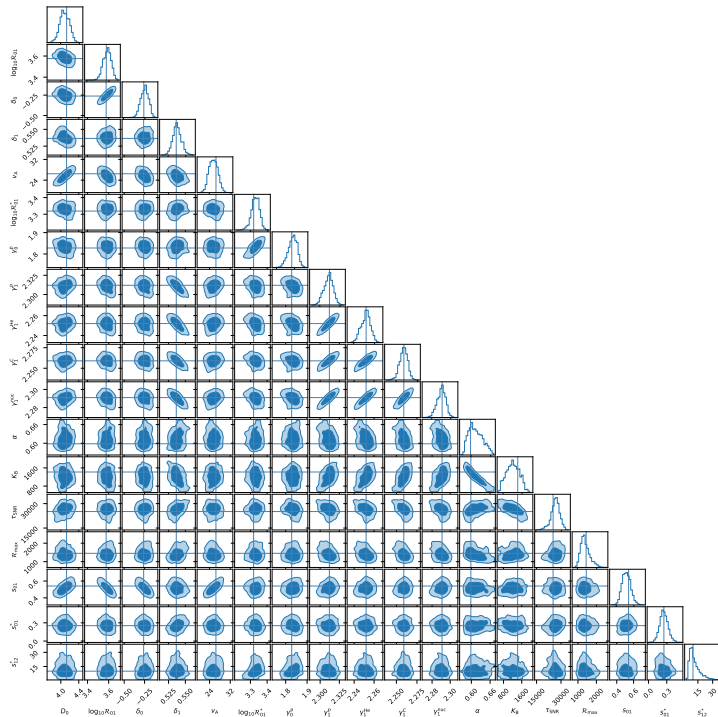
Numerical codes

- Observations can constrain free parameters, e.g.: D_0 , δ , v_A , z_h , ρ_{br}
- BUT: possible inconsistencies
- example: Johannesson *et al.* (2017)



MCMC analysis

Mertsch, Vittino, Sarkar (2021)



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- ⑭ Summary & Conclusions

Outline

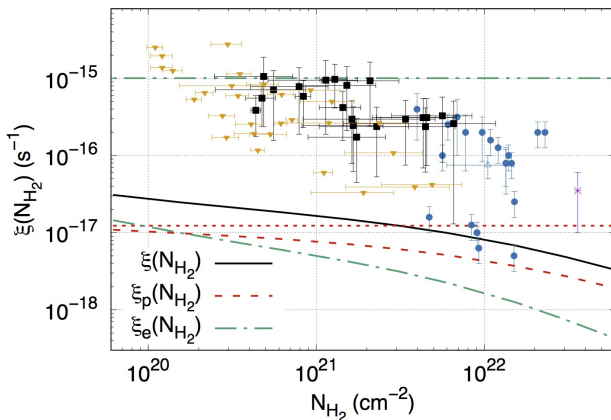
- ① Motivation
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 - Anisotropy
 - Composition
- ③ Key insights
 - Grammage
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 - Rigidity-dependence
 - Source candidates
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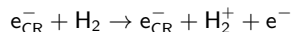
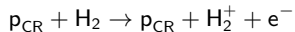
The ionisation puzzle



- Cosmic rays are ionising dense molecular clouds
- But the observed ionisation is much higher than expected
- Maybe the flux of cosmic rays elsewhere is different from what it is here?!

Phan, Morlino, Gabici (2018)





$\zeta_{\text{H}_2} \equiv$ (production rate of H_2^+ by cosmic rays)

Measurements

- Absorption of light from background star
- Line ratios sensitive to production of H_2^+
 - In diffuse clouds ($N_{\text{H}} \lesssim 10^{22} \text{ cm}^{-2}$):
 H_3^+ produced by charge exchange
 - In dense clouds ($N_{\text{H}} \gtrsim 10^{22} \text{ cm}^{-2}$):
 H_3^+ reacts to HCO^+ , DCO^+ , N_2H^+

Computation

- Galactic propagation:
 - Source distribution, spectrum
 - Propagation parameters
 - Interstellar intensity $j_{\text{ISM}} = v/(4\pi)\psi$
- Transport into cloud:
 - Energy losses
 - Ballistic or diffusive Skilling & Strong (1976); Padovani *et al.* (2009); Morlino & Gabici (2015); Phan *et al.* (2018); Ivlev *et al.* (2018)
 - Intensity averaged over cloud \bar{j}

