LOCAL OPERATIONS AND FIELD-MEDIATED ENTANGLEMENT WITHOUT A LOCAL TENSOR PRODUCT STRUCTURE

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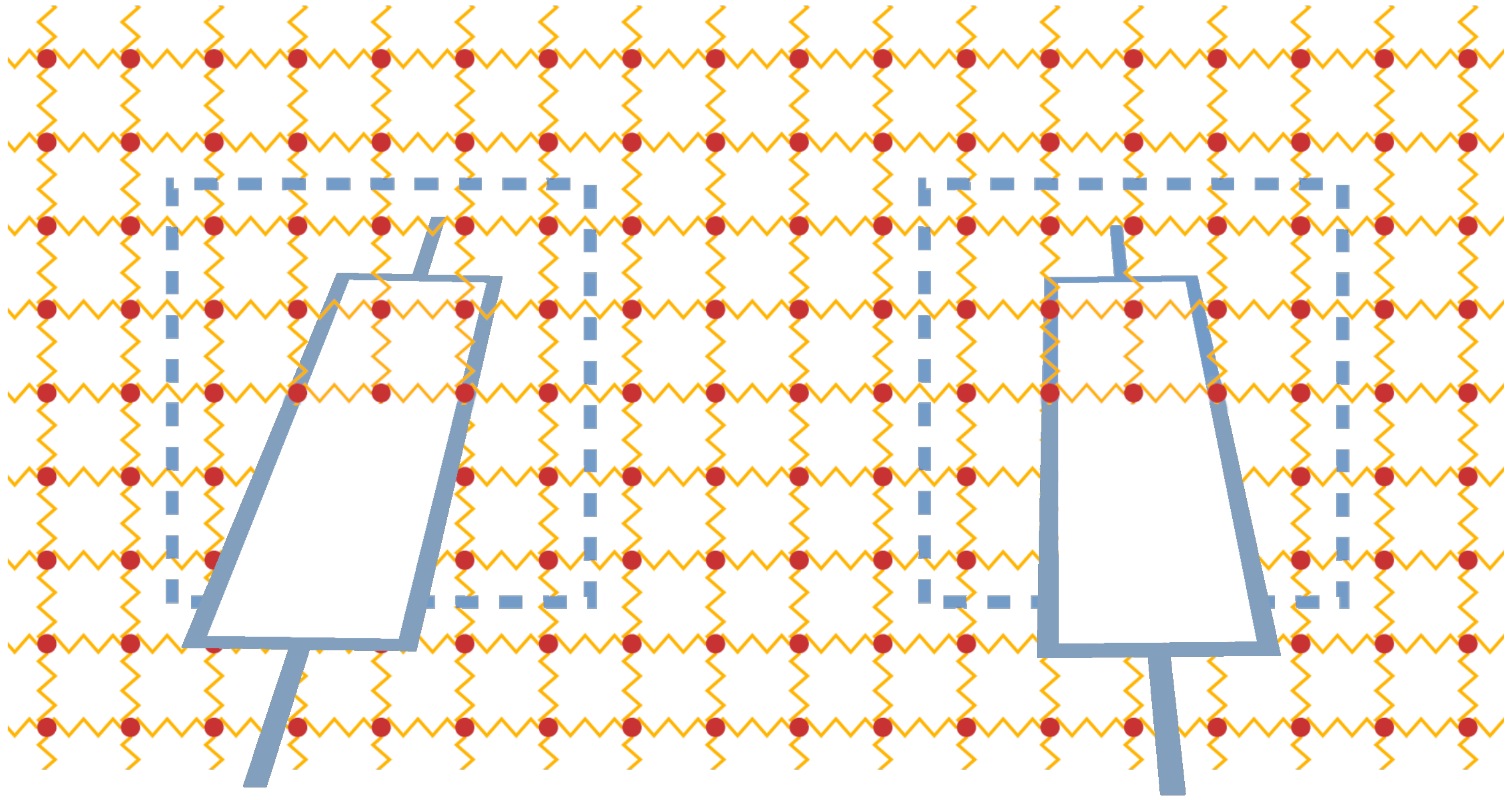
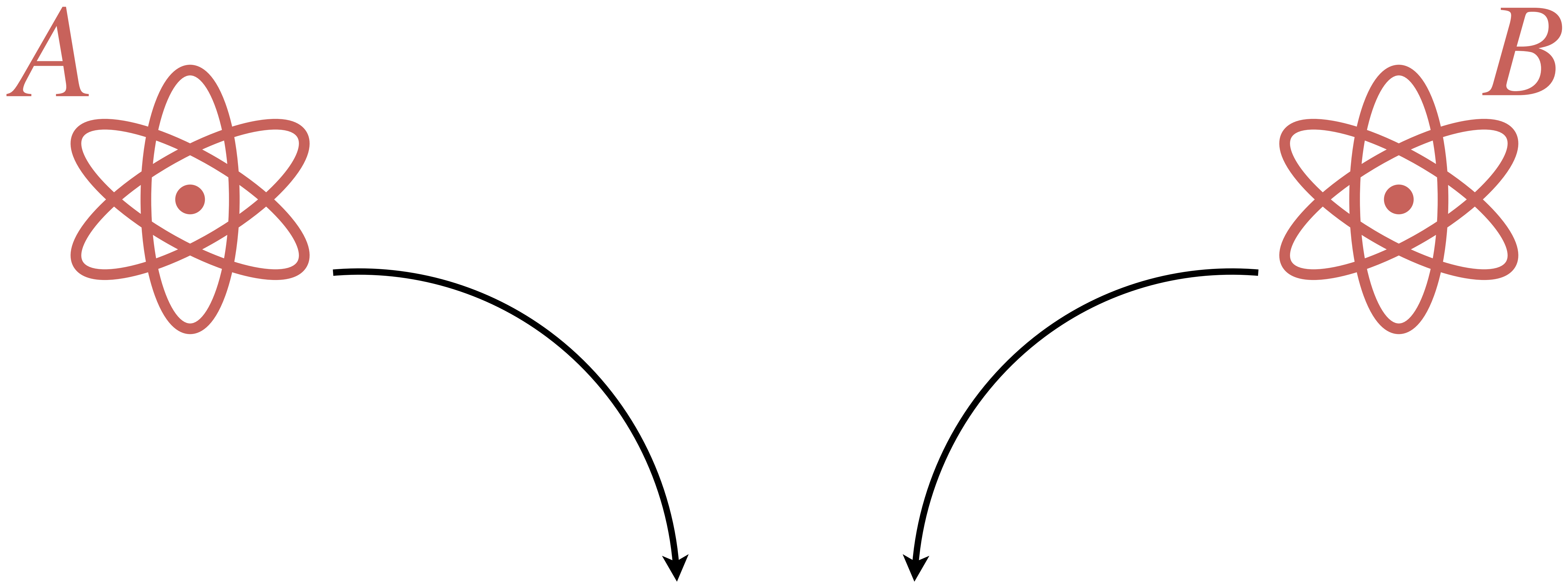
joint work with A. Spalvieri, S. G. Garmier, arXiv 2512.19806



Quantum Gravity Meets Quantum Information – Foundations, Frameworks and Frontiers

