ETH zürich

WOST Exploring Quantum Without SpaceTime

Introduction to Quantum Reference Frames

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material reference frames

problem of time

asymptotic symmetries

edge modes

quantum gravity

quantum hole argument

dressed observables

philosophy



quantum field theory

empirical significance of symmetries

type reduction

quantum information

accessibility of observables

Super-selection sectors

frame-dependence of quantum properties

tools for prediction

perspectival approach

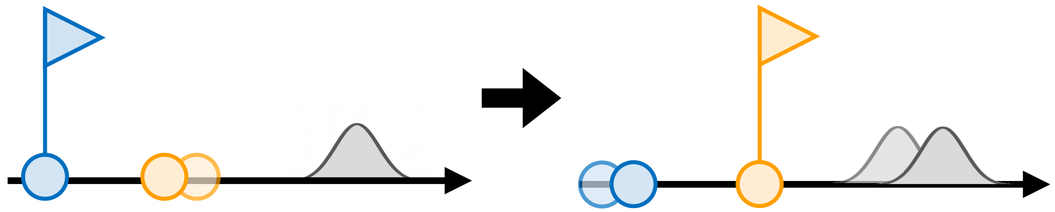
perspective-neutral approach



“extra-particle” approach

operational approach

quantum information





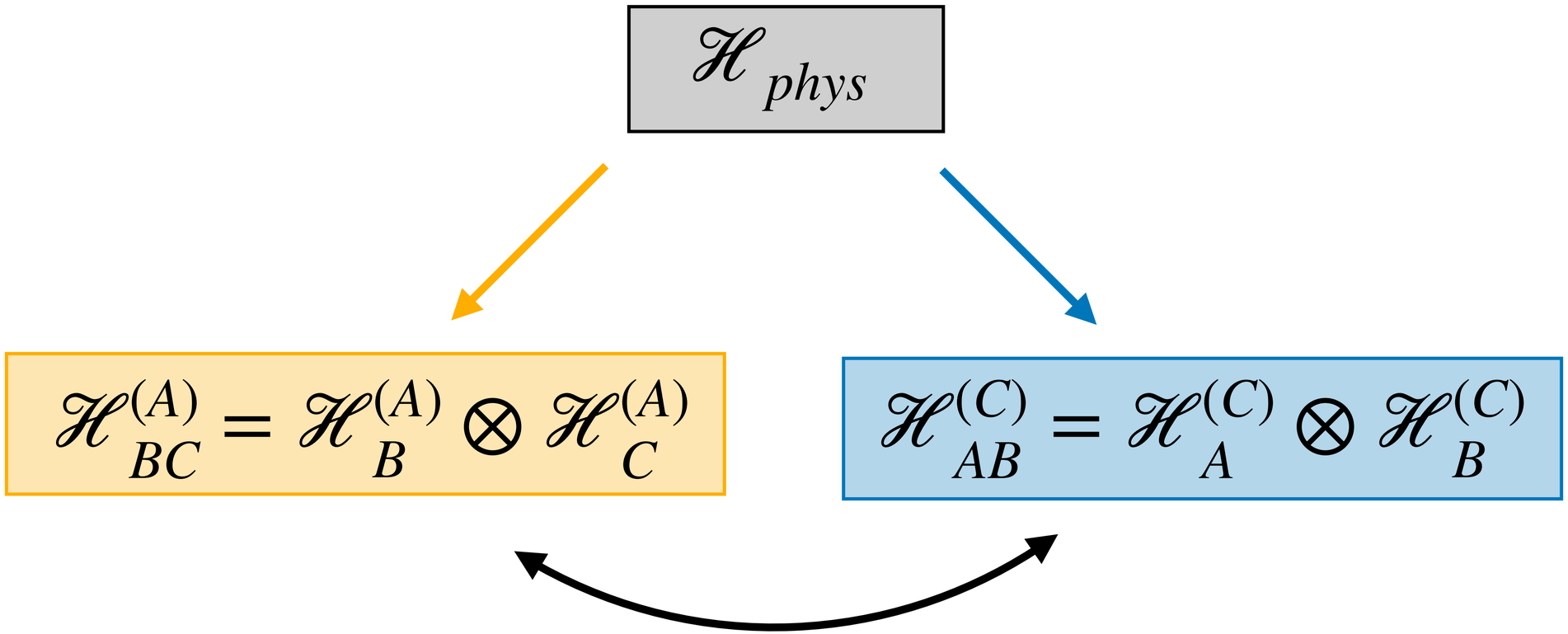


Outline

- ✦ **Quantum Reference Frames**
  - Perspective-Neutral & Perspectival Approach
  - Extra-Particle & Operational Approach
- ✦ **Implications & Applications**
  - Relativity of Superposition
  - Subsystem Relativity
- ✦ **More/Open Questions**

Quantum Reference Frames

Perspectival & Perspective-Neutral Approach



Quantum Reference Frames

Perspective-Neutral Approach

$$\mathscr{H}_{kin} = \mathscr{H}_A \otimes \mathscr{H}_B \otimes \mathscr{H}_C$$

But now introduce symmetries!



Quantum Reference Frames

Perspective-Neutral Approach

$$\mathscr{H}_{kin} = \mathscr{H}_A \otimes \mathscr{H}_B \otimes \mathscr{H}_C$$

$$\Pi_{phys} = \frac{1}{|G|} \int dg U(g)$$

"coherent G-Twirl"

$$\mathscr{H}_{phys}$$

Can only measure invariant quantities.

Vanrietvelde, Höhn, Giacomini, Castro-Ruiz (2020)
de la Hamette, Galley, Höhn, Loveridge, Müller (2021)

Quantum Reference Frames

Perspective-Neutral Approach

$$\mathscr{H}_{kin} = \mathscr{H}_A \otimes \mathscr{H}_B \otimes \mathscr{H}_C$$

$$\Pi_{phys} = \frac{1}{|G|}\int dg\, U(g)$$

"coherent G-Twirl"

$\mathscr{H}_{phys}$

condition on A being the identity

$\langle id|_A$

$\langle id|_C$

condition on C being the identity

$$\mathscr{H}_{BC}^{(A)} = \mathscr{H}_B^{(A)} \otimes \mathscr{H}_C^{(A)} \qquad \mathscr{H}_{AB}^{(C)} = \mathscr{H}_A^{(C)} \otimes \mathscr{H}_B^{(C)}$$

$\hat{S}_{C\to A}$

Vanrietvelde, Höhn, Giacomini, Castro-Ruiz (2020)
de la Hamette, Galley, Höhn, Loveridge, Müller (2021)

Quantum Reference Frames

Perspective-Neutral Approach

$$|\Psi\rangle_{kin} = \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right) \otimes |y\rangle_B \otimes |z\rangle_C$$

$$\Pi_{phys} = \int dx U(x) = \int dx e^{i(\hat{p}_A + \hat{p}_B + \hat{p}_C)x}$$

$$|\Psi\rangle_{phys} = \frac{1}{\sqrt{2}} \int dx \left(|x_1 - x\rangle_A + |x_2 - x\rangle_A\right) |y - x\rangle_B |z - x\rangle_C$$

$\langle 0|_A$

$\langle 0|_C$



$$|\Psi\rangle^{(A)}_{BC} = \frac{1}{\sqrt{2}}\left(|y - x_1\rangle_B |z - x_1\rangle_C + |y - x_2\rangle_B |z - x_2\rangle_C\right)$$

$$\hat{S}_{C\to A} = \mathscr{P}_{CA} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