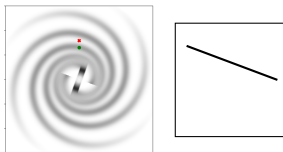
- Ionisation rate:

$$\zeta_{\text{H}_2} = 4\pi \int_{E_1}^{\infty} dE \sigma_{\text{ion}}(E) (1 + \phi(E)) \bar{j}(E)$$

- Transport equation:
$$\underbrace{\frac{\partial}{\partial t} \psi - \nabla \cdot (\kappa \cdot \nabla - \mathbf{u}) \psi - \frac{\partial}{\partial p} (\dot{p} \psi)}_{\equiv \mathcal{L} \psi} = q$$

- Green's function:
$$\mathcal{L} \mathcal{G} = \delta^{(3)}(\mathbf{x} - \mathbf{x}') \delta(t - t') \delta(p - p')$$

- Source term:
$$q = q(\mathbf{x}, t, p) = \underbrace{s(\mathbf{x}, t)}_{\text{src. density}} \cdot \underbrace{s_p(p)}_{\text{spectrum}}$$

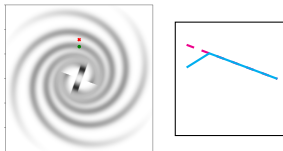


- Solution:
$$\psi(\mathbf{x}, t, p) = \int d\mathbf{x}' dt' dp' \mathcal{G}(\mathbf{x} - \mathbf{x}', t - t', p, p') \underbrace{s(\mathbf{x}', t')}_{\text{src. density}} \underbrace{s_p(p')}_{\text{spectrum}}$$

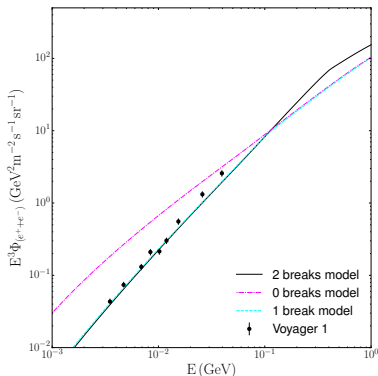
Voyager data

- Source term:

$$q = q(\mathbf{x}, t, p) = \underbrace{s(\mathbf{x}, t)}_{\text{src. density}} \cdot \underbrace{s_p(p)}_{\text{spectrum}}$$



Vittino, PM, Gast, Schael, PRD **100** (2019) 043007



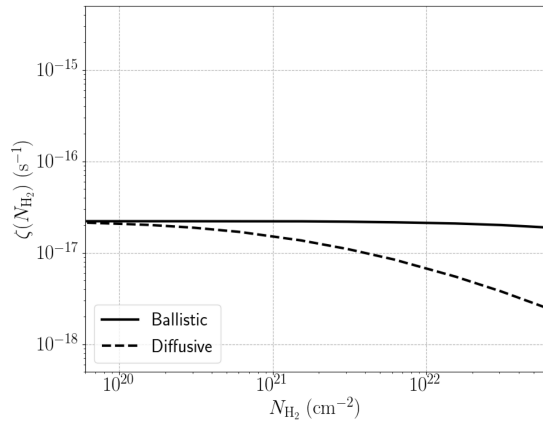
Problem # 1

Reproducing Voyager $e^+ + e^-$
requires unmotivated break in source spectrum
at few hundred MeV

Cumming *et al.* (2016); Orlando (2018); Boschini *et al.* (2018);
Jóhannesson *et al.* (2018); Bisschoff *et al.* (2019)

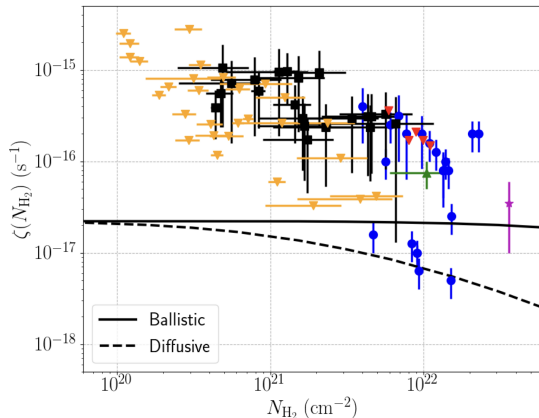
The ionisation puzzle

Padovani *et al.* (2009); Indriolo (2012); Phan, Morlino, Gabici (2018); Gabici (2021)