7-9 July 2026



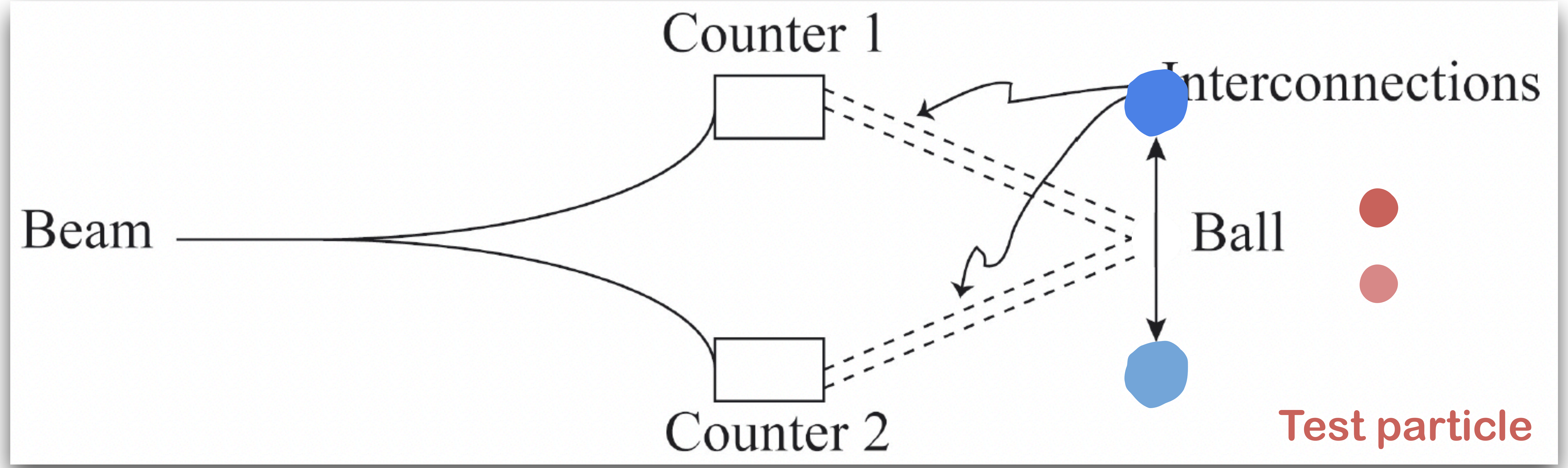
$$\underbrace{\mathscr{H}_A \otimes \mathscr{H}_B}_{\mathscr{H}}$$

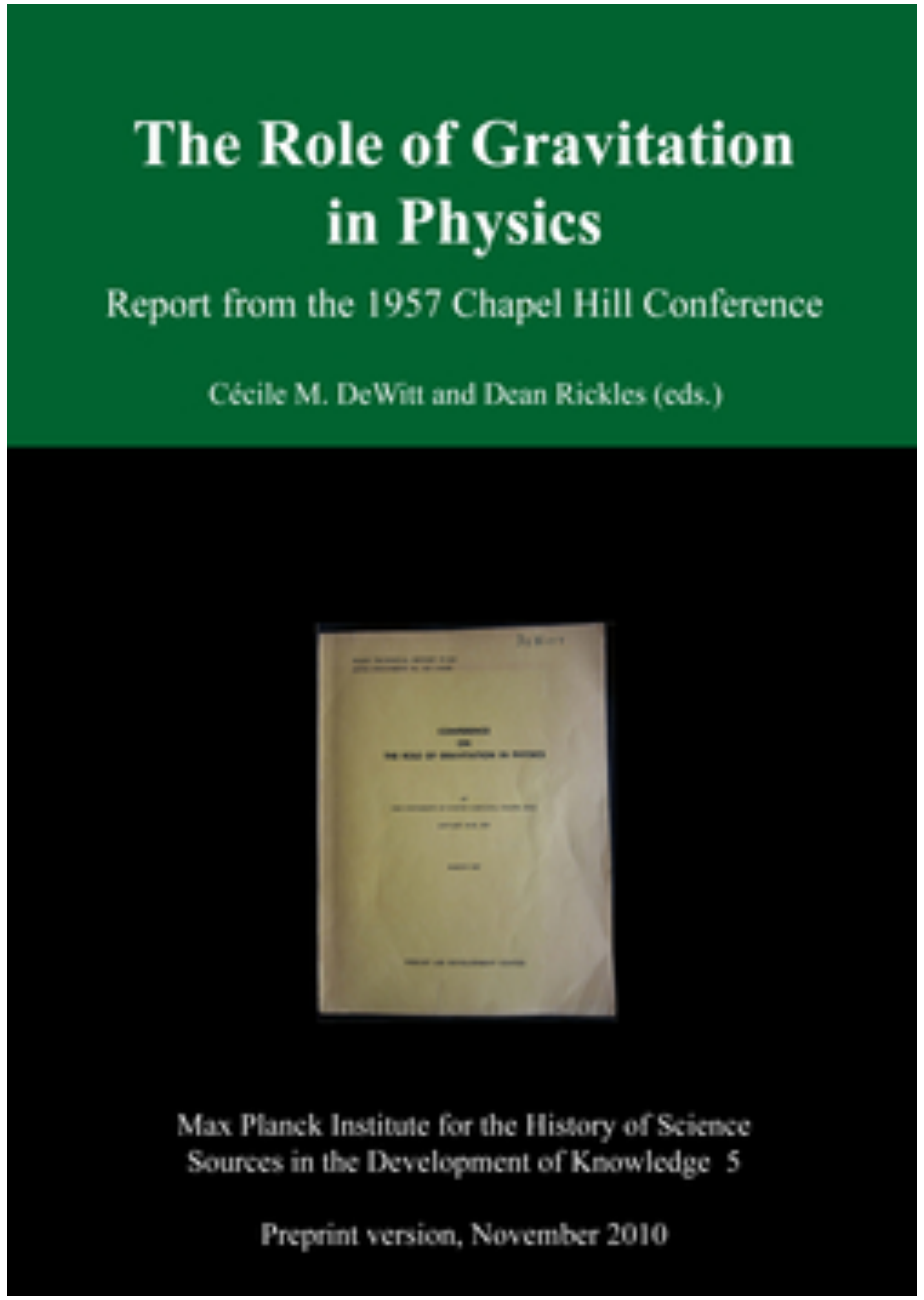
$$\hat{O}_A \otimes \mathbb{I}_B \qquad \mathbb{I}_A \otimes \hat{O}_B$$

FEYNMAN'S THOUGHT EXPERIMENT (1957)

"… it seems to me that we are in trouble if we believe in quantum mechanics but do not quantize gravitational theory"

