
Reduced Quantum-Reference-Frame Channels for Open Quantum Systems

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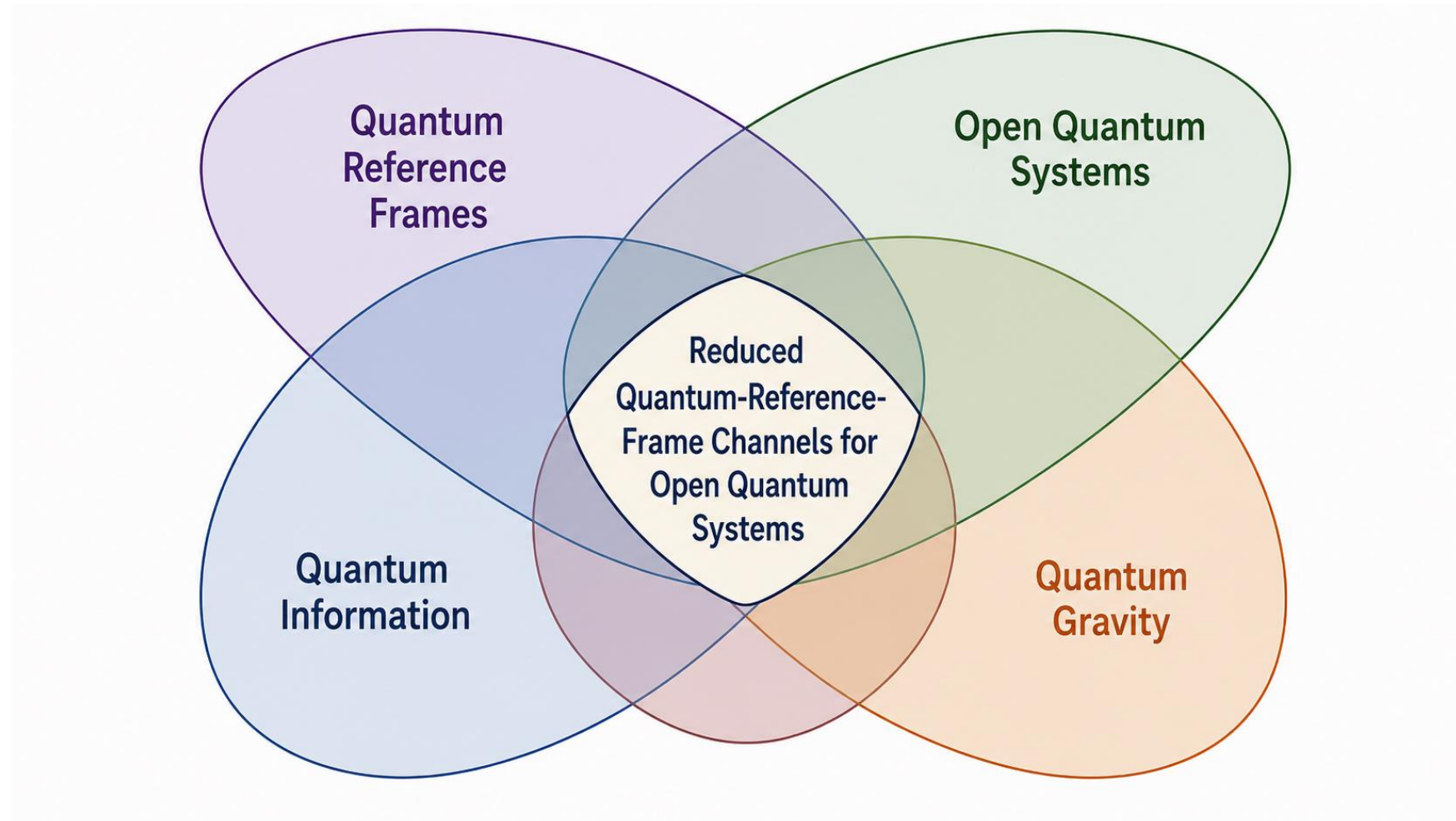
Viktoria Kabel
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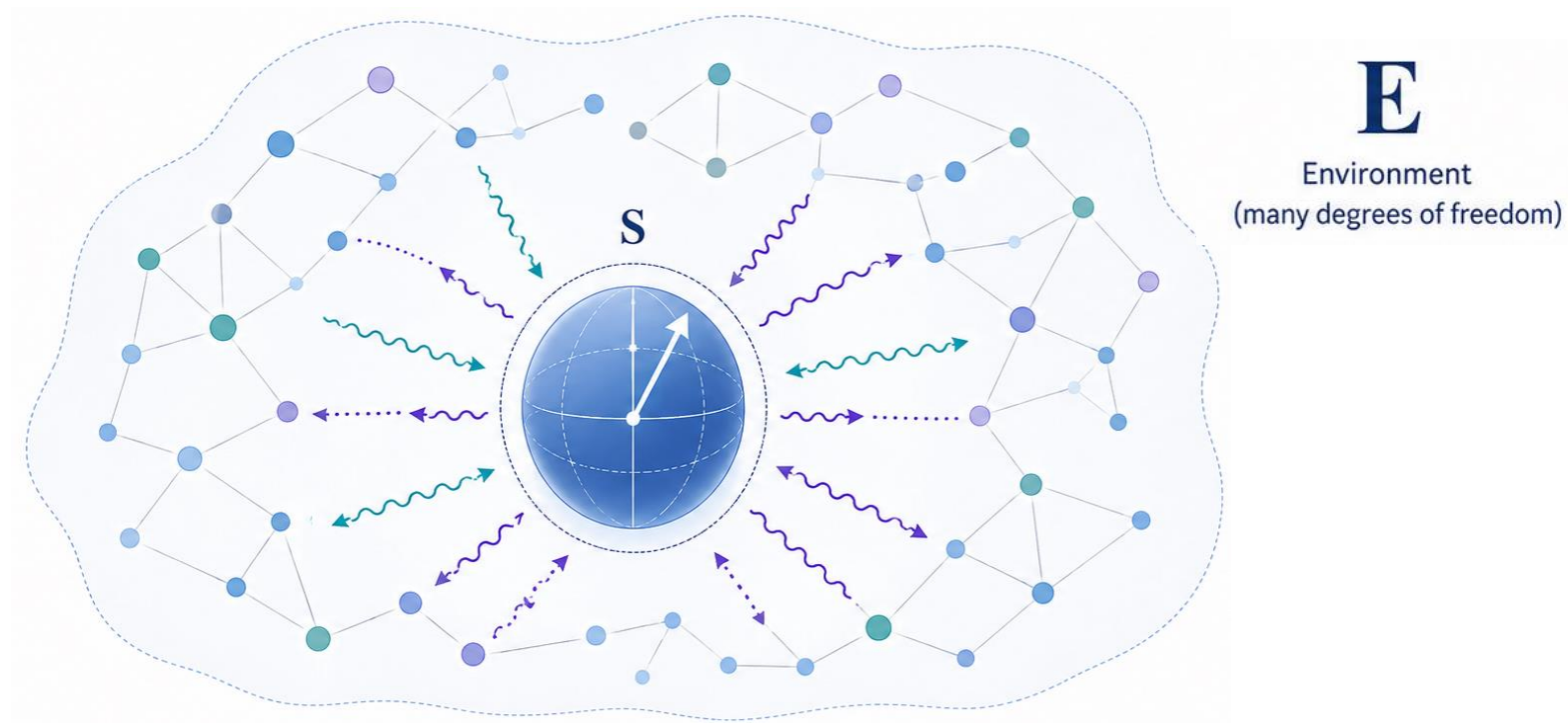
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PART I