The ionisation puzzle

Padovani *et al.* (2009); Indriolo (2012); Phan, Morlino, Gabici (2018); Gabici (2021)



Caselli *et al.* (2018); Williams *et al.* (1998); Maret *et al.* (2007);

Indriolo *et al.* (2012); Bialy *et al.* (2022)

Problem # 2

Excess and large scatter in measured ionisation rate

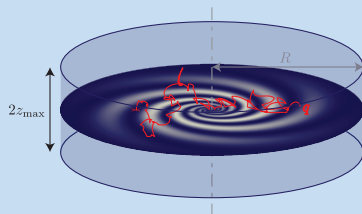
Maybe the Voyager spectrum is not representative for the spectra elsewhere?

Indriolo (2012)

Source discreteness

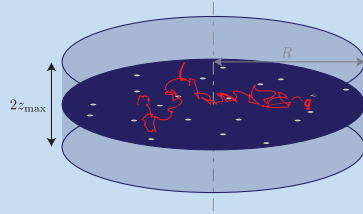
Phan, Schulze, Mertsch, Recchia, Gabici (2021)

Continuous source density



$$\psi(\mathbf{x}, t, p) = \int d\mathbf{x}' dt' dp' \mathcal{G}(\mathbf{x} - \mathbf{x}', t - t', p, p') \underbrace{s(\mathbf{x}', t')}_{\text{src. density}} s_p(p')$$

Discrete sources



$$\psi(\mathbf{x}, t, p) = \int dp' \sum_n \mathcal{G}(\mathbf{x} - \mathbf{x}_n, t - t_n, p, p') s_p(p')$$

- Assuming smooth source density is good approximation if
(diffusion-loss length) \gg (average source separation)
- However, energy losses can severely limit diffusion-loss length
- Examples:
 - High-energy e^\pm Malyshev *et al.* (2009) Blasi & Amato (2010), PM (2011, 2018), Manconi *et al.* (2017, 2019, 2020); Cholis *et al.* (2018, 2021), Evoli *et al.* (2020), Orusa *et al.* (2021)
 - Low-energy CRs

Monte Carlo simulation

Phan, Schulze, Mertsch, Recchia, Gabici (2023)

- Numerically solve transport equation for point-like sources
- Solutions on a grid in distance, age and momentum
- Interpolation allows approximating $\mathcal{G}(\mathbf{x} - \mathbf{x}', t - t', p, p')$

- Draw random distances and ages from $\frac{s(\mathbf{x}, t)}{\text{src. density}}$

- Compute

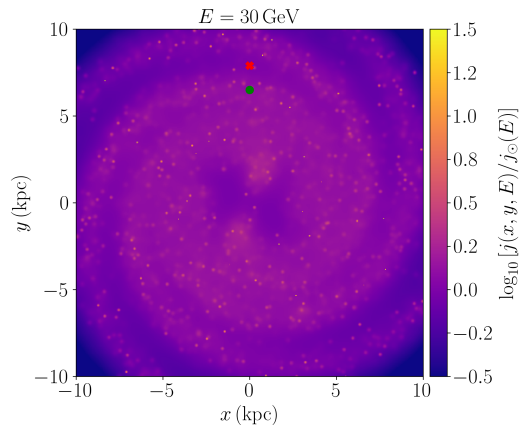
$$\psi(\mathbf{x}, t, p) = \int d p' \sum_n \mathcal{G}(\mathbf{x} - \mathbf{x}_n, t - t_n, p, p') s_p(p')$$

for each member of ensemble of realisations

→ Ensemble of intensities ψ

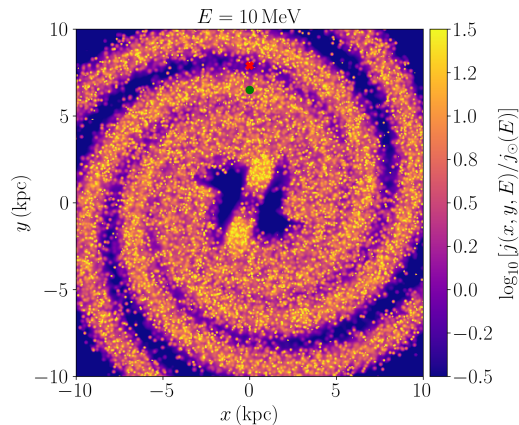
GeV vs MeV

Phan, Schulze, Mertsch, Recchia, Gabici (2023)