R. Feynman, Chapel Hill Conference

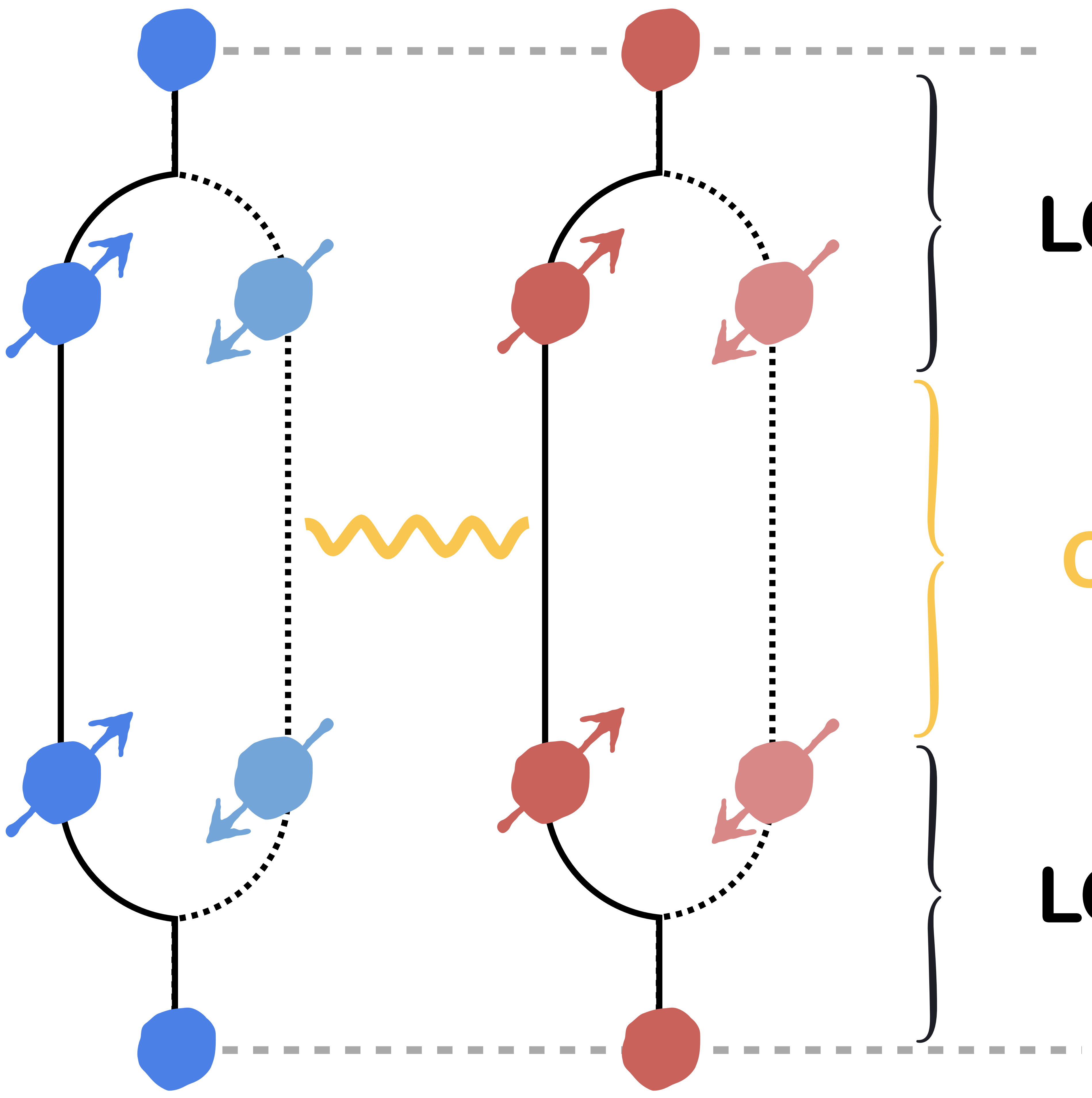




"If you believe in quantum mechanics up to any level then you have to believe in gravitational quantization in order to describe this experiment."

GRAVITATIONALLY INDUCED ENTANGLEMENT (GIE)

Bose et al. PRL (2017)
Marletto, Vedral PRL (2017)



$|\psi_f\rangle_{AB} = ?$

LO

C

LOCC ?

LO

$|\psi_i\rangle_{AB} = |\phi\rangle_A |\phi\rangle_B$

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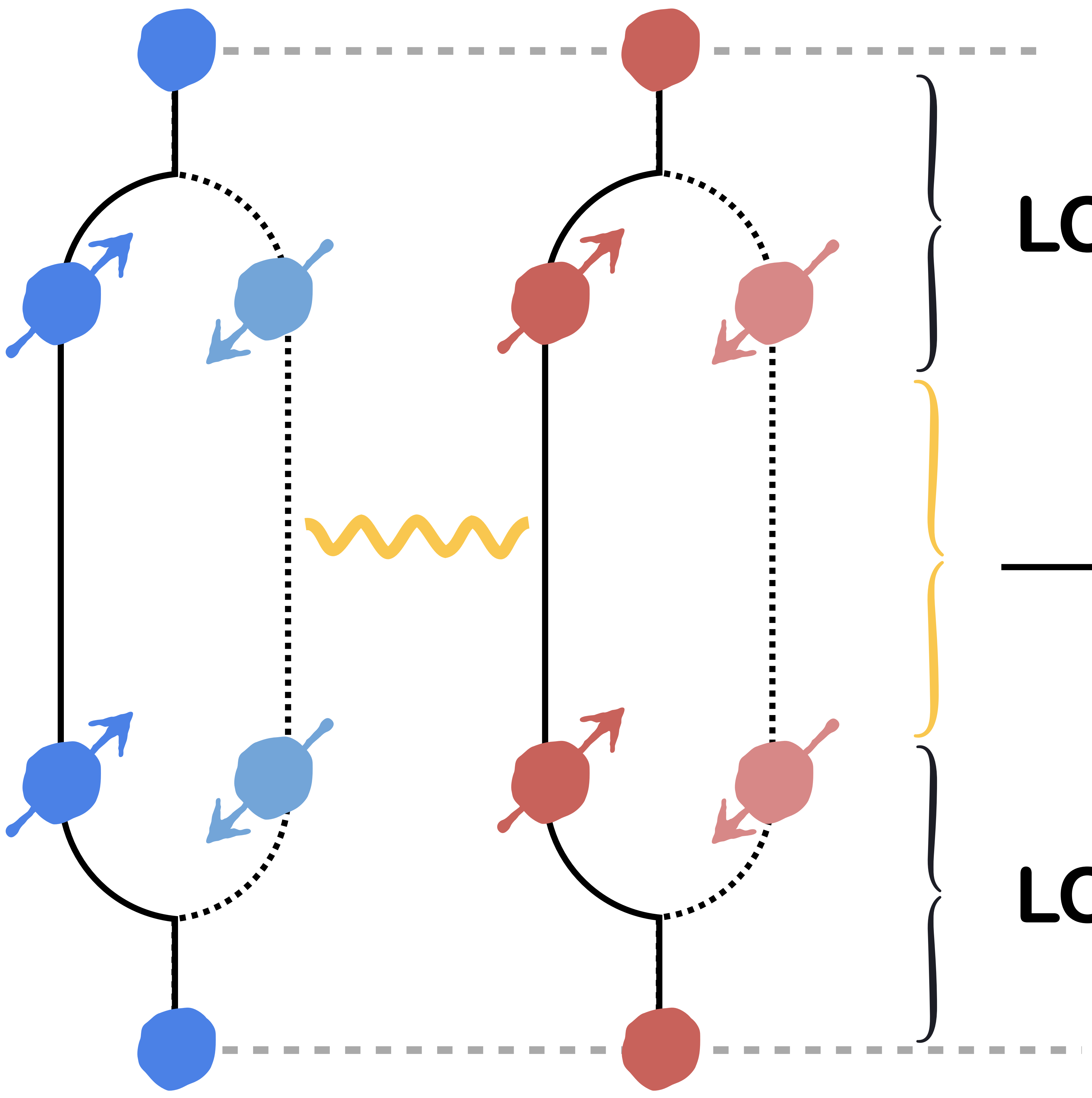


$$|\psi_f\rangle_{AB} = ?$$

LO?

Quantum theory

$$e^{-\frac{i}{\hbar}\frac{m_A m_B}{|\hat{x}_A - \hat{x}_B|}t}$$

CC?

LO?

$$|\psi_i\rangle_{AB} = |\phi\rangle_A |\phi\rangle_B$$

HILBERT SPACE STRUCTURE IN GAUGE THEORIES

Classical electrodynamics $\mathscr{G} := \nabla \cdot E(x) = 0 \qquad \forall x \in \mathbb{R}^3$

Quantum electrodynamics $\nabla \cdot \hat{E}(x) \,|\, \Psi\rangle_{ph} = 0 \qquad \forall x \in \mathbb{R}^3$

$$\mathscr{H} = \cdots \otimes \underbrace{\mathscr{H}^E_{x-\delta} \otimes \mathscr{H}^E_x}_{\nabla} \otimes \underbrace{\phantom{\mathscr{H}^E_x}}_{\nabla} \mathscr{H}^E_{x+\delta} \otimes \cdots$$

$$\mathscr{H}_{ph} \neq \otimes_x \mathscr{H}^E_x$$

Local Operations? Entanglement?