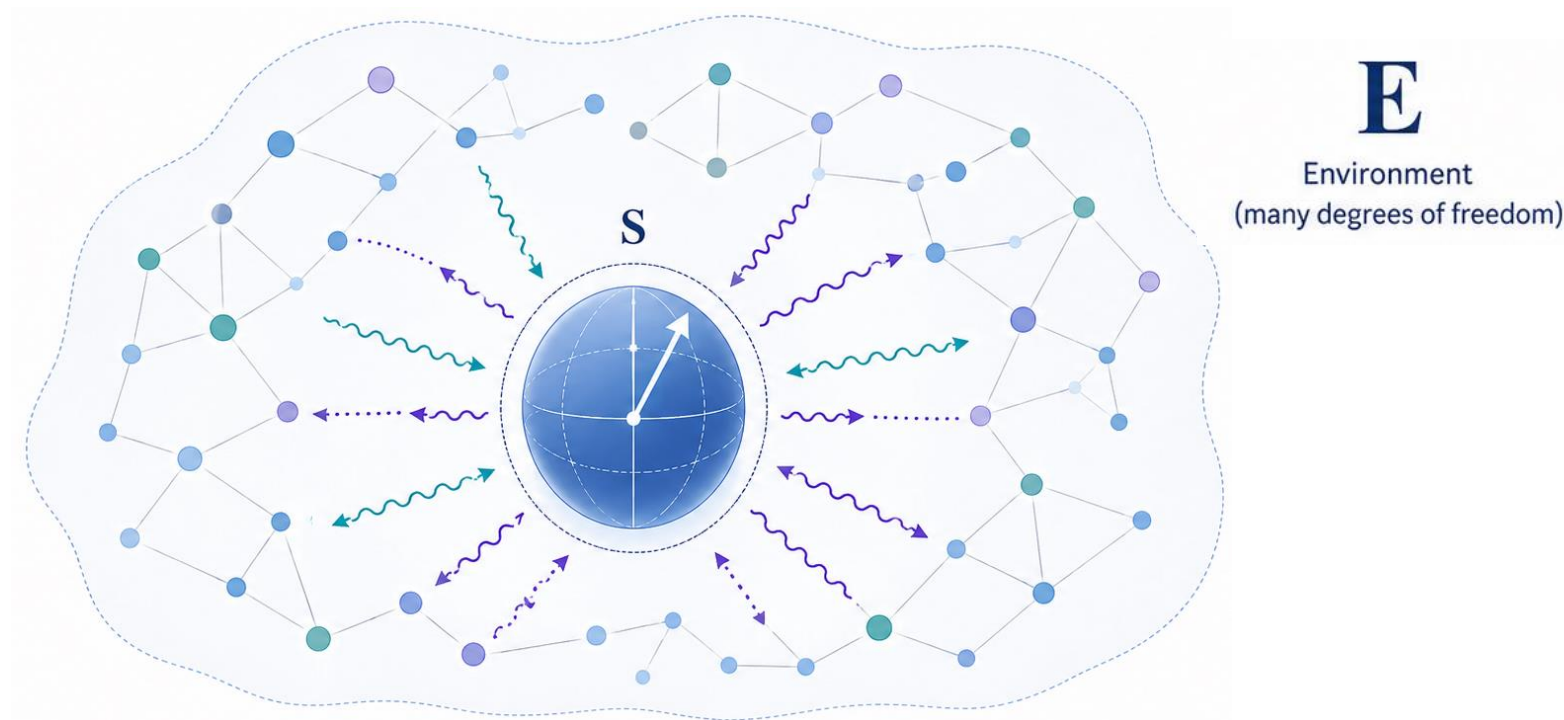
Setup: open quantum systems and quantum reference frames