(diffusion-loss length) \gg (average source separation)

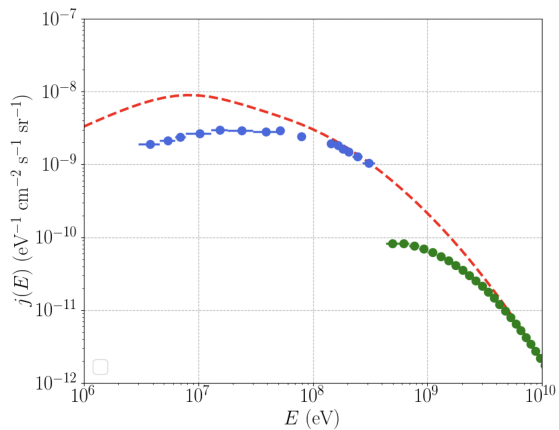
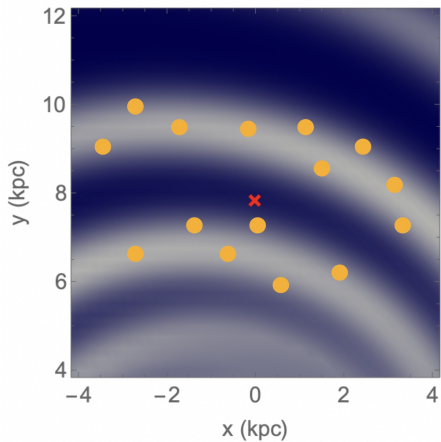
\Rightarrow little fluctuation
 \Rightarrow smooth approximation is good



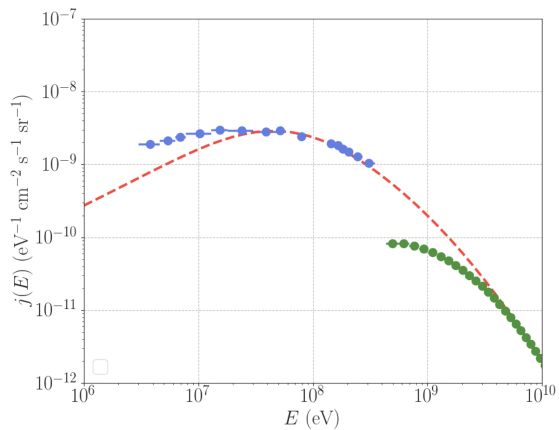
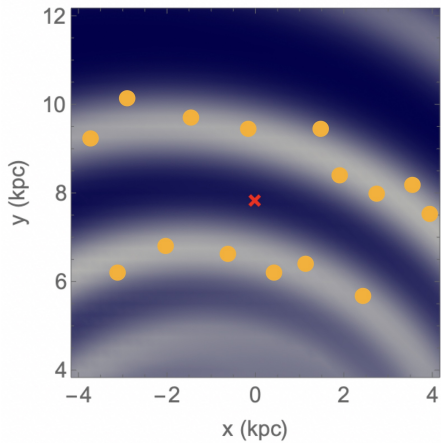
(diffusion-loss length) \ll (average source separation)