Entanglement entropy in gauge theories/ entanglement in lattice gauge theories

Bianchi, Casini, Dittrich, Donnelly, Lloyd, Riello, Stottmeister, Vestraete, Wall, Zanardi, …

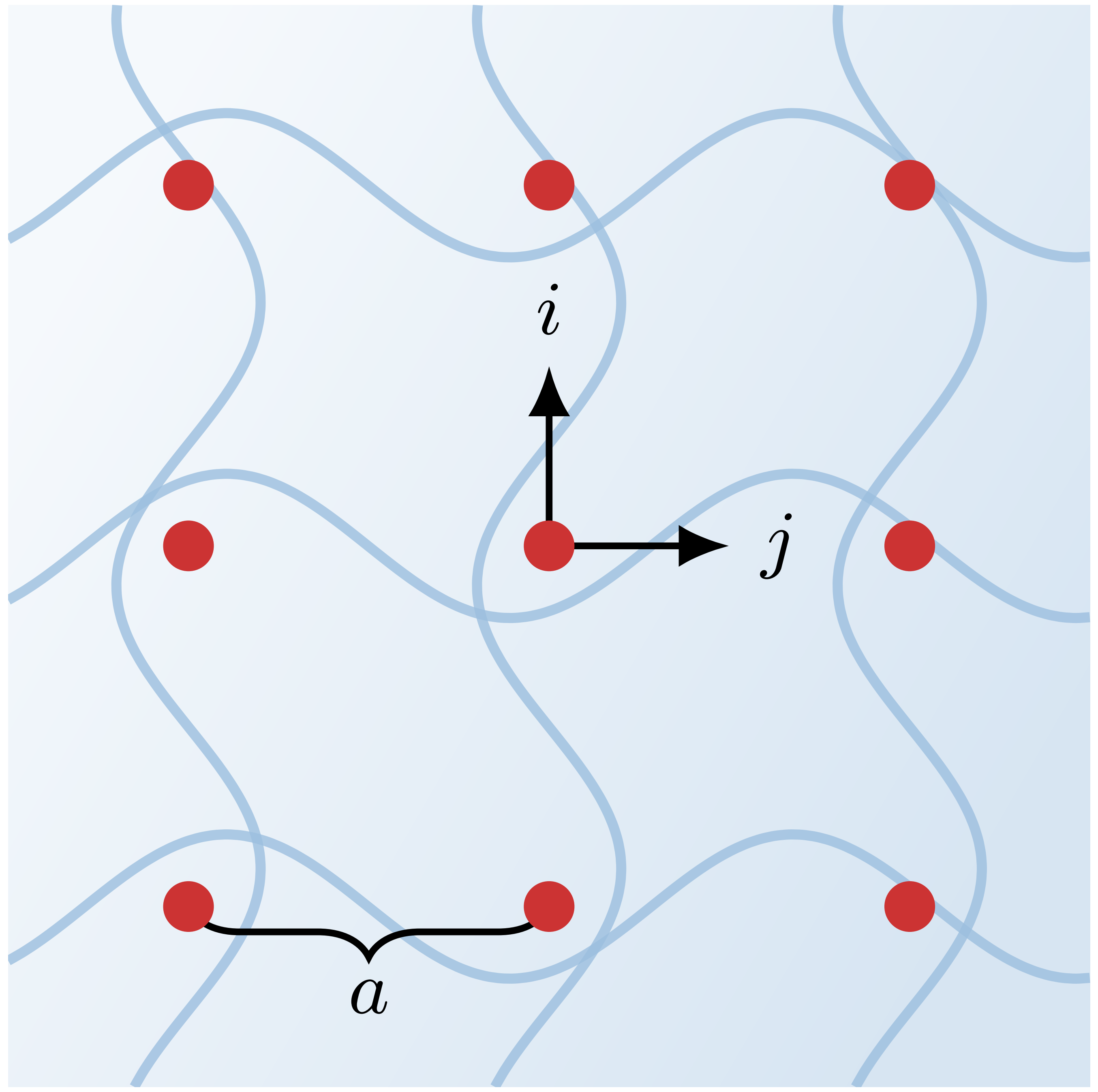
TWO MAIN CRITICISMS

1. The LOCC framework and the corresponding theorem cannot be applied

2. The mechanism for entanglement generation is unclear

How is entanglement generated in a theory that does not have a spacetime-local tensor product structure?
How can we generalise QI tools to gauge theories?

A TOY MODEL FOR 2+1 QUANTUM ELECTRODYNAMICS

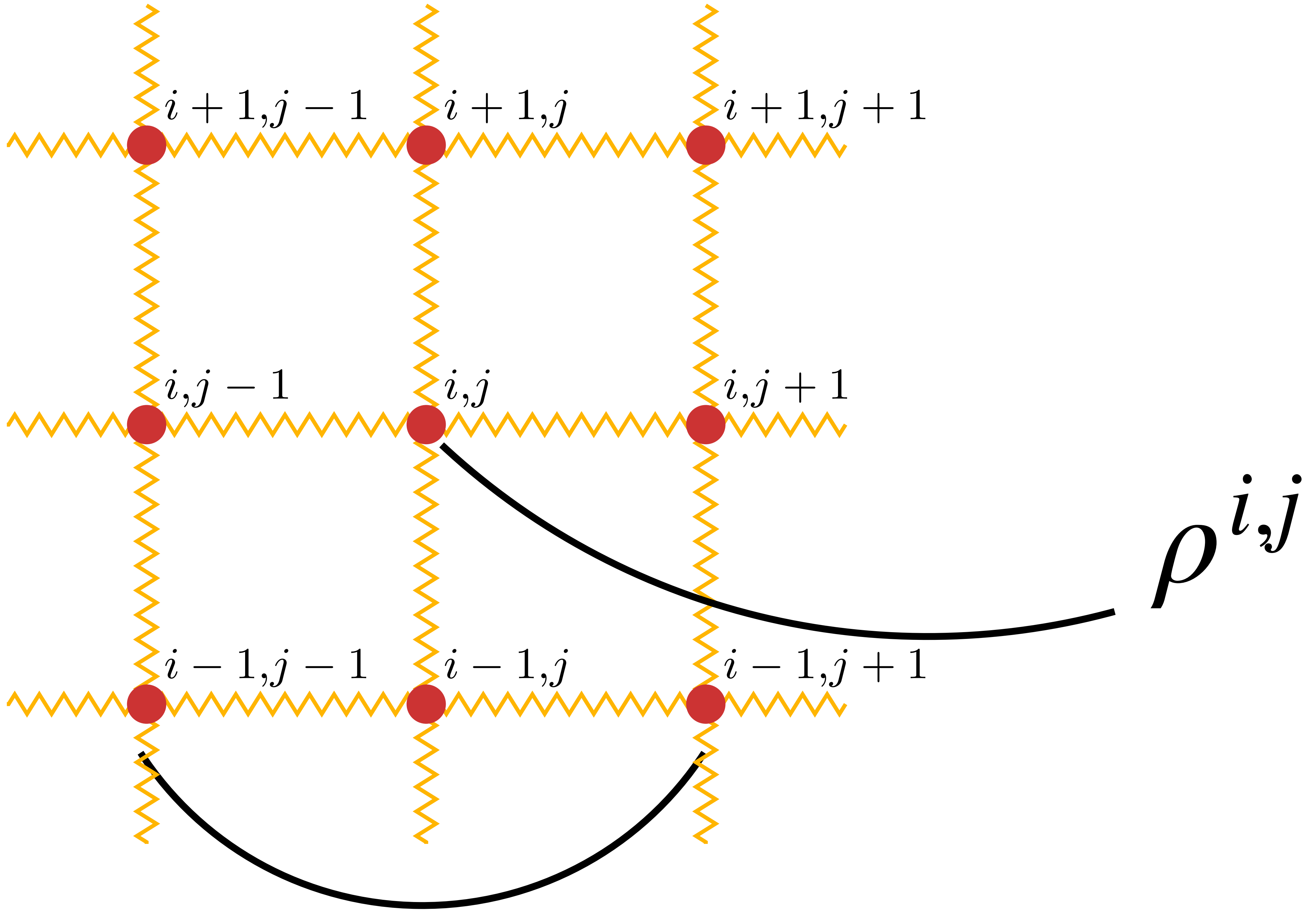


Matter $\rightarrow m$

Field $\rightarrow f$

Gravitationally induced entanglement (GIE)
$\rightarrow$ **Field Mediated Entanglement (FME)**