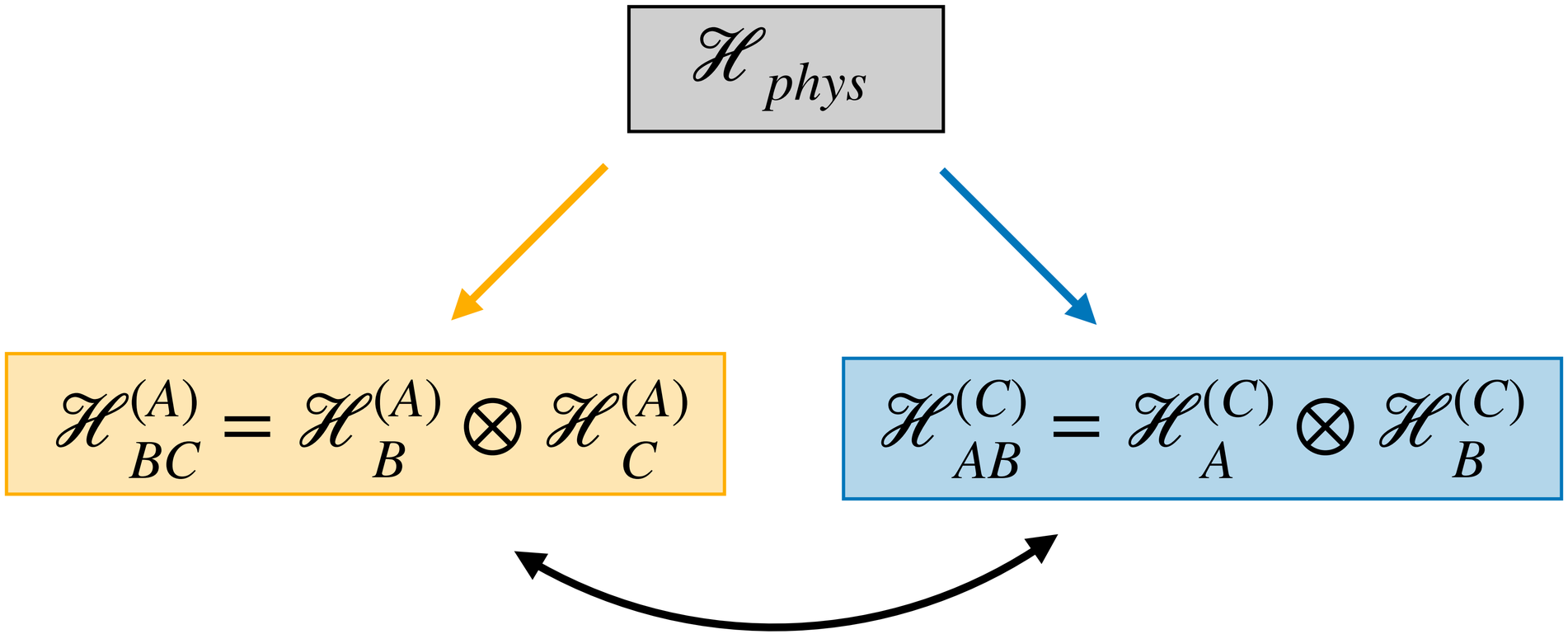
$$|\Psi\rangle^{(C)}_{AB} = \frac{1}{\sqrt{2}}\left(|x_1 - z\rangle_A + |x_2 - z\rangle_A\right) |y - z\rangle_B$$



Vanrietvelde, Höhn, Giacomini, Castro-Ruiz (2020)

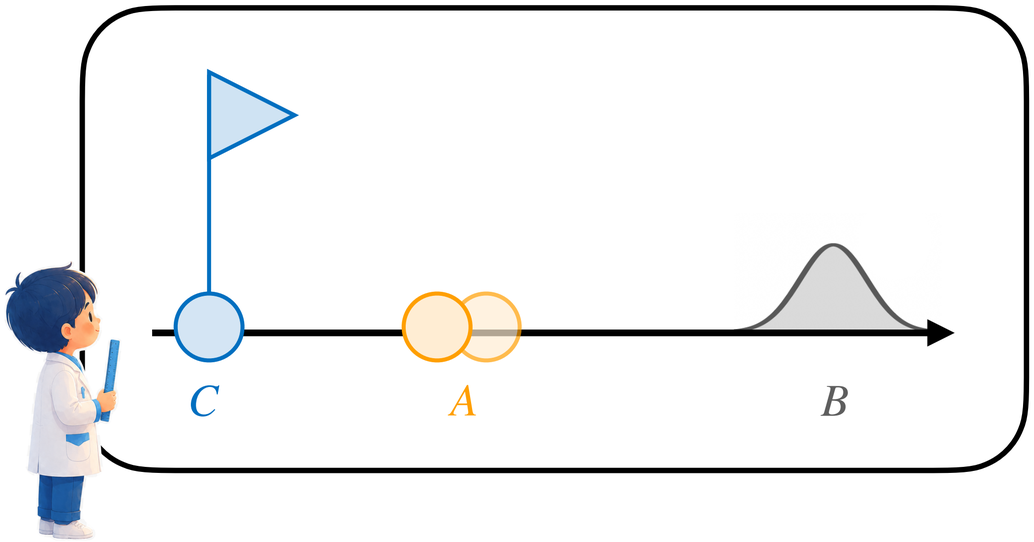
Quantum Reference Frames

Perspectival & Perspective-Neutral Approach



Quantum Reference Frames

Perspectival Approach



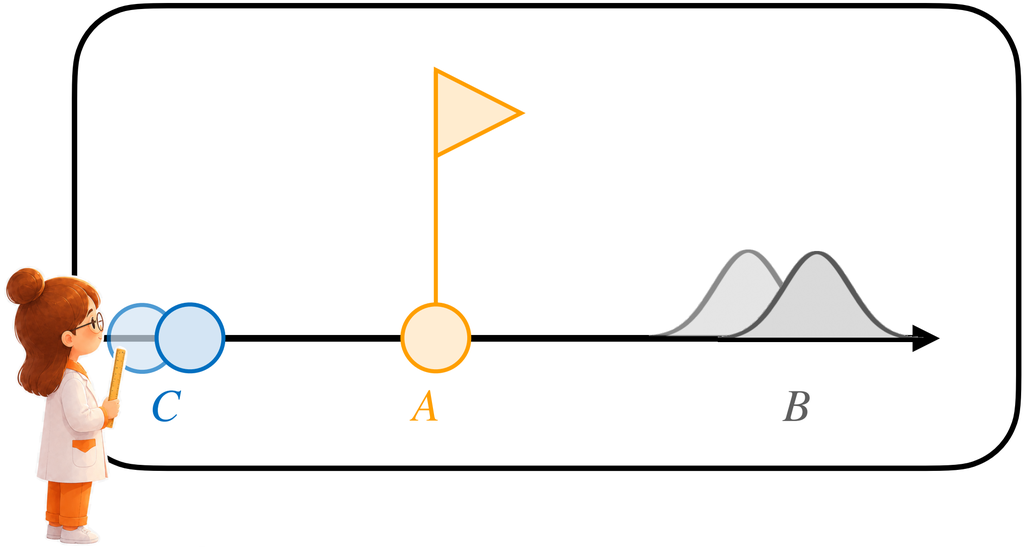
- **Canonical variables:**

$$(\hat{x}_A, \hat{p}_A) \qquad (\hat{x}_B, \hat{p}_B)$$

- Commutation relations:

$$[\hat{x}_B, \hat{p}_B] = -i\hbar \qquad [\hat{x}_A, \hat{p}_A] = -i\hbar$$

$$[\hat{x}_B, \hat{p}_A] = 0 \qquad [\hat{x}_C, \hat{p}_A] = 0$$



- **Canonical variables:**

$$(\hat{q}_B = \hat{x}_B - \hat{x}_A, \boxed{?}) \qquad (\hat{q}_C = -\hat{x}_A, \boxed{?})$$

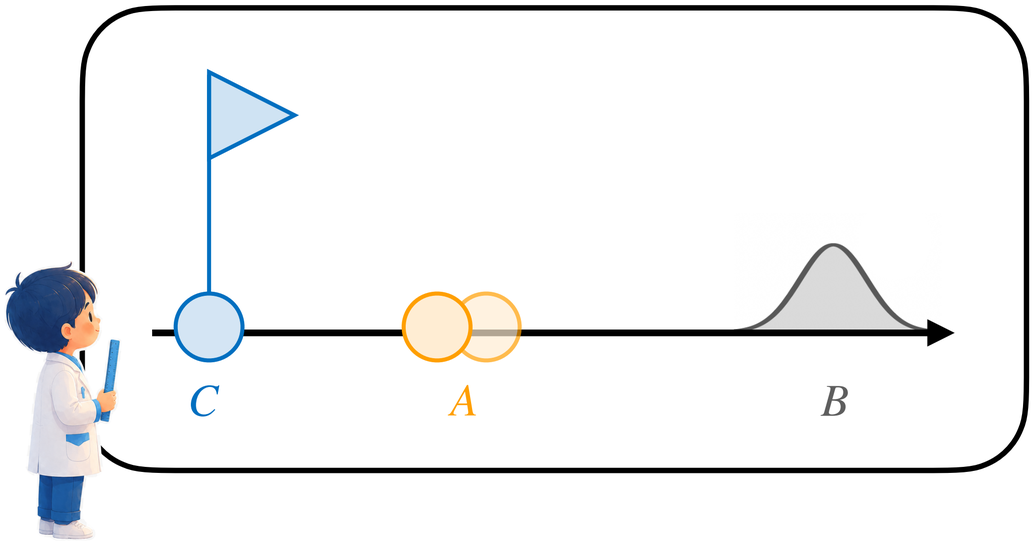
- Commutation relations imply

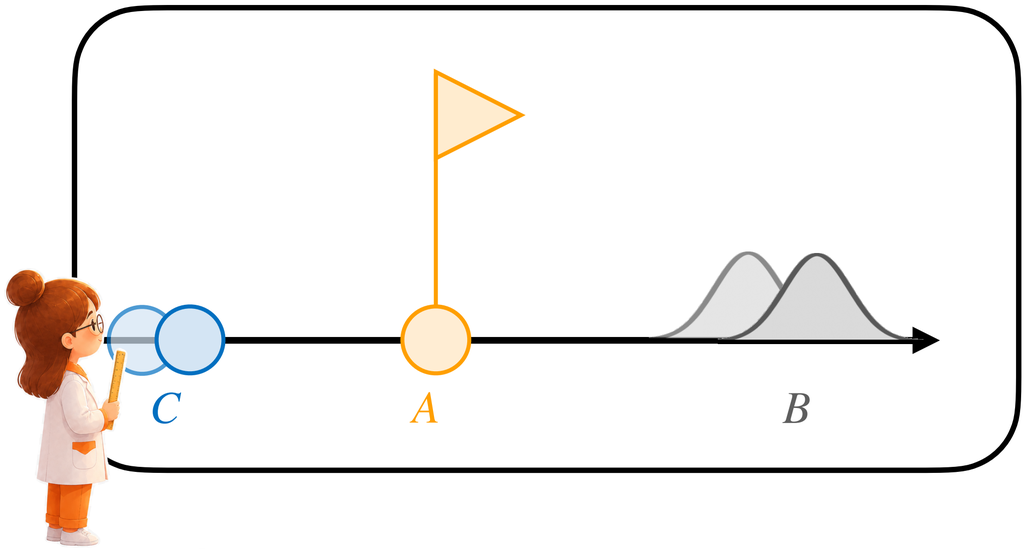
$$\boxed{\hat{\pi}_B = \hat{p}_B} \qquad \boxed{\hat{\pi}_C = -\hat{p}_A - \hat{p}_B}$$