\Rightarrow sizeable fluctuations
 \Rightarrow smooth approximation is bad

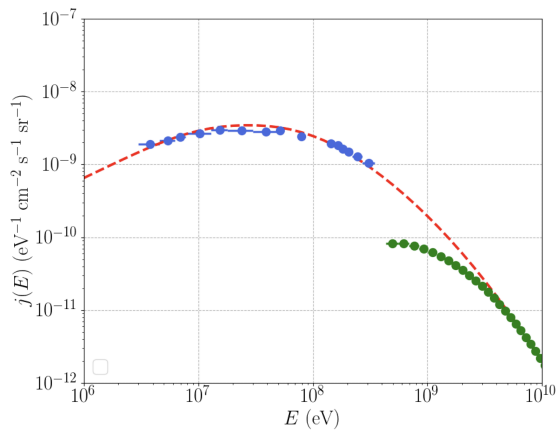
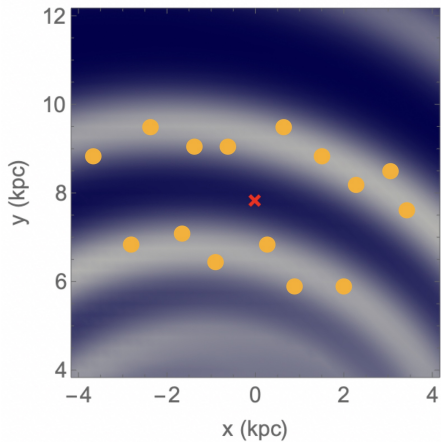
Importance of nearby sources



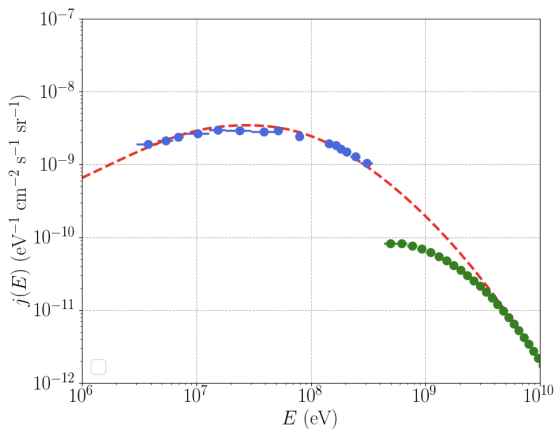
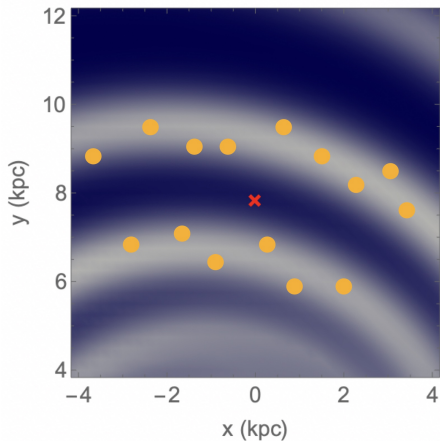
Importance of nearby sources



Importance of nearby sources



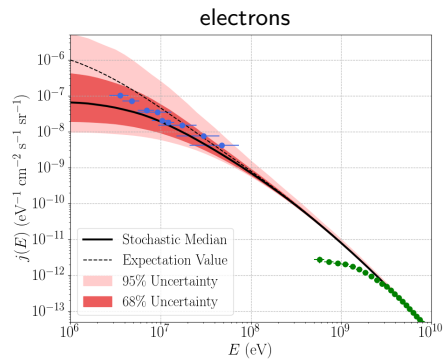
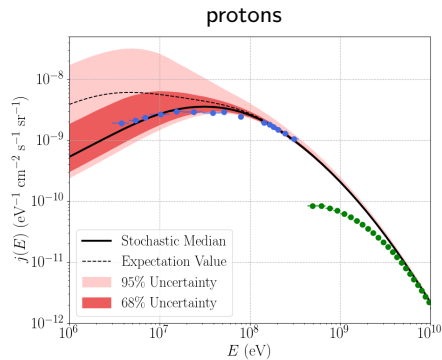
Importance of nearby sources



Cosmic ray flux is a stochastic quantity

Results: protons & electrons

Phan, Schulze, Mertsch, Recchia, Gabici (2021)



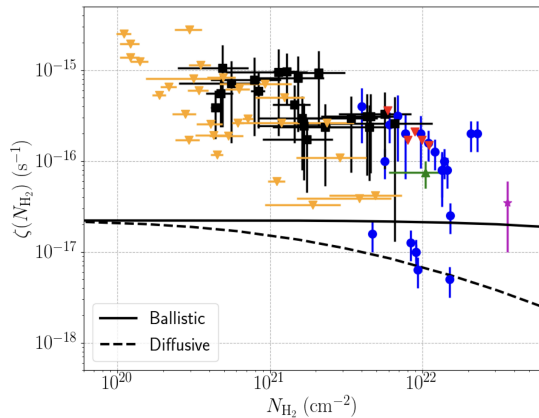
- Voyager 1 data inside uncertainty band
- Source discreteness effects important

Result # 1

- No need for unmotivated break in source spectrum!