THE CLASSICAL TOY MODEL



$$\bar{\partial}_x f^{i,j} = \frac{1}{2a}\left(f^{i,j+1} - f^{i,j-1}\right)$$

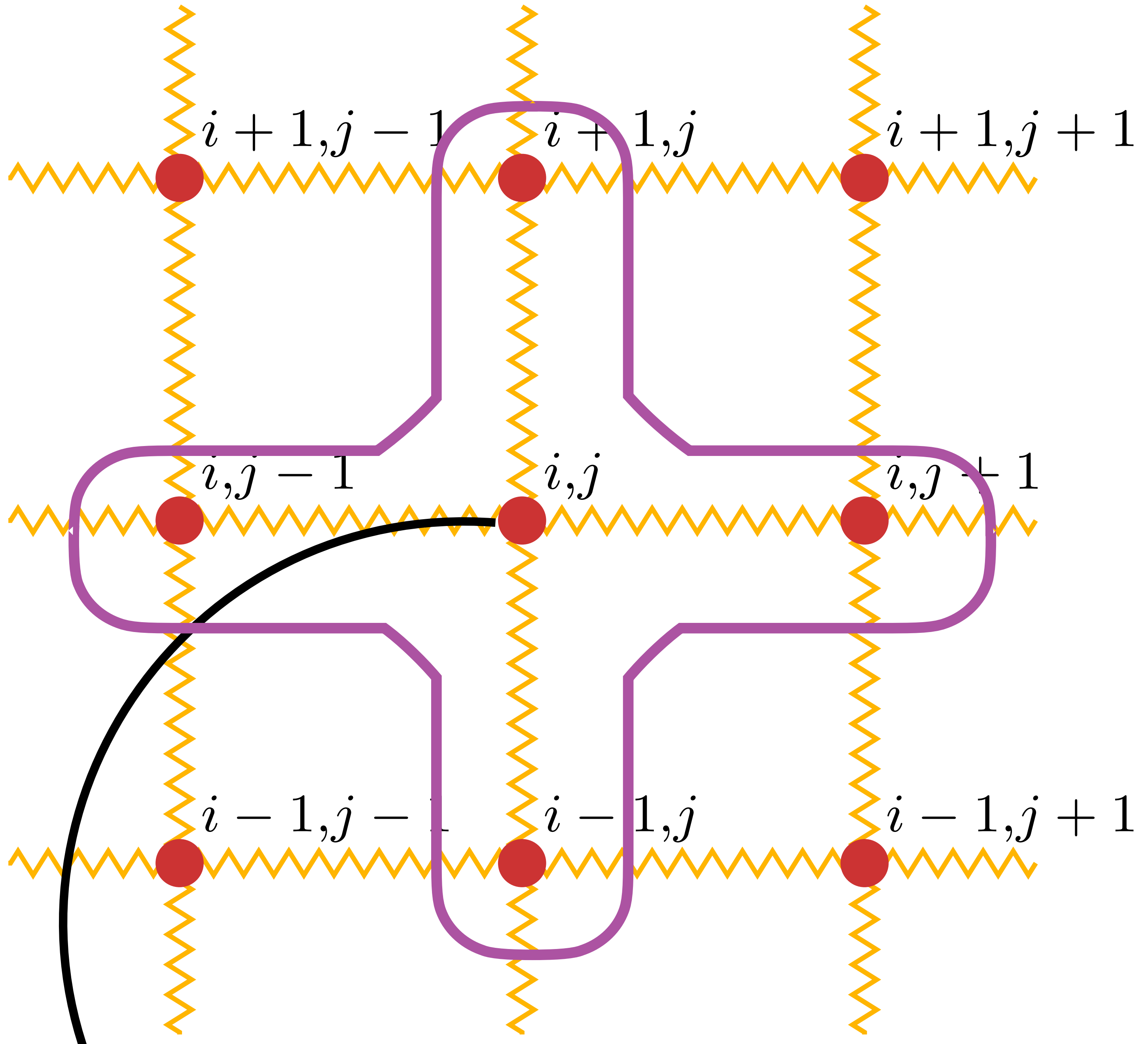
Electromagnetism	Toy model
$\mathbf{A}(\mathbf{x})$	$\mathbf{q}^{i,j} = (q_x^{i,j}, q_y^{i,j})$
$-\mathbf{E}(\mathbf{x})$	$\mathbf{p}^{i,j} = (p_x^{i,j}, p_y^{i,j})$
$\mathcal{G} := \nabla \cdot \mathbf{E}(\mathbf{x})$	$\mathcal{C}^{i,j} = \bar{\partial}_x p_x^{i,j} + \bar{\partial}_y p_y^{i,j}$
$\mathbf{B}(\mathbf{x}) = \nabla \times \mathbf{A}(\mathbf{x})$	$b^{i,j} = \bar{\partial}_x q_y^{i,j} - \bar{\partial}_y q_x^{i,j}$
$H = \frac{1}{2}\int d\mathbf{x}(\lvert\mathbf{E}(\mathbf{x})\rvert^2 + \lvert\mathbf{B}(\mathbf{x})\rvert^2)$	$H = \frac{1}{2}\sum_{i,j}[(p_x^{i,j})^2 + (p_y^{i,j})^2 + (b^{i,j})^2]$

Static source case

$$H_\rho = H$$

$$\mathscr{C}_\rho^{i,j} = \mathscr{C}^{i,j} + \rho^{i,j} = 0$$

QUANTIZATION OF THE TOY MODEL



$$\hat{\rho}^{i,j} = |1\rangle_{i,j}\langle 1|$$

$$\sum_s c(s)|s\rangle \qquad |s\rangle = \otimes_{i,j} |0 \vee 1\rangle_{i,j}$$

$$[\hat{q}_s^{i,j}, \hat{p}_r^{n,m}] = i\hbar\, \delta_{s,r}\delta^{i,n}\delta^{j,m}$$

$$\hat{C}^{i,j}|\psi\rangle_{ph} = \bar{\partial}\cdot \hat{\mathbf{p}}^{i,j}|\psi\rangle_{ph} = 0$$