Giacomini, Castro-Ruiz, and Brukner (2019)

Quantum Reference Frames

Perspectival Approach





- **Canonical variables:**

$$(\hat{x}_A, \hat{p}_A)$$

$$(\hat{x}_B, \hat{p}_B)$$

$$\hat{S}_{C\to A} = \mathscr{P}_{CA} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

- **Canonical variables:**

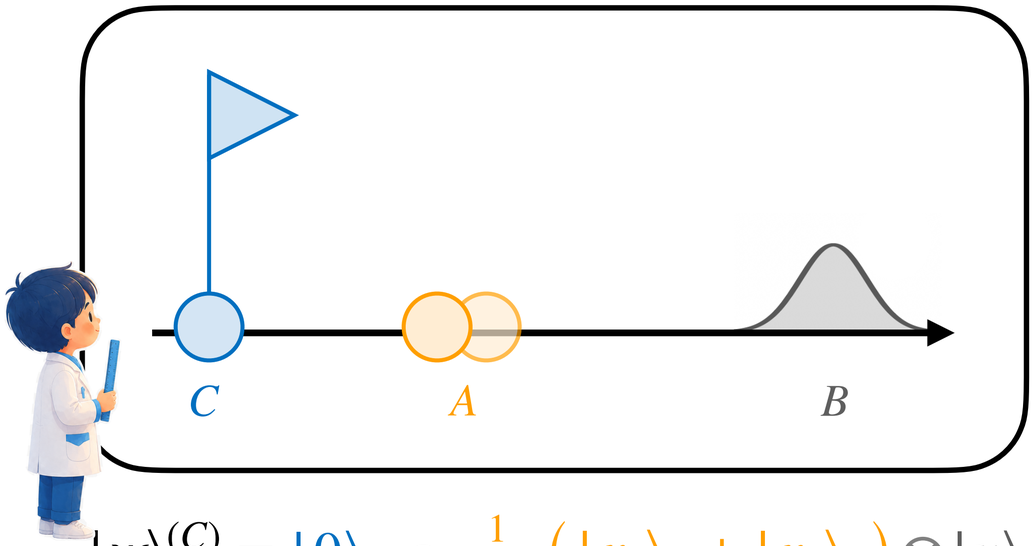
$$(\hat{q}_B = \hat{x}_B - \hat{x}_A,\ \hat{\pi}_B = \hat{p}_B\ )$$

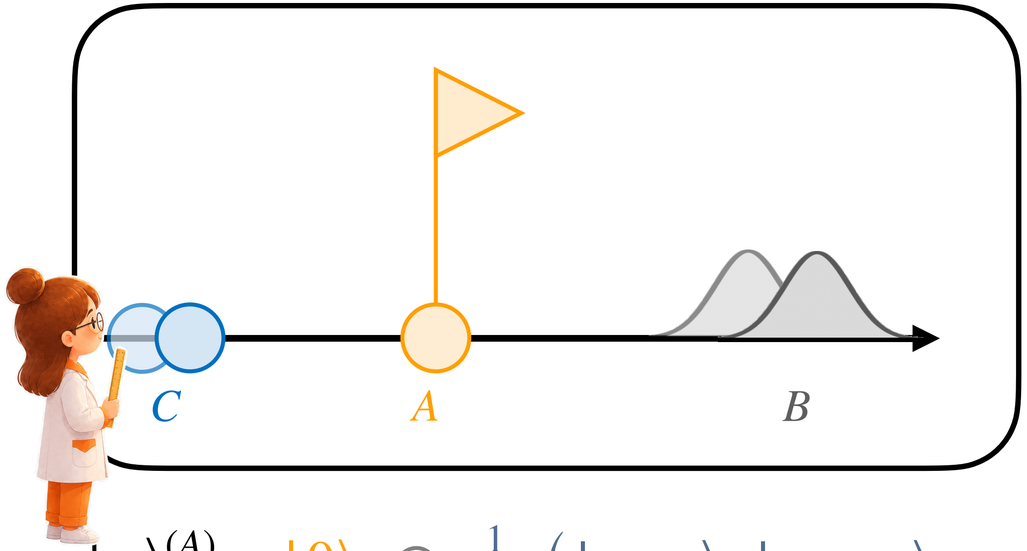
$$(\hat{q}_C = -\hat{x}_A,\ \hat{\pi}_C = -\hat{p}_A - \hat{p}_B\ )$$

Giacomini, Castro-Ruiz, and Brukner (2019)

Quantum Reference Frames

Perspectival Approach





$$|\psi\rangle^{(C)} = |0\rangle_C \otimes \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right) \otimes |y\rangle_B$$

$$|\psi\rangle^{(A)} = |0\rangle_A \otimes \frac{1}{\sqrt{2}}\left(|-x_1\rangle_C |y-x_1\rangle_B + |-x_2\rangle_C |y-x_2\rangle_B\right)$$

$$\hat{S}_{C\to A} = \mathscr{P}_{CA} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

For ideal QRFs, the PN and perspectival approach lead to the same perspectives.

Giacomini, Castro-Ruiz, and Brukner (2019)

Quantum Reference Frames

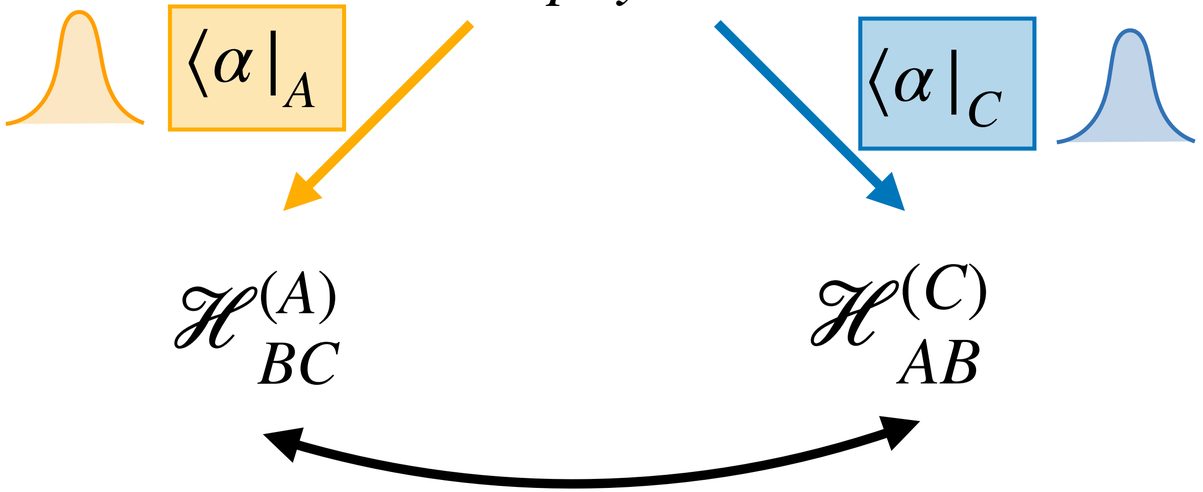
Perspective-Neutral Approach ... with non-ideal QRFs

$$\mathcal{H}_{kin} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

$$\Pi_{phys} = \frac{1}{|G|}\int dg\, U(g)$$

"coherent G-Twirl"

$$\mathcal{H}_{phys}$$



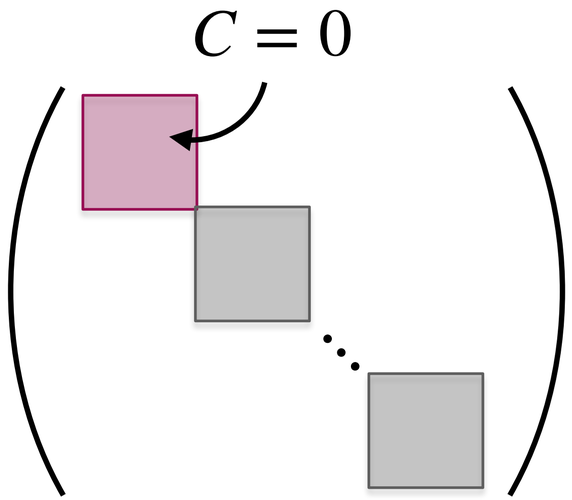
de la Hamette, Galley, Höhn, Loveridge, Müller (2021)