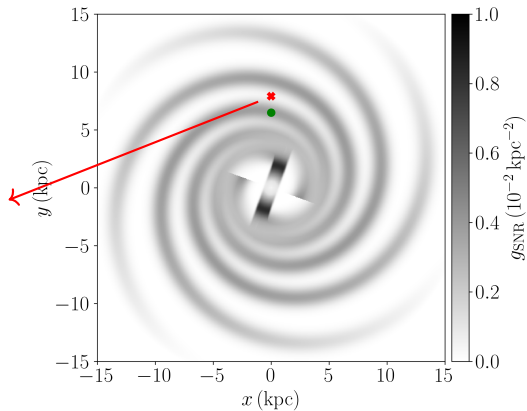
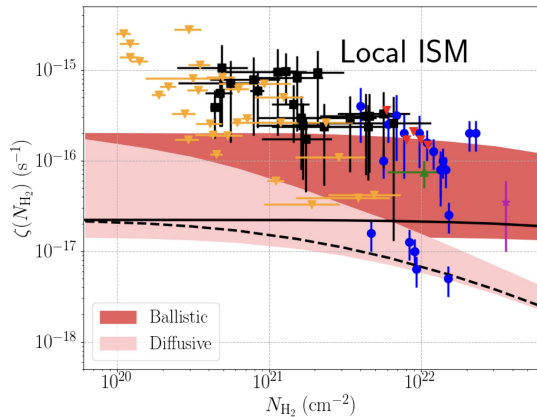
The ionisation puzzle

Phan, Schulze, Mertsch, Recchia, Gabici (2023)



The ionisation puzzle

Phan, Schulze, Mertsch, Recchia, Gabici (2023)

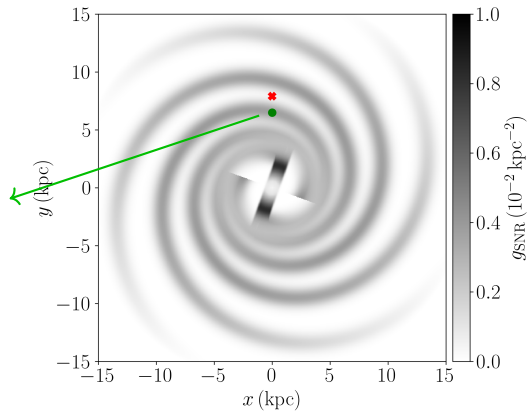
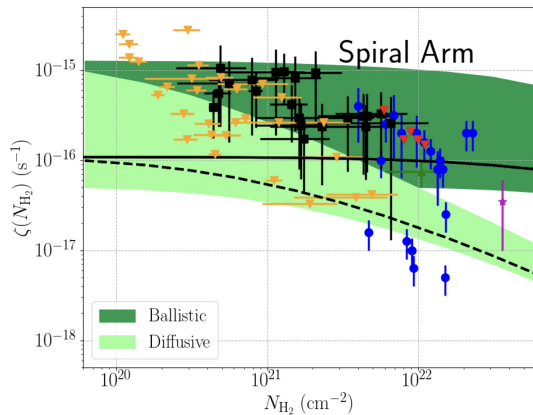


Result # 2

- Local ISM: improvement, but still too low

The ionisation puzzle

Phan, Schulze, Mertsch, Recchia, Gabici (2023)



Result # 2

- Local ISM: improvement, but still too low
- Spiral Arm: systematic shift up

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Summary & Conclusions

Acceleration

Particles gain energy while scattering in converging flow

Galactic propagation

High precision experiments → significantly increased parameter space

Open questions

- e^+ excess
- Self-confinement
- A swiss-cheese galaxy
- Ionisation puzzle
- Small-scale anisotropies
- Diffuse emission

Any questions?