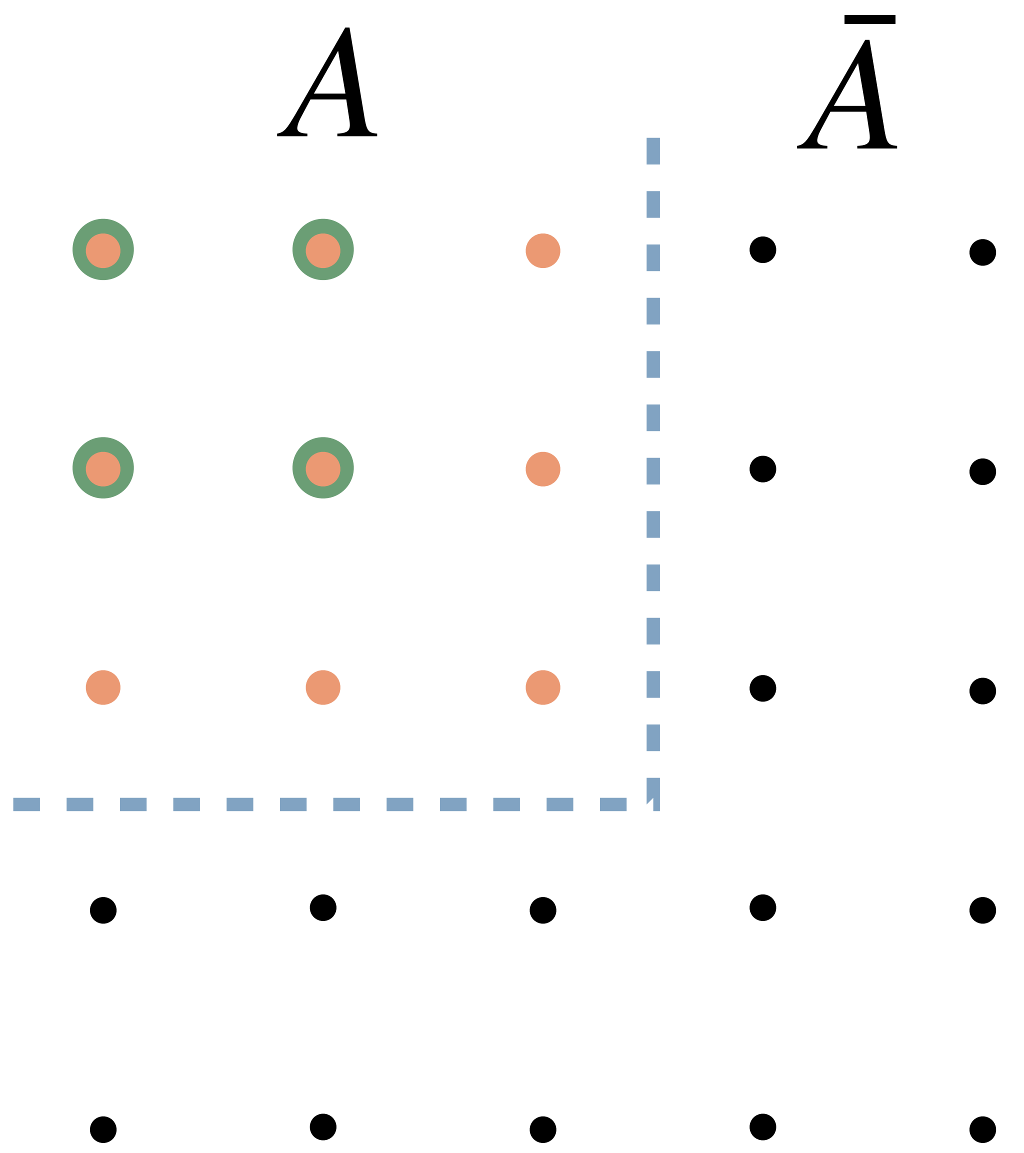
$$\boxed{\mathscr{H}_{ph} \neq \otimes_{i,j}\mathscr{H}_{i,j}}$$

Ground state (no source): $\Psi_0(p)$

Ground state (with source): $\Psi_{0,\rho(s)} = T_{\rho(s)}(p)\,\Psi_0(p)$

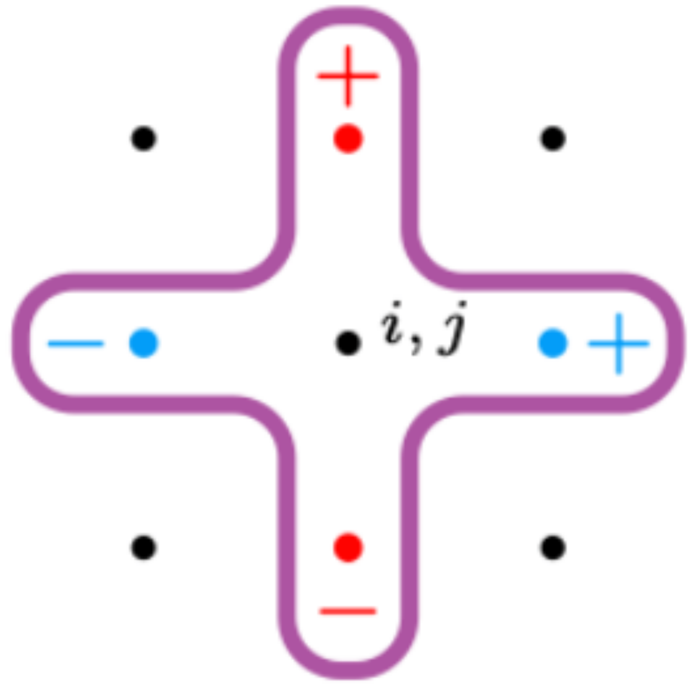
$$|\Phi\rangle = N\sum_s \alpha_s |s\rangle_m |\Psi_{0,\rho(s)}\rangle_f$$

Chen, Giacomini, Rovelli, Quantum (2023)

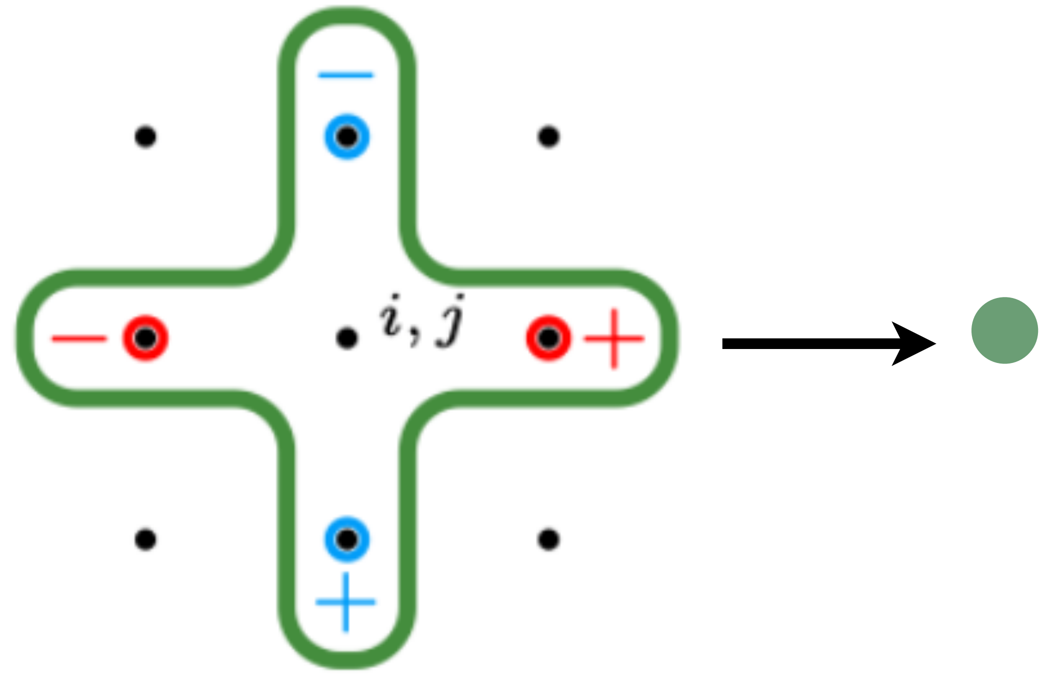


$\hat{p}_x^{i,j}$ $\hat{p}_y^{i,j}$ $\hat{p}_x^{i,j}, \hat{p}_y^{i,j}$ $\hat{q}_x^{i,j}$ $\hat{q}_y^{i,j}$

$$[\hat{O}, \hat{\mathscr{C}}^{i,j}] = 0$$



$$\hat{\mathscr{C}}^{i,j} = \frac{1}{2a}(\hat{p}_x^{i,j+1} - \hat{p}_x^{i,j-1} + \hat{p}_y^{i+1,j} - \hat{p}_y^{i-1,j})$$



$$\hat{b}^{i,j} = \frac{1}{2a}(\hat{q}_y^{i,j+1} - \hat{q}_y^{i,j-1} - \hat{q}_x^{i+1,j} + \hat{q}_x^{i-1,j})$$