Quantum Reference Frames

Comparison with other Approaches

1. Perspectival & Perspective-Neutral Approach

- restriction to **invariant wavefunction** (“coherent G-twirl”)
- restriction to **zero-charge** sector
- observables $O \in \mathscr{B}(\mathscr{H}_{AB}^{(C)})$



$$\Pi_{phys}\,|\psi\rangle = \frac{1}{|G|}\int dg\, U(g)\,|\psi\rangle$$

Castro-Ruiz, Galley, Loveridge; de la Hamette, VK, Brukner; Doat, Vanrietvelde (2025)

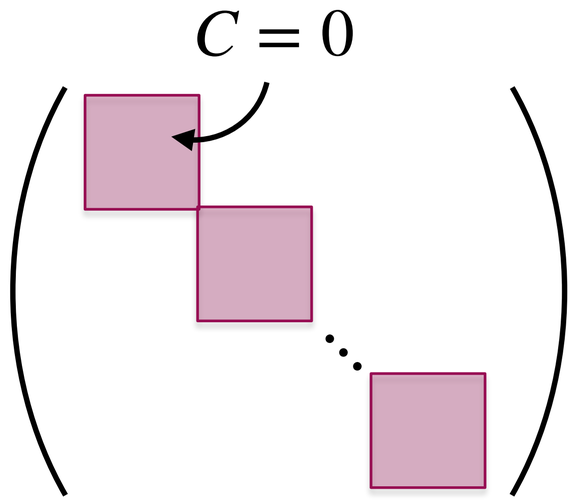
Quantum Reference Frames

Comparison with other Approaches

1. Perspectival & Perspective-Neutral Approach

2. **“Extra-Particle” Approach**

   - requires **commutation** $[\hat{C}, \hat{O}] = 0$
   - restriction to direct sum of **all charge sectors** via the “incoherent G-twirl”



$$\Pi(\rho) = \frac{1}{|G|} \int dg U(g) \rho U^{\dagger}(g)$$



What is the “extra particle”?

For abelian theories: extra-particle is the **total charge**.

Castro-Ruiz, Oreshkov (2021)

Castro-Ruiz, Galley, Loveridge; de la Hamette, VK, Brukner; Doat, Vanrietvelde (2025)

Quantum Reference Frames

Comparison with other Approaches

1. Perspectival & Perspective-Neutral Approach
2. “Extra-Particle” Approach
3. **Operational Approach**

- QRF perspective restricted to operators of the form

$$\mathcal{O}_{AB}^{(C)} = \int dx\; U_{AB}^{P}(x)\big(\,|y\rangle\langle y|_A \otimes O_B\big)\, U_{AB}^{P}(x)^{\dagger} \otimes |x\rangle\langle x|_C\,.$$



- Treats configurations that cannot be distinguished by such operators as **equivalent**.



Alice cannot uniquely infer the state of Charlie – only that it is within a certain equivalence class.

Carette, Głowacki, Loveridge (2023)

Castro-Ruiz, Galley, Loveridge; de la Hamette, VK, Brukner; Doat, Vanrietvelde (2025)

Quantum Reference Frames

Comparison with other Approaches

1. **Perspectival & Perspective-Neutral Approach**
   - all observables $O \in \mathscr{B}(\mathscr{H}_{AB}^{(C)})$
2. **“Extra-Particle” Approach**
   - all observables such that $[\hat{C}, \hat{O}] = 0$
3. **Operational Approach**
   - all observables of the form

$$\mathcal{O}_{AB}^{(C)} = \int dx \; U_{AB}^{P}(x)\left(|y\rangle\langle y|_A \otimes O_B \right) U_{AB}^{P}(x)^{\dagger} \otimes |x\rangle\langle x|_C .$$

Different approaches assume different accessible observables!



Castro-Ruiz, Galley, Loveridge; de la Hamette, VK, Brukner; Doat, Vanrietvelde (2025)

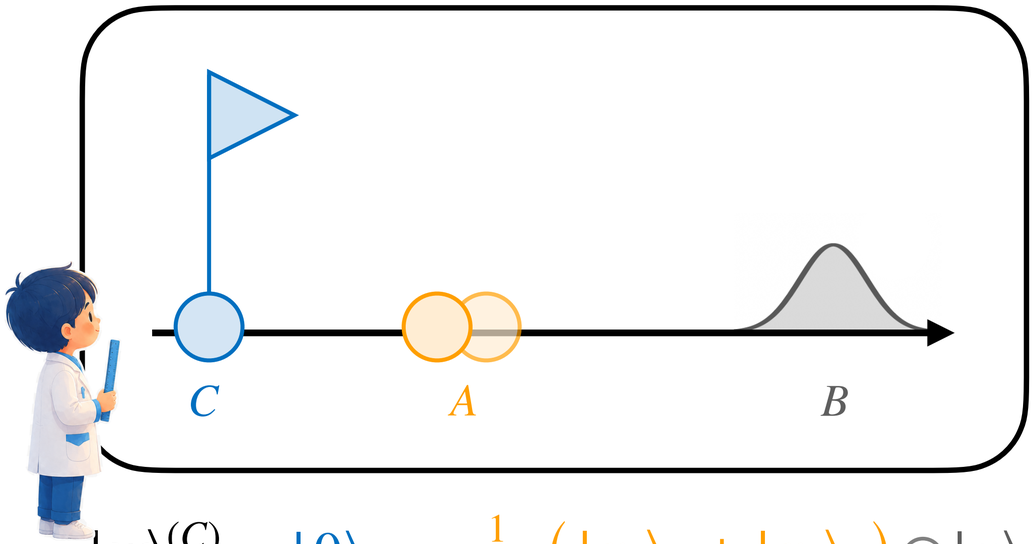


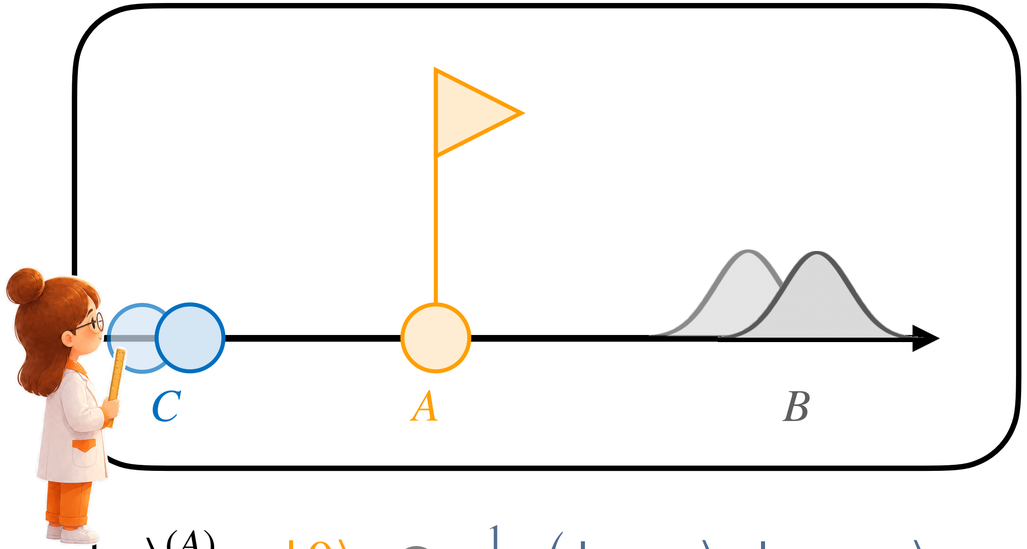
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  - Relativity of Superposition
  - Subsystem Relativity
- ✦ **More/Open Questions**

Relativity of Superposition

Perspectival Approach





$$|\psi\rangle^{(C)} = |0\rangle_C \otimes \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right) \otimes |y\rangle_B$$

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$$\hat{S}_{C\to A} = \mathscr{P}_{CA} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

Giacomini, Castro-Ruiz, and Brukner (2019)