Open Quantum Systems (OQSs)



Any realistic quantum system S interacts with a surrounding environment E .

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Total **global unitary evolution**:

$$\rho_{SE}(t) = U(t) \rho_S(0) \otimes \rho_E(0) U^\dagger(t)$$

System evolution:

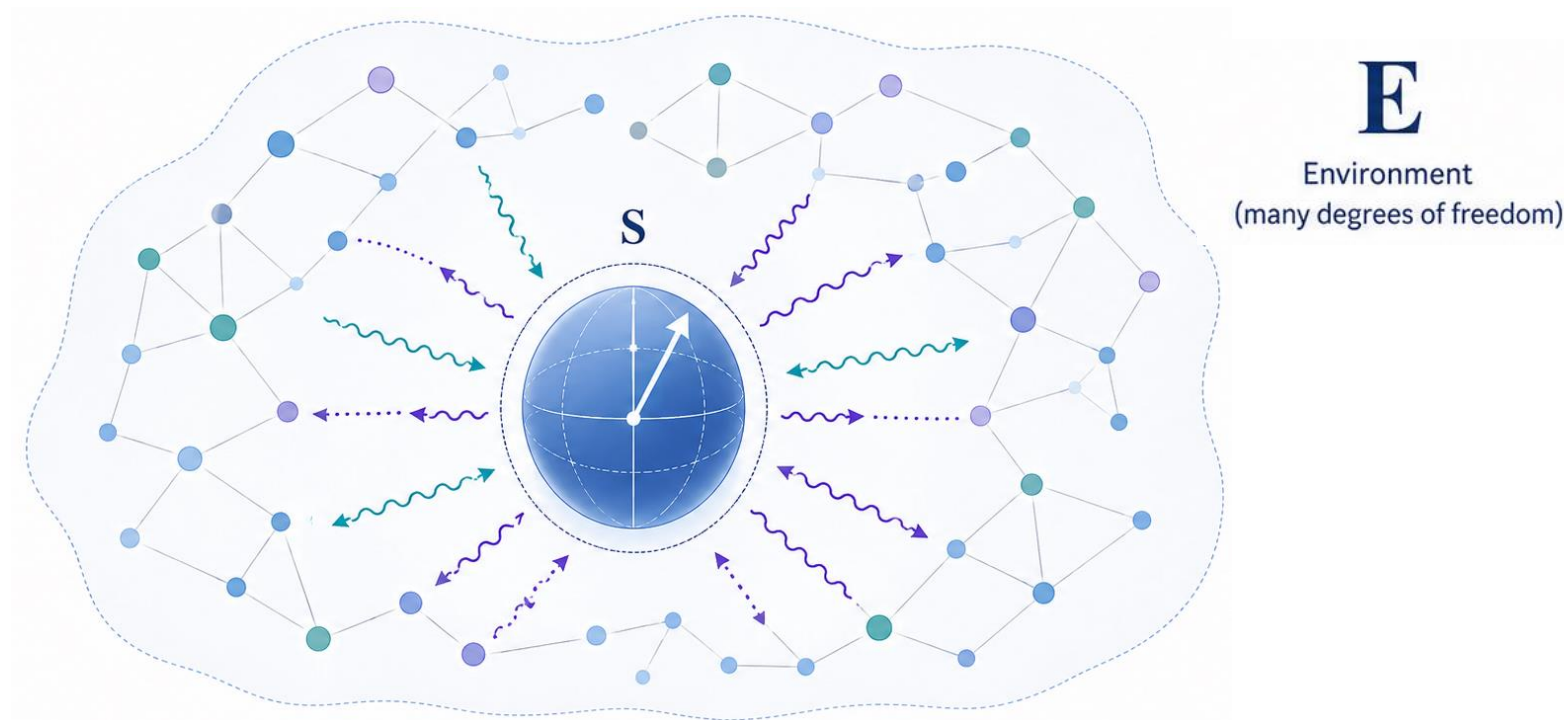
$$\rho_S(t) = \text{Tr}_E [U(t) \rho_S(0) \otimes \rho_E(0) U^\dagger(t)] = \Phi_t \rho_S(0)$$