LOCAL ALGEBRA

- $\hat{p}_x^{i,j}, \hat{p}_y^{i,j}$
- $\hat{b}^{i,j}$

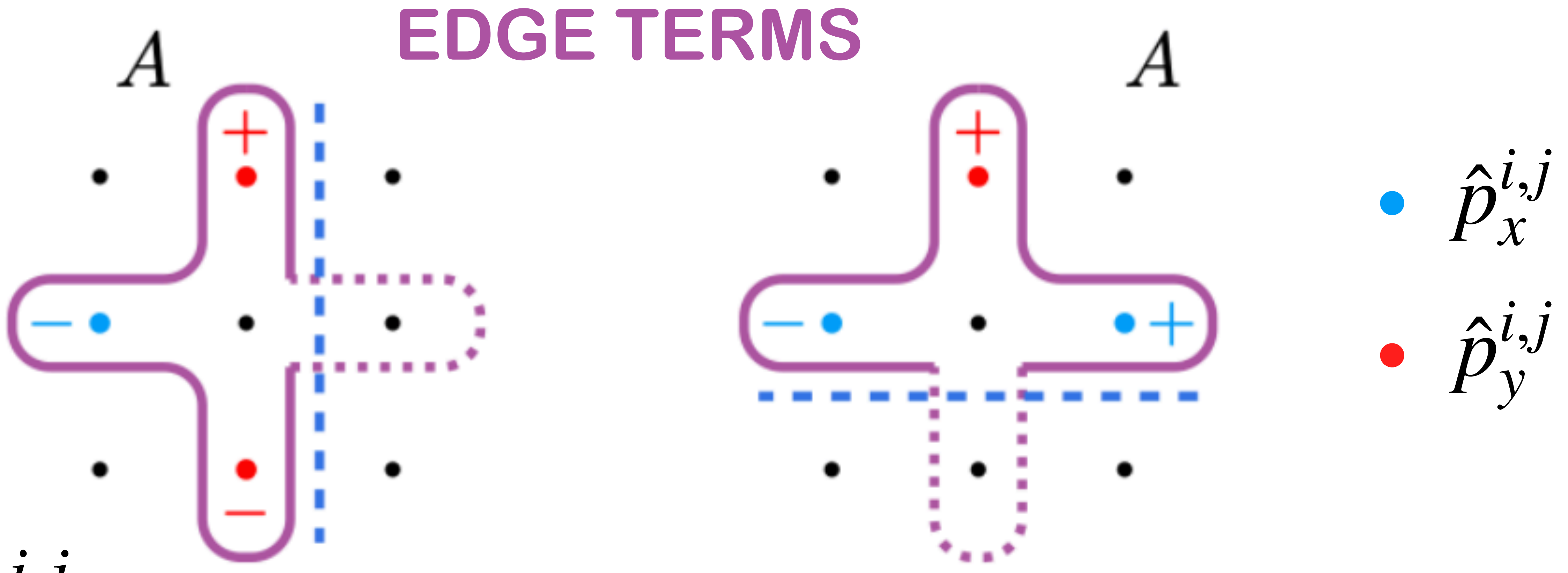
Van Acoleyen, et al., Phys.Rev.Lett (2016); Bianchi, Dona, Kumar, SciPost Phys. (2024)

$\mathscr{Z}_A$: elements of the local algebra $\mathscr{A}_A$ that commute with all its elements

$\mathscr{Z}_A$ trivial $\implies \mathscr{H}_A \otimes \mathscr{H}_{\bar{A}}$ **Usual setup of QI!**

$\mathscr{Z}_A$ non-trivial $\implies \oplus_k \mathscr{H}^k_A \otimes \mathscr{H}^k_{\bar{A}}$ **Gauge theories!**

"Truncated" constraints are part of $\mathscr{Z}_A$!



LOCAL ALGEBRA

- $\hat{p}^{i,j}_x, \hat{p}^{i,j}_y$
- $\hat{b}^{i,j}$

$$\hat{\mathscr{C}}^{i,j} |\Psi\rangle_{ph} = 0 \implies \underbrace{\hat{p}^{i,j+1}_x |\Psi\rangle_{ph}}_{k^{i,j}_x} = (\hat{p}^{i,j-1}_x - \hat{p}^{i+1,j}_y + \hat{p}^{i-1,j}_y)|\Psi\rangle_{ph}$$

Van Acoleyen, et al., Phys.Rev.Lett (2016); Bianchi, Dona, Kumar, SciPost Phys. (2024)

MATTER CASE - TWO LOCAL DECOMPOSITIONS

OPERATIONAL DECOMPOSITION	SPLIT DECOMPOSITION
$\mathscr{H}_{ph} = \oplus_k \mathscr{H}^k_A \otimes \mathscr{H}^k_{\bar{A}}$	$\mathscr{H}_{ph} = \oplus_s \left[\lvert s \rangle^m \otimes (\oplus_k \mathscr{H}^s_{k,A} \otimes \mathscr{H}^s_{k,\bar{A}}) \right]$
Operational notion of local operations in a subregion	Describes how the quantum field depends on the state of the source
Useful to define LO and entanglement	Useful to describe the entangling mechanism between two sources
Interaction between matter and field not explicit	Lacks immediate operational counterpart

GENERALISING THE LOCC THEOREM

GENERALISED LO defined according to the OPERATIONAL DECOMPOSITION

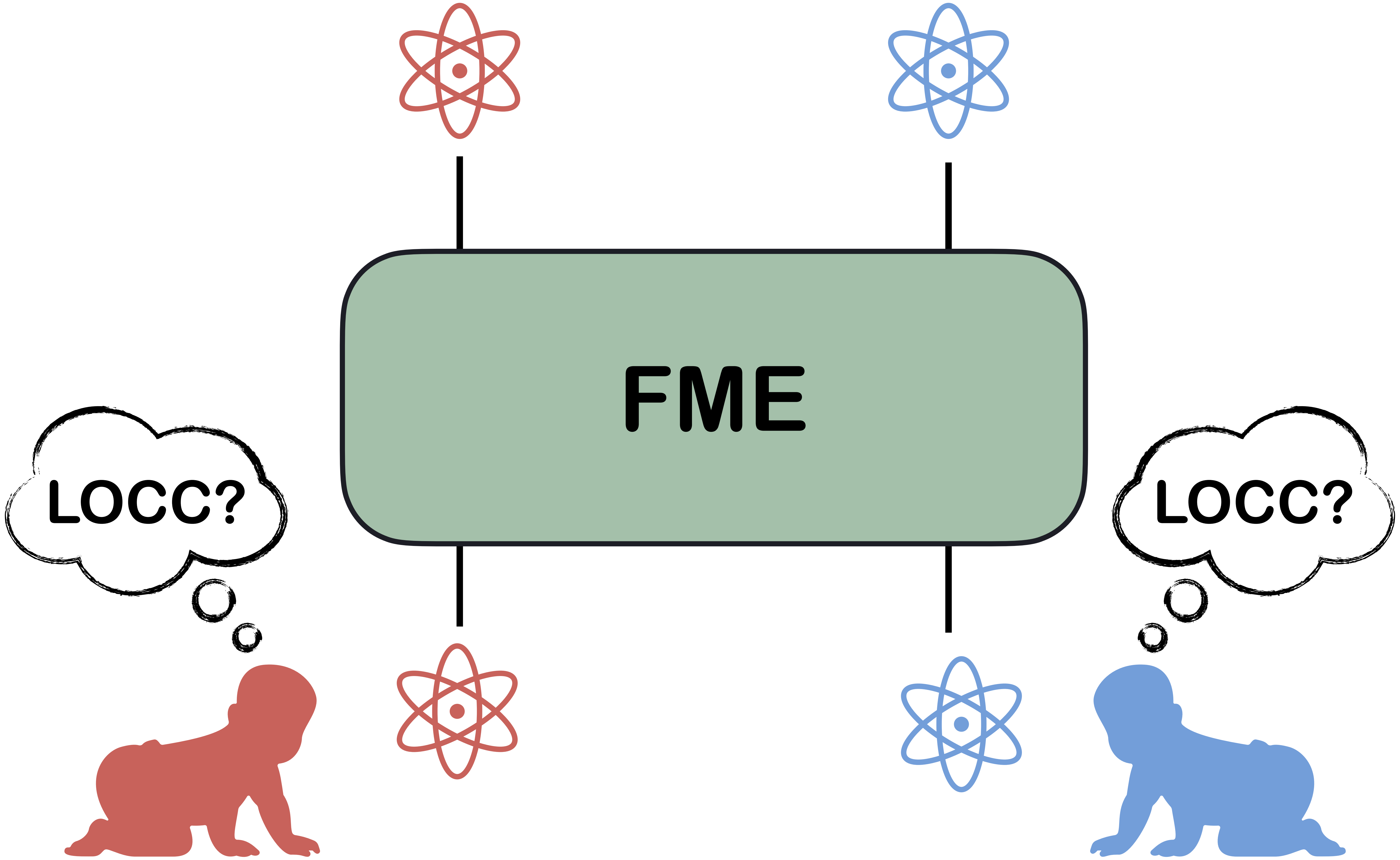
$$\mathscr{A}_A = \bigoplus_k \mathcal{O}_A^k \otimes \mathbb{I}_{\bar{A}}^k \qquad \mathscr{A}_{\bar{A}} = \bigoplus_k \mathbb{I}_A^k \otimes \mathcal{O}_{\bar{A}}^k$$