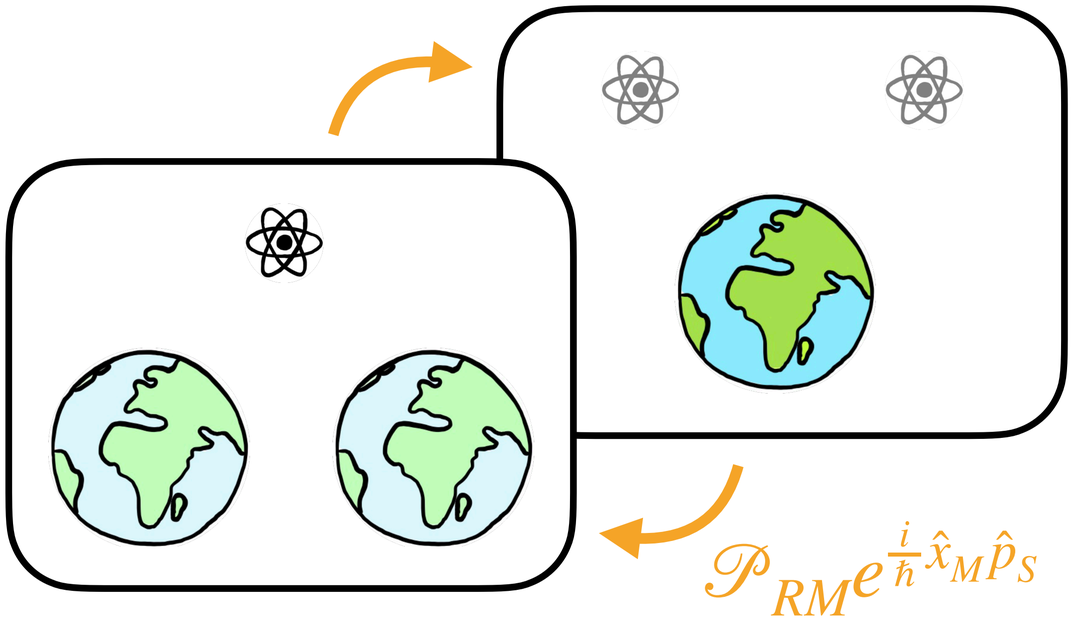
Relativity of Superposition

QRFs as Tool for Predictions

Goal: Describe motion of test particle in presence of a gravitational source in superposition.

Strategy

✦ Change into QRF in which the gravitational source is definite.



Relativity of Superposition

QRFs as Tool for Predictions

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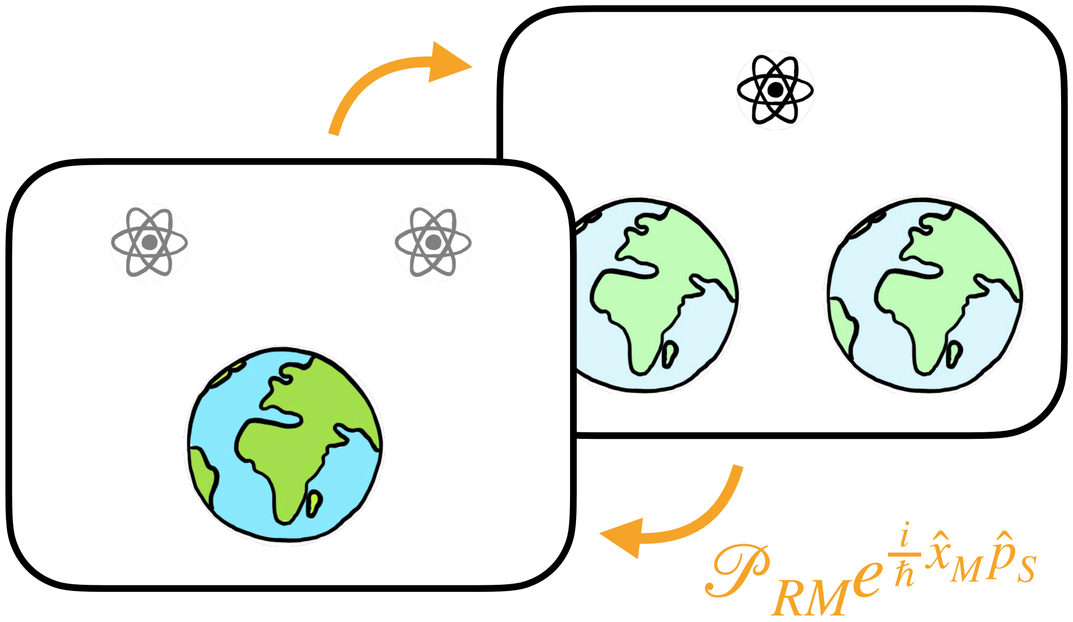
- ✦ Change into QRF in which the gravitational source is definite.
- ✦ Solve problem in the new frame.

geodesic motion

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

quantum phase

$$\Phi^{(i)} = \int_{A^{(i)}}^{B^{(i)}} m_S \sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$$



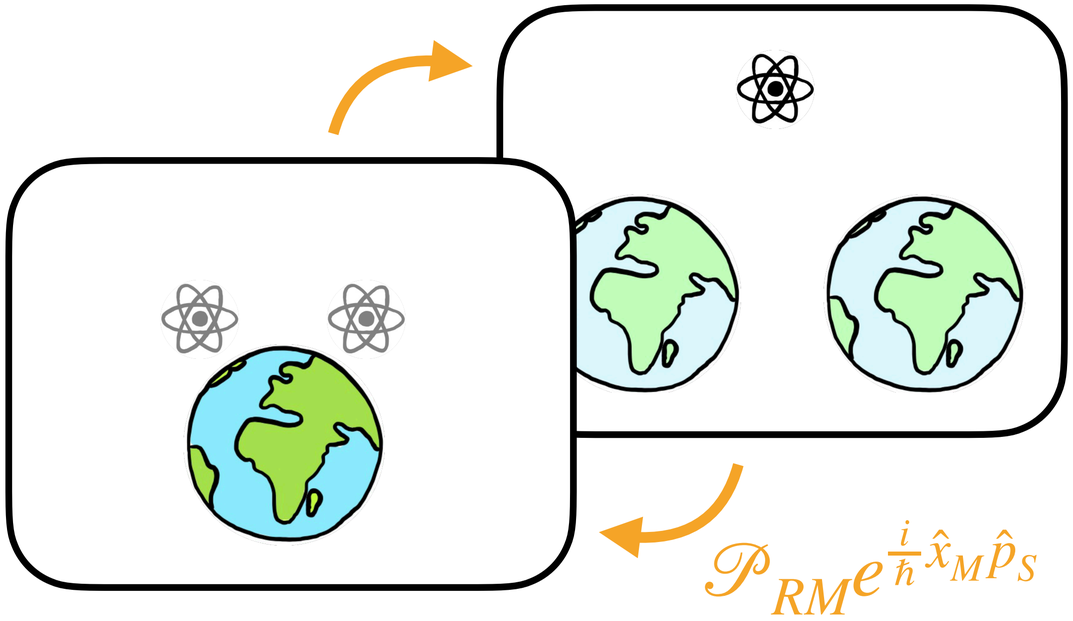
Relativity of Superposition

QRFs as Tool for Predictions

Goal: Describe motion of test particle in presence of a gravitational source in superposition.

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- ✦ Change into QRF in which the gravitational source is definite.
- ✦ Solve problem in the new frame.
- ✦ Transform back to infer the dynamics *assuming that the change of QRF is a symmetry of the equations of motion*.



Relativity of Superposition

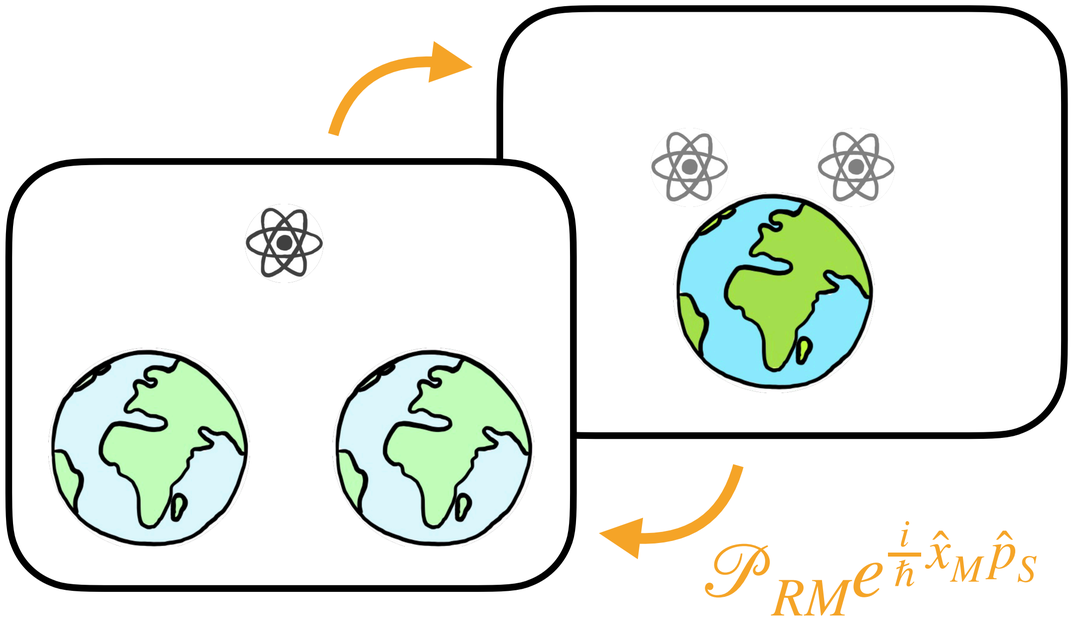
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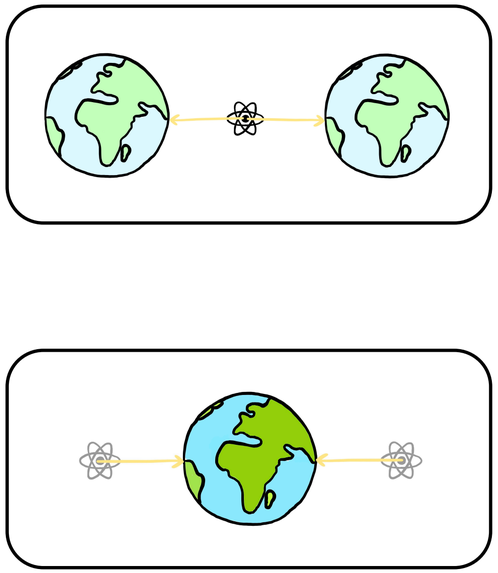
- ✦ Change into QRF in which the gravitational source is definite.
- ✦ Solve problem in the new frame.
- ✦ Transform back to infer the dynamics *assuming that the change of QRF is a symmetry of the equations of motion*.