Tr_E partial trace of E : an average over the environment's degrees of freedom.

Nonunitary



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A **quantum dynamical map** Φ_t is:

- Completely positive ($\Phi_t \otimes \mathbb{I}$ is positive);
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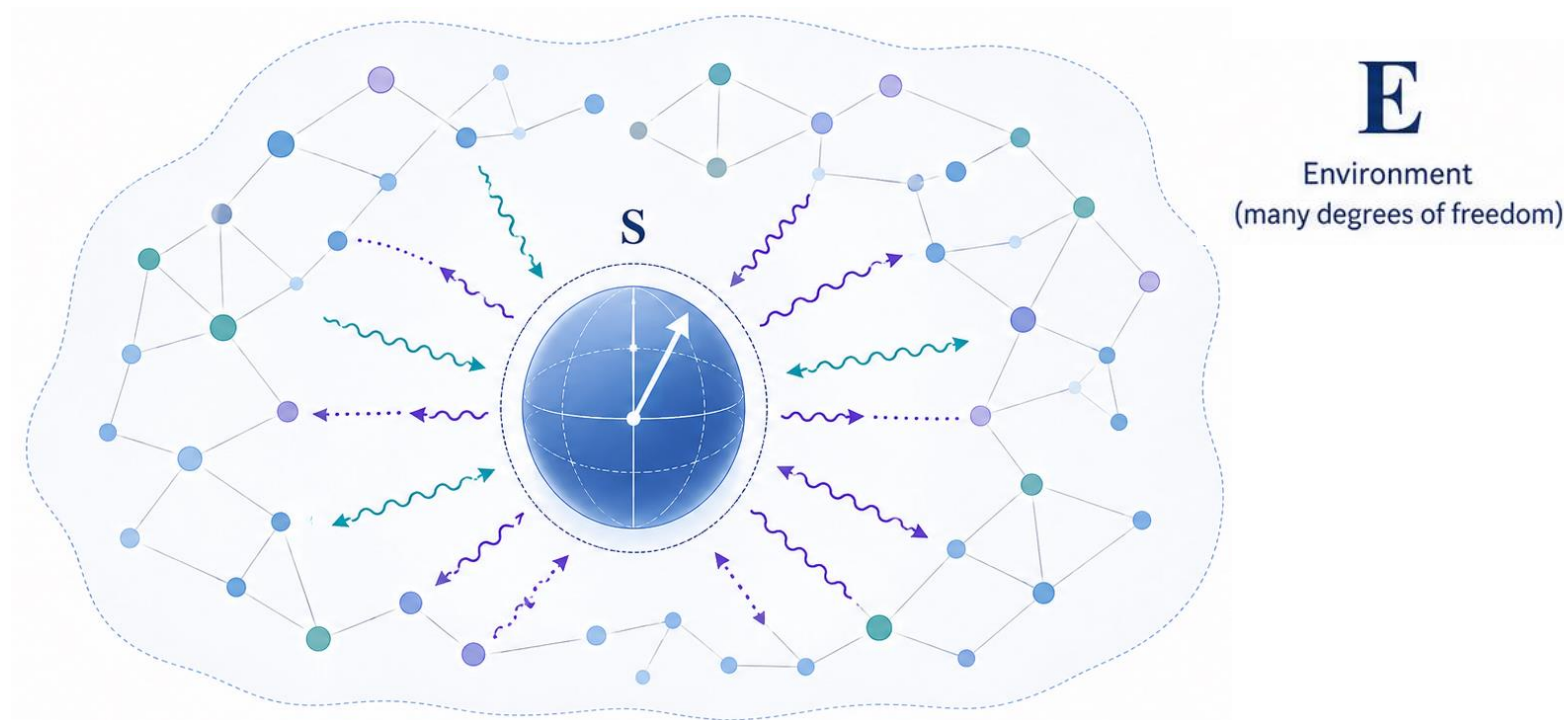
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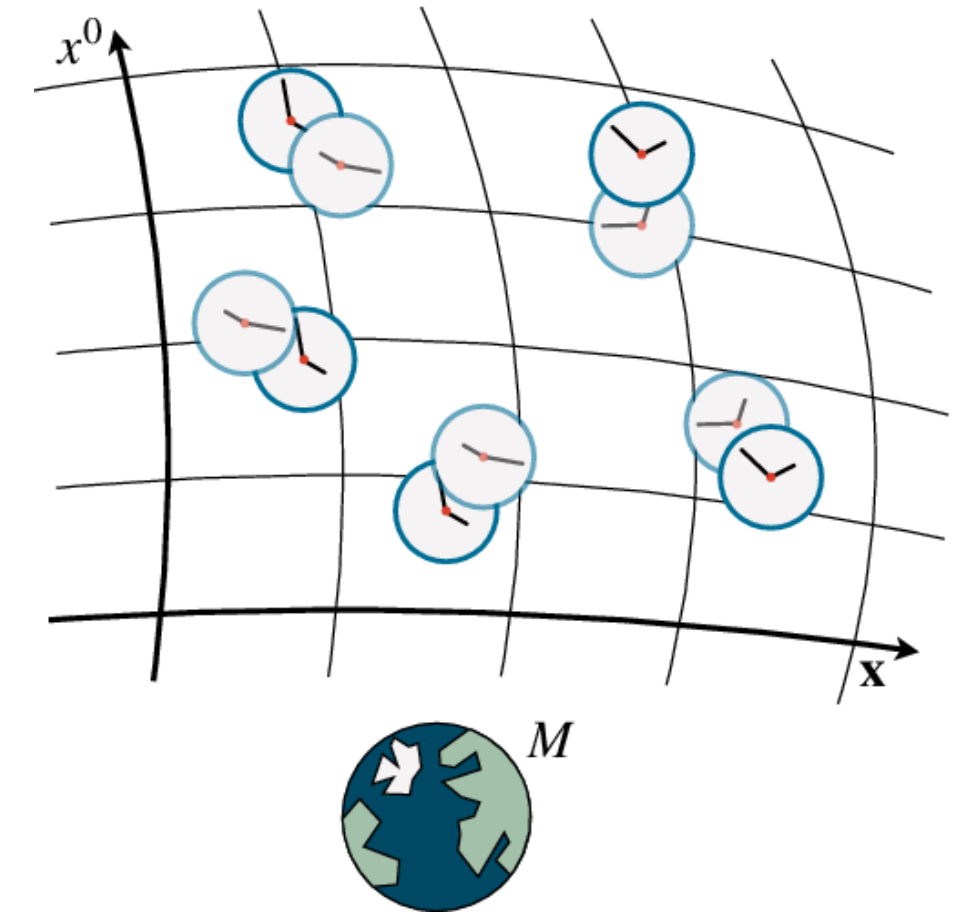
New effects such as **decoherence**, **dissipation** and **memory effects**.