OPERATIONALLY ACCESSIBLE ENTANGLEMENT within each k block
$=$ standard entanglement

GENERALISED LOCC THEOREM

No operationally accessible entanglement can be generated through the exclusive use of classical communication and generalised LO, belonging to the local algebra accessible to a given spatial region.

FME ARGUMENT REVISITED

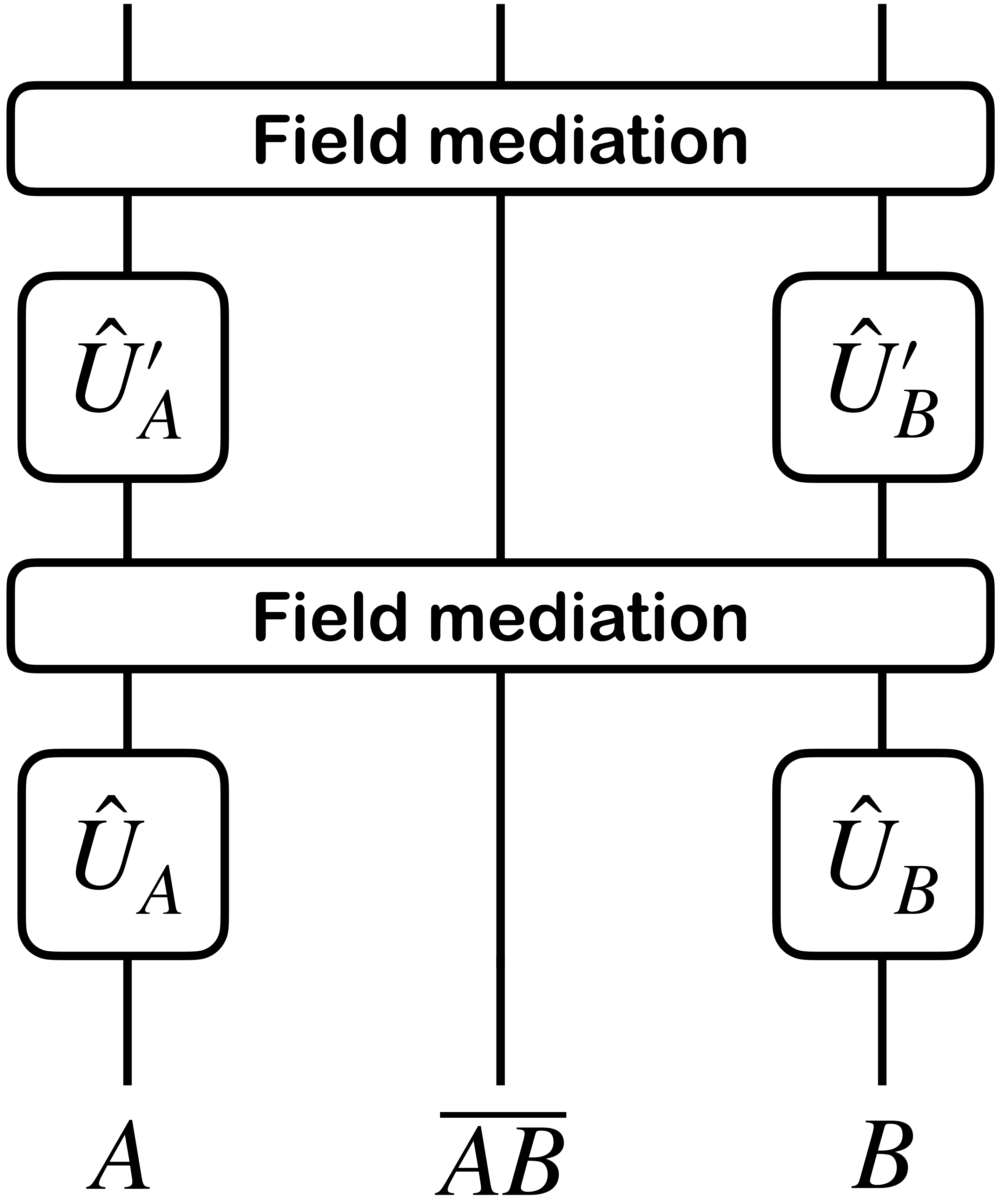


$$|\chi_f\rangle^\sigma = ?$$

$$|\psi_0\rangle^m|\chi_0\rangle^\sigma = |\phi\rangle_A|\phi\rangle_B|+\rangle_A^\sigma|+\rangle_B^\sigma$$

FME ARGUMENT REVISITED

$|s\rangle^m |\Psi_{0,s}\rangle_f |\chi\rangle^\sigma_{AB}$



$|s\rangle^m |\Psi_{0,s}\rangle_f |+\rangle^\sigma_A |+\rangle^\sigma_B$

If ent$(|\chi\rangle^\sigma_{AM}) > 0$

Non classical field!

Hamiltonian evolution of the field

$$\hat{H} \sim \int d^3x(|\hat{E}(x)|^2 + |\hat{B}(x)|^2)$$

$$\mathscr{A}_{AB} = \oplus_k \mathscr{O}^k_A \otimes \mathscr{O}^k_B \otimes \mathbb{I}^k_{\overline{AB}}$$

"field dressing"

SUMMARY

1. LOCC can be extended to gauge models with no spacetime-local tensor product structure

2. Explicit mechanism for entanglement generation in accordance with the Coulomb limit

How about electromagnetism?

Toy model $\xrightarrow{\text{continuum}}$ 2D+1 QED

Chen, Giacomini, Rovelli, Quantum (2023)

RELATIONS WITH OTHER TALKS OF THE WORKSHOP

THIS TALK

How is quantum information encoded in gauge theories?
No typical "subsystem" structure

(Quantum) nature of gravity in table-top experiments
Max Joseph Fahn
Eduardo Martín Martínez

Entanglement in quantum fields
Francesco Sartini (relational)
Alexander Stottmeister

"LOCALISE" INFORMATION

QRFs in linearized QG
Lin-Qing Chen

Mutual Information in QRFs
Viktoria Kabel

RELATIONAL APPROACH

Quantum Reference Fields
Fabio Mele

Carlo Rovelli

Toy model for the gravitational field

Quantum delocalised sources

Chen, Giacomini, PRX (2025)

Connection with quantum reference frames

THANK YOU!

Exploring Quantum Without SpaceTime

BridgeQG

Swiss National Science Foundation

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