"extended symmetry principle"



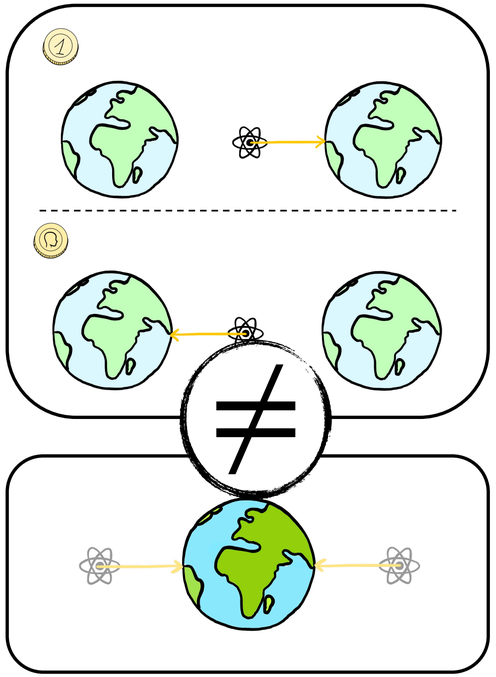
Relativity of Superposition

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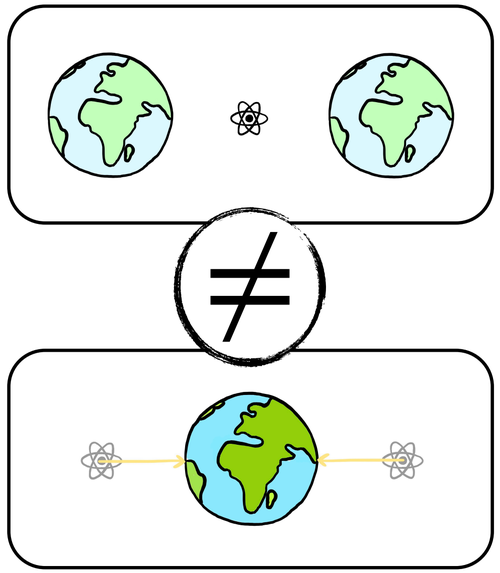


Extended Symmetry Principle

Consistent with gravitational field in superposition.



Collapse Models



Semi-Classical Gravity

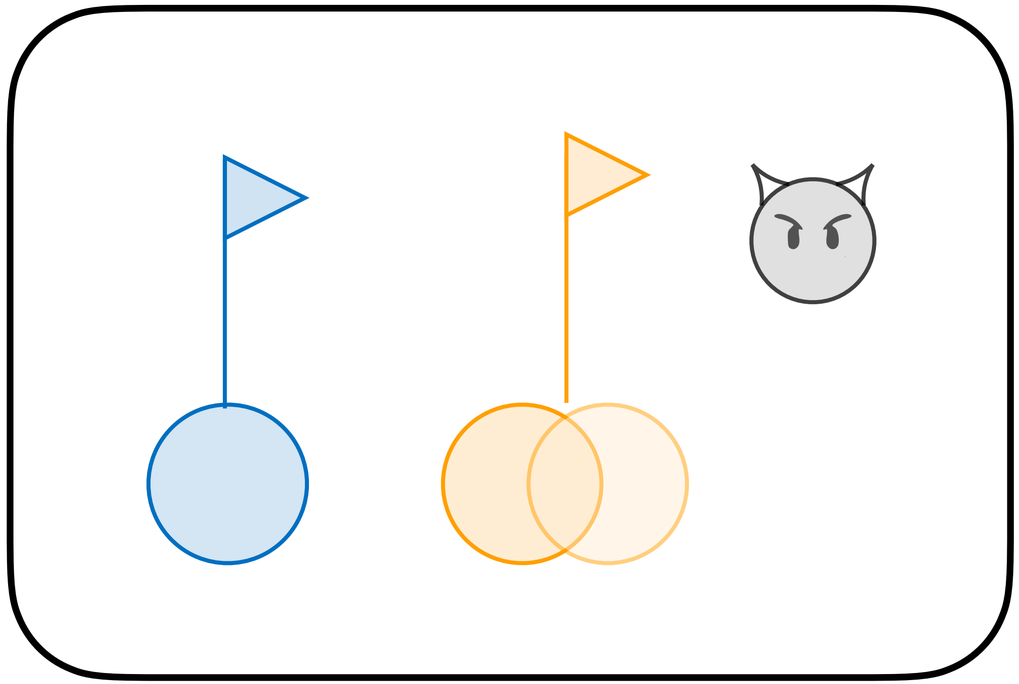


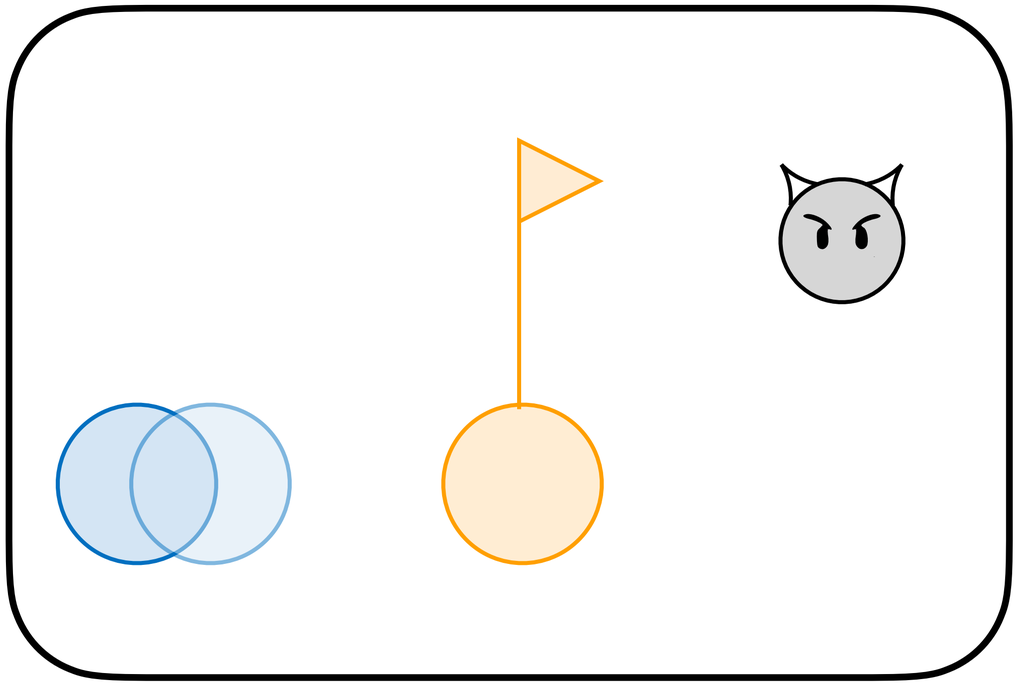
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Subsystem Relativity

Motivation: Paradox of the Third Particle





$$|\psi\rangle^{(B)}_{ABC} = |0\rangle_B \frac{1}{\sqrt{2}}\left(|-x^{(1)}\rangle_A |\phi^{(1)}\rangle_C + |-x^{(2)}\rangle_A |\phi^{(2)}\rangle_C\right) \neq |\psi'\rangle^{(B)}_{ABC} = |0\rangle_B \frac{1}{\sqrt{2}}\left(|-x^{(1)}\rangle_A + |-x^{(2)}\rangle_A\right) \otimes |\phi\rangle_C$$