Quantum Reference Frames (QRFs)

A **reference frame** is **physical** and therefore can be **quantum**.

Time in quantum mechanics: In quantum gravity there is no absolute time; QRFs let us define dynamics relative to quantum clocks.

Gauge fixing: In background-free quantum theories, choosing a reference frame is effectively a gauge fixing: QRFs provide a fully quantum and operational way to implement and transform between different gauge choices.



F.Giacomini "Spacetime Quantum Reference Frames and superpositions of proper times" in Quantum

OQSS and QRFs?

How do open **quantum system dynamics** change under QRFs transformations?

Which features of the **open-system** dynamics remain **invariant** under a change of QRF?

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In **broader gravity-related settings**, questions about information loss further motivate a consistent treatment of open quantum systems without relying on an external classical frame.

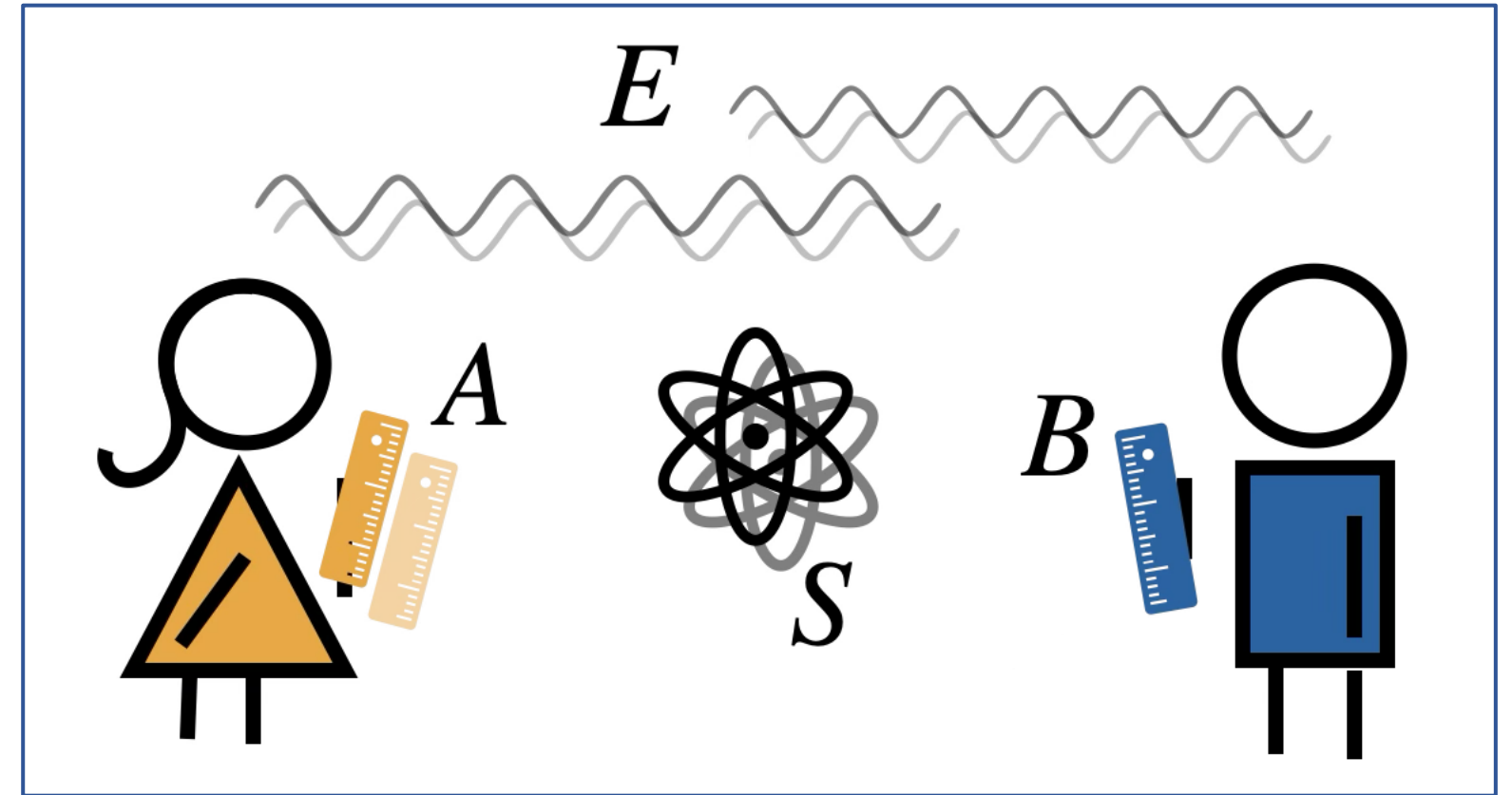
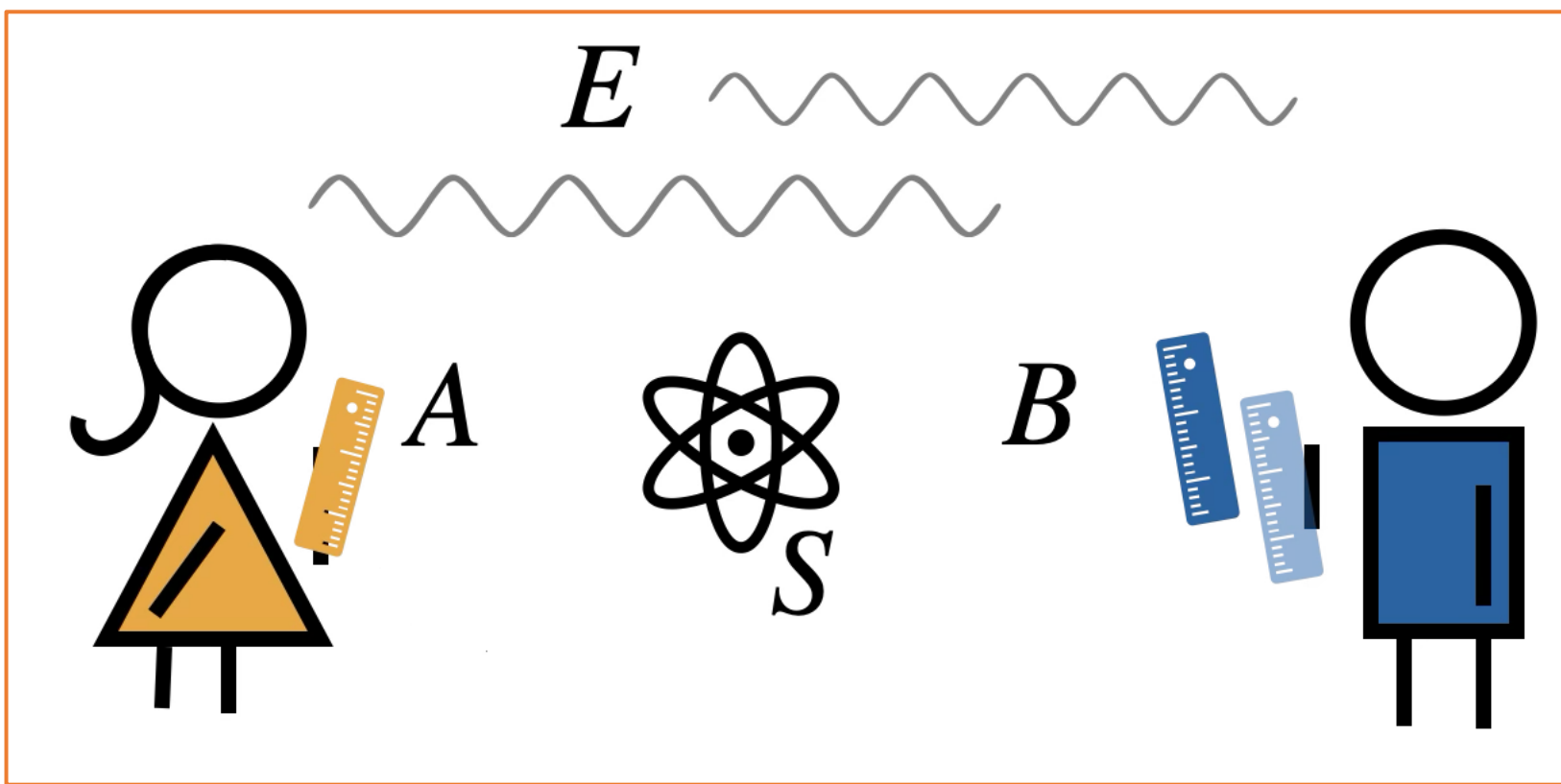
PART II