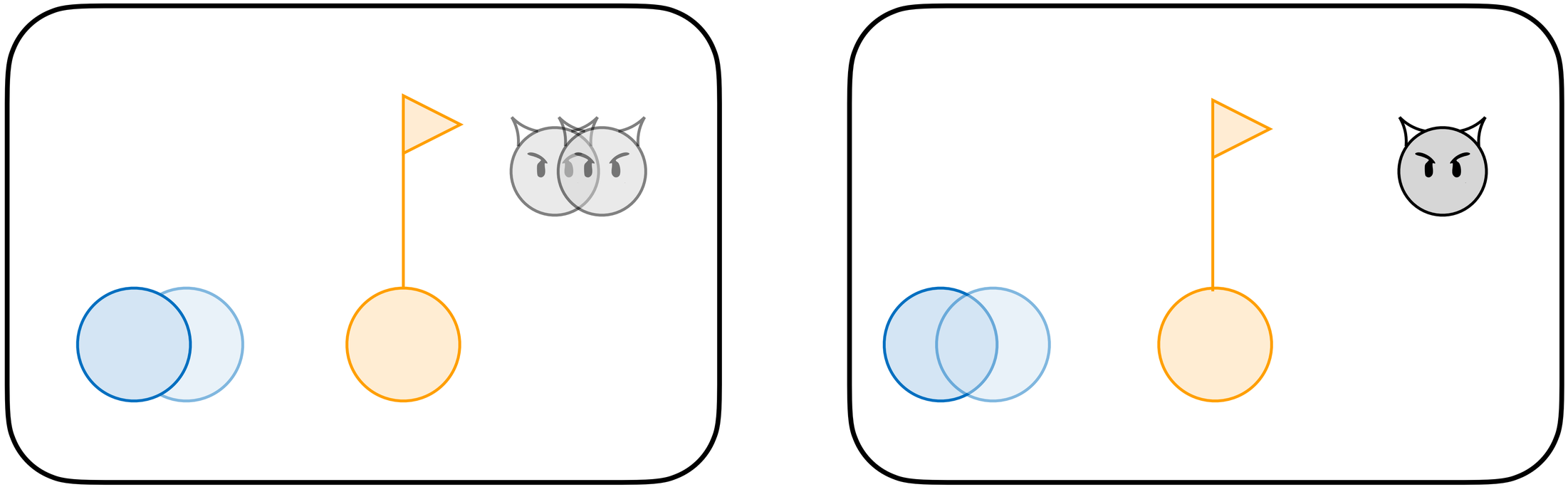
$$\rho_A^{(B)} = \frac{1}{2}(|-x^{(1)}\rangle\langle -x^{(1)}| + |-x^{(2)}\rangle\langle -x^{(2)}|)$$

$$\rho_A^{(B)} = |\psi'\rangle_B^{(B)}\langle\psi'|$$

Krumm, Höhn, Müller (2021)

Subsystem Relativity

Motivation: Paradox of the Third Particle



$$|\psi\rangle^{(B)}_{ABC} = |0\rangle_B \frac{1}{\sqrt{2}} \left(|-x^{(1)}\rangle_A |\phi^{(1)}\rangle_C + |-x^{(2)}\rangle_A |\phi^{(1)}\rangle_C \right) \neq |\psi'\rangle^{(B)}_{ABC} = |0\rangle_B \frac{1}{\sqrt{2}} \left(|-x^{(1)}\rangle_A + |-x^{(2)}\rangle_A \right) \otimes |\phi\rangle_C$$

Adding the third particle in **different QRFs** leads to **different states**.

Krumm, Höhn, Müller (2021)

Subsystems in Quantum Theory

In QM, the **tensor-product structure** provides the subsystem structure:

> **Postulate 4**: The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. Moreover, if we have systems numbered 1 through n, and system number i is prepared in the state $|\psi_i\rangle$, then the joint state of the total system is $|\psi_1\rangle \otimes |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$.

Nielsen and Chuang (2010)

But what does this mean operationally?



Subsystems in Quantum Theory

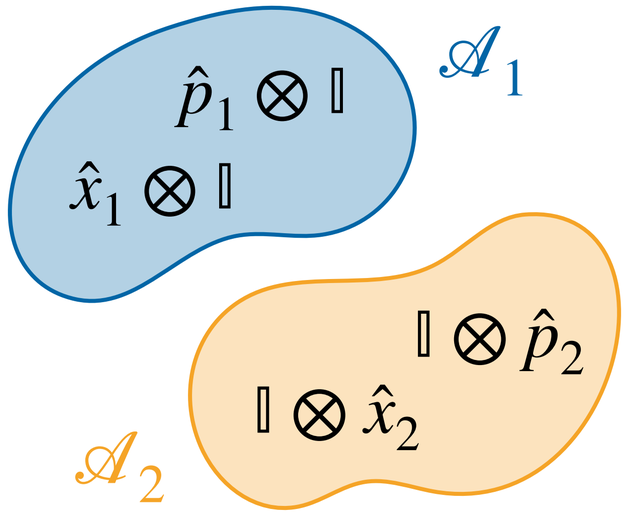
VOLUME 92, NUMBER 6 PHYSICAL REVIEW LETTERS week ending 13 FEBRUARY 2004

Quantum Tensor Product Structures are Observable Induced

Paolo Zanardi,[1,2] Daniel A. Lidar,[3] and Seth Lloyd[1]

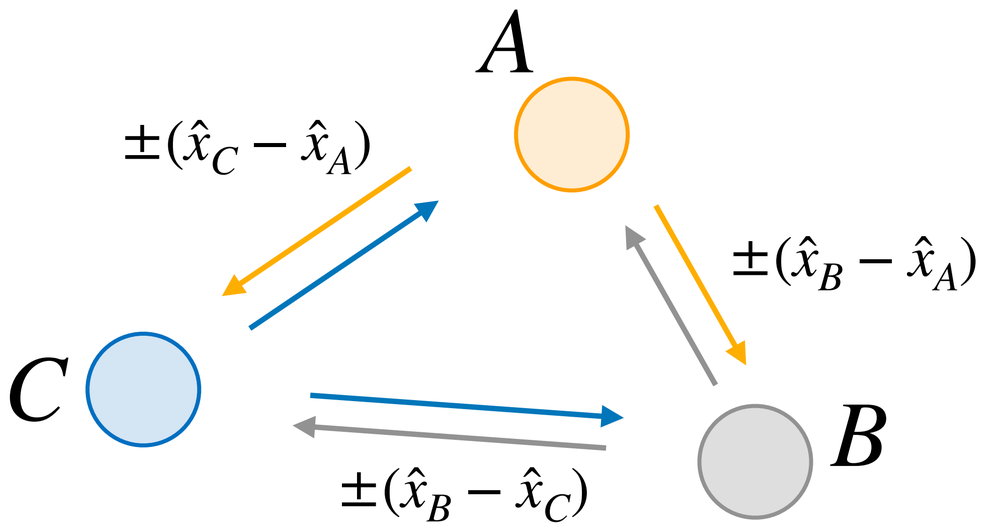
Idea: A tensor product structure is induced by independent **algebras of observables** satisfying:

1. **Local accessibility:** Each $\mathscr{A}_i$ corresponds to a set of controllable observables.
2. **Subsystem independence:** $[\mathscr{A}_i, \mathscr{A}_j] = 0 \quad \forall i \neq j$
3. **Completeness:** $\bigvee_{i=1}^{n} \mathscr{A}_i \simeq \bigotimes_{i=1}^{n} \mathscr{A}_i \simeq \mathrm{End}(\mathscr{C})$



QRF-induced Tensor Product Structure

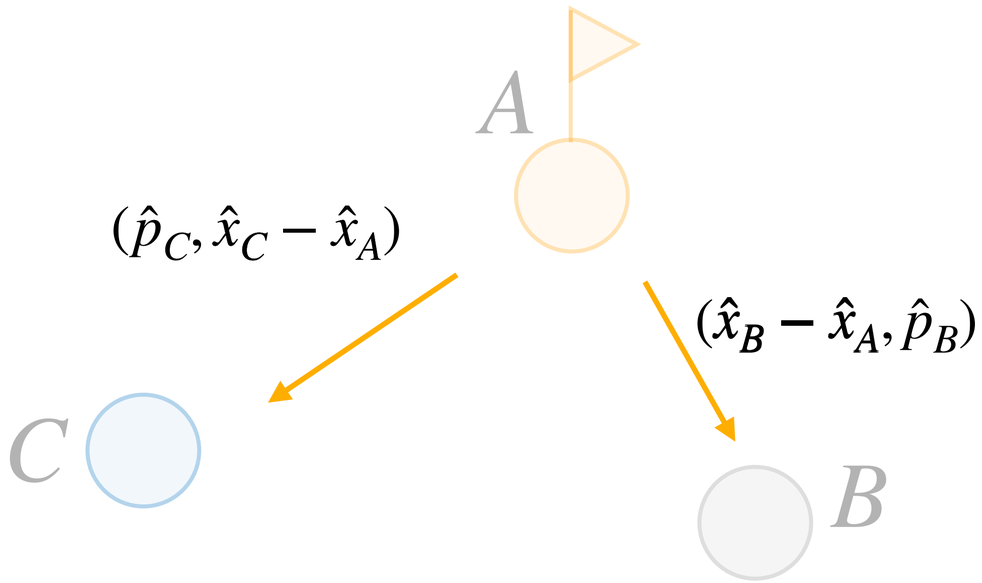
- A tensor product structure is induced by independent algebras of **observables**.
- The **choice of QRF** singles out particular commuting subalgebras.