The reduced QRF channel and its structure

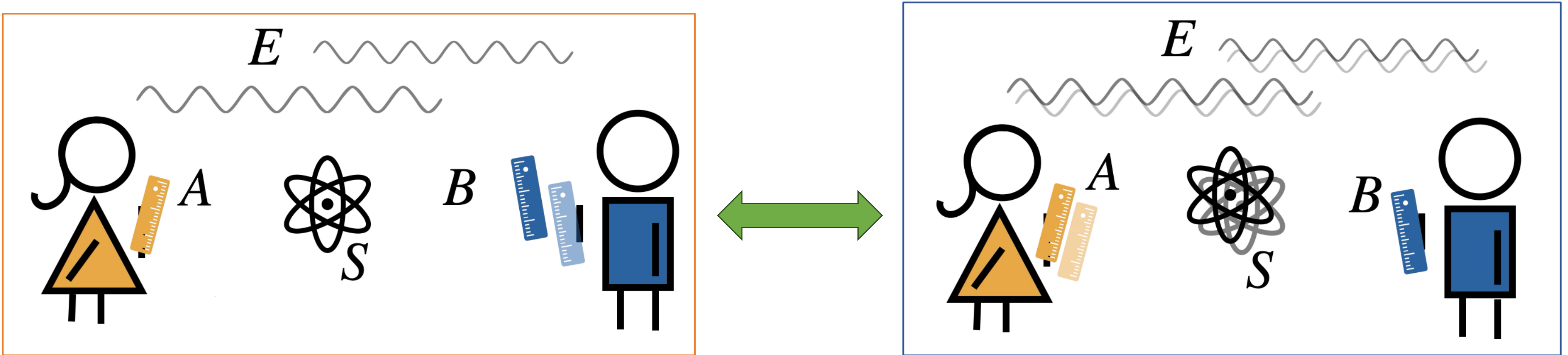
QRF transformation

Perspectival approach



QRF transformation

Perspectival approach



$$\rho_{SEA}^{(B)}(t) = S_{A \rightarrow B} \rho_{SEB}^{(A)}(t) S_{A \rightarrow B}^\dagger$$

Unitary transformation:

$$S_{A \rightarrow B} = \int dg \left| g_A^{-1} \right\rangle \langle g_B | \otimes V_{ES}^\dagger(g)$$

$\{|g\rangle\}_{g \in G}$: basis states of the QRF

$V_{ES}(g)$: unitary representation

Reduced QRF channel

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$$\rho_S^{(B)}(t) = Q \left[\rho_{SEB}^{(A)}(t) \right] := \text{Tr}_{AE} \left[S_{A \rightarrow B} \rho_{SEB}^{(A)}(t) S_{A \rightarrow B}^\dagger \right]$$

$$Q: \mathcal{H}_{SEB}^{(A)} \rightarrow \mathcal{H}_S^{(B)}$$

Q is a **CPTP** map

$$\text{Dual map } Q^\dagger: \mathcal{H}_S^{(B)} \rightarrow \mathcal{H}_{SEB}^{(A)}$$

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$$\begin{array}{ccc} \rho_{SBE}^{(A)}(0) & \xrightarrow{\mathcal{U}_{SBE}^{(A)}(t)} & \rho_{SBE}^{(A)}(t) \\ \downarrow \text{Tr}_{BE} & & \downarrow Q \\ \rho_S^{(A)}(0) & \xrightarrow{\Lambda_t} & \rho_S^{(B)}(t) \end{array}$$

$$\text{When } \rho_{SEB}^{(A)}(0) = \rho_S^{(A)}(0) \otimes \rho_{EB}^{(A)}(0)$$

$\exists$ a dynamical map:

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Block-preserving reduced QRF channels

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- Fix a PVM on S in A $\{P_m^{(A)}\}_m$ and a PVM on S in B $\{\tilde{P}_n^{(B)}\}_n$ (e.g energy projectors)

Q is **block preserving** if $\forall n$:

$$Q^\dagger \left(\tilde{P}_n^{(B)} \right) = \sum_m P_m^{(A)} \otimes \Pi_{nm}^{(A)}$$

$\{\Pi_{nm}^{(A)}\}_n$ **POVM** on BE

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$$p_n^{(B)}(t) = \text{Tr}_S \left[\tilde{P}_n^{(B)} \rho_S^{(B)}(t) \right]$$

$$= \text{Tr}_S \left[\tilde{P}_n^{(B)} Q(\rho_{SEB}^{(A)}(t)) \right]$$

$$= \text{Tr}_S \left[Q^\dagger(\tilde{P}_n^{(B)}) \rho_{SEB}^{(A)}(t) \right]$$

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$$Q^\dagger \left(\tilde{P}_n^{(B)} \right) = \sum_m P_m^{(A)} \otimes \Pi_{nm}^{(A)}$$



$$\tilde{\Delta}_S^{(B)} \circ Q = \tilde{\Delta}_S^{(B)} \circ Q \circ \Delta_S^{(A)}$$

$$\{\Pi_{nm}^{(A)}\}_n \text{ **POVM** on } BE$$

Pinching maps:

$$\Delta_S^{(A)}(X) = \sum_m P_m^{(A)} \otimes \mathbb{I}_{BE} X P_m^{(A)} \otimes \mathbb{I}_{BE}$$

$$\tilde{\Delta}_S^{(B)}(X) = \sum_n \tilde{P}_n^{(B)} X \tilde{P}_n^{(B)}$$

Dephase first or change frame first : the output block populations are the same.

Strong preservation

$$Q^{\dagger} \left(\tilde{P}_n^{(B)} \right) = P_{\pi(n)}^{(A)} \otimes \mathbb{I}_{BE}^{(A)}$$

Output populations are a permutation of the input populations.

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Output populations are a permutation of the input populations.

Sum of Entanglement ε and Coherence $\mathcal{C}$ is invariant:

$$\varepsilon^{(A)} + \mathcal{C}^{(A)} = \varepsilon^{(B)} + \mathcal{C}^{(B)} \quad (1)$$



Frame B's
coherence **Frame A's**
coherence

(1) C. Cepollaro et al “Sum of Entanglement and Subsystem Coherence Is Invariant under Quantum Reference Frame Transformations”. In: Phys. Rev. Lett.

Strong preservation

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Sum of Entanglement ε and Coherence C is invariant:

$$\varepsilon^{(A)} + \underset{\substack{\text{Frame B's} \\ \text{coherence}}}{C^{(A)}} = \varepsilon^{(B)} + \underset{\substack{\text{Frame A's} \\ \text{coherence}}}{C^{(B)}} \quad (1)$$

Under **strong preserving** Q channel:

$$\varepsilon^{(A)} + C_S^{(A)} = \varepsilon^{(B)} + C_S^{(B)}$$

$C_S^{(A)}$: **system's** coherence in A

$C_S^{(B)}$: **system's** coherence in B

System coherence and entanglement can be redistributed, but their **sum** is **invariant**.

- Invariant at the system level;
- It does not rely on a specific dynamics of S .

Classical misalignment — and how to witness its failure

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$$\rho_S^{(B)} = Q \left[\rho_{SBE}^{(A)} \right] = \int dg \, p(g) V_S^\dagger(g) \rho_S^{(A)} V_S(g)$$

Classical uncertainty about the parameter g

$$p(g) \geq 0