Höhn, Lock, Ahmad, Smith, Galley (2021)

QRF-induced Tensor Product Structure

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QRF-induced Tensor Product Structure

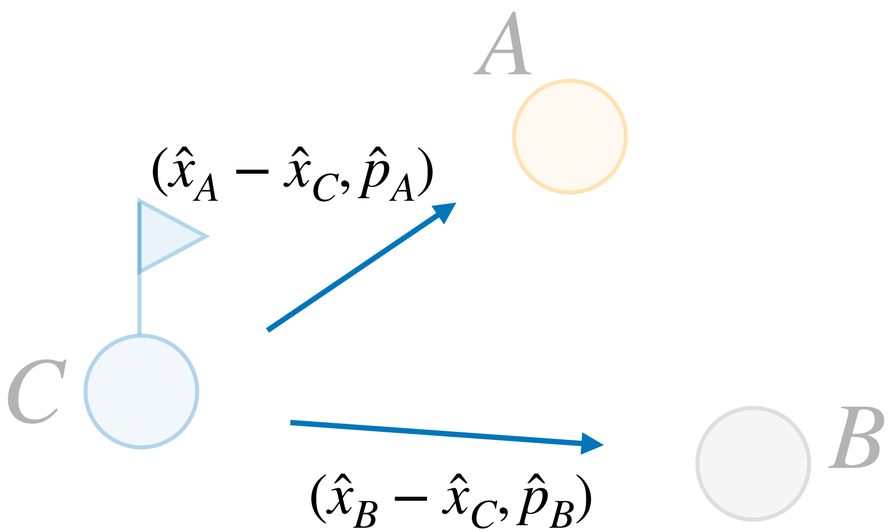
- A tensor product structure is induced by independent algebras of **observables**.
- The **choice of QRF** singles out particular commuting subalgebras.
- It thus **induces** a tensor product structure

$$\mathscr{H}_{AB}^{(C)} = \mathscr{H}_{A}^{(C)} \otimes \mathscr{H}_{B}^{(C)}$$

can measure $\hat{x}_B - \hat{x}_C, \hat{p}_B$

can measure $\hat{x}_A - \hat{x}_C, \hat{p}_A$

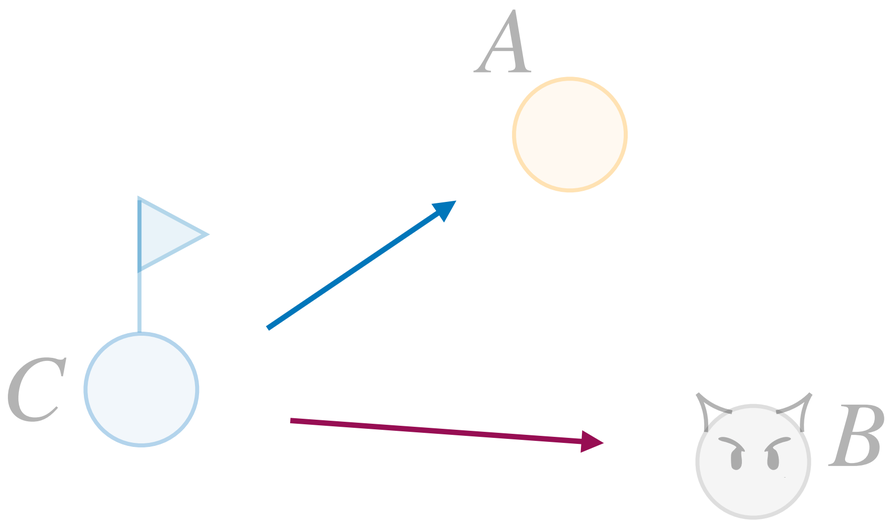
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Höhn, Lock, Ahmad, Smith, Galley (2021)

QRF-induced Tensor Product Structure

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- The **choice of QRF** singles out particular commuting subalgebras.
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- Allows to make sense of the **paradox of the third particle**…



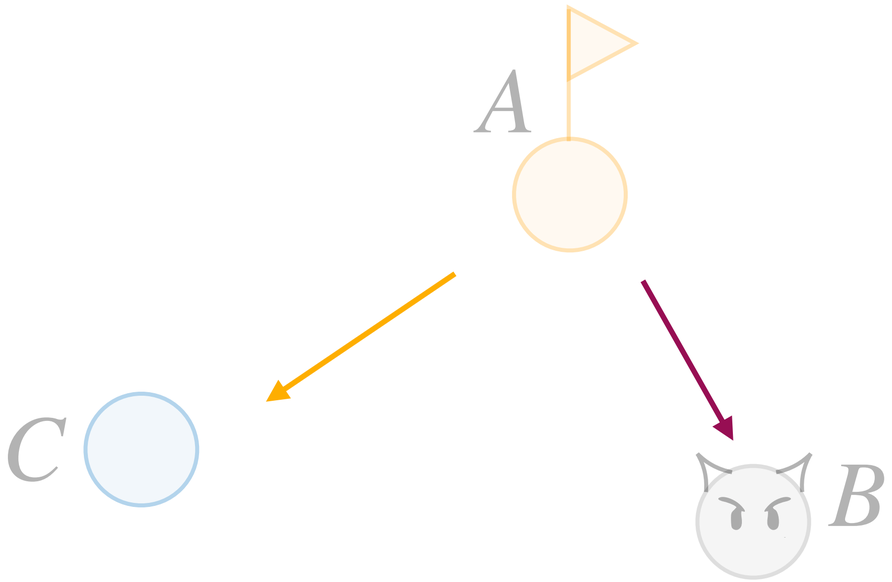
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We are adding different subsystems!



Höhn, Lock, Ahmad, Smith, Galley (2021)

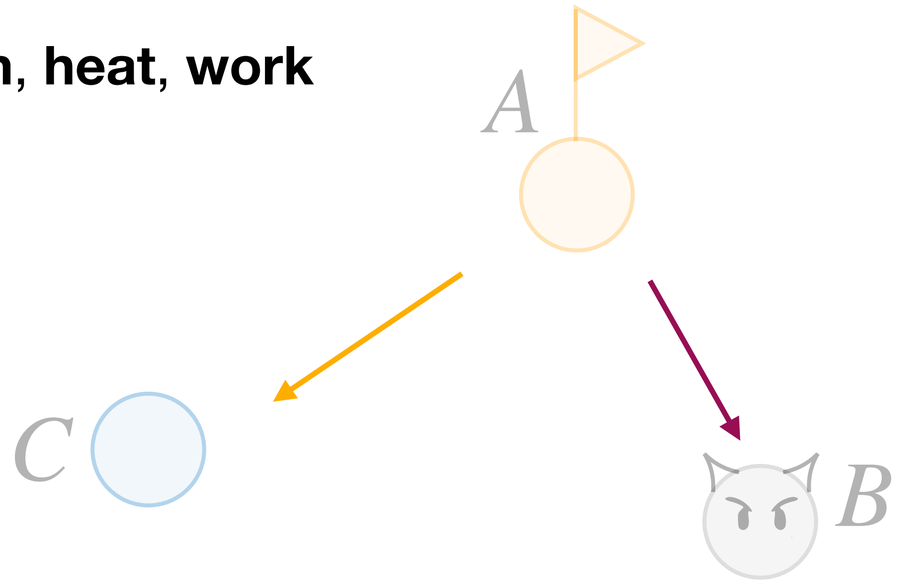
Consequences of Subsystem Relativity
(a selection)

The relativity of subsystems under QRF transformations leads to the relativity of

- **Entanglement**
- **Entropy** and **coherence**

} sum of entropy and coherence is invariant

- Thermodynamic quantities, such as **thermal equilibrium**, **heat**, **work**
- Decoherence and **open quantum systems** dynamics



Cepollaro et al. (2024); Höhn, Kotecha, Mele (2023); Luppi, VK, Giacomini, Smirne (2026)



Outline

- ✦ **Quantum Reference Frames**
  - Perspective-Neutral & Perspectival Approach
  - Extra-Particle & Operational Approach
- ✦ **Implications & Applications**
  - Relativity of Superposition
  - Subsystem Relativity
- ✦ **More/Open Questions**

More/Open Questions

- **Relation** between the different approaches.
- Implications of **non-ideal** QRFs.
- Operational meaning - what does it mean to **measure** from within a given QRF?
- Extension to **quantum field theory** and **gravity**.
- **Connection** to ongoing work in QG and HEP.





Your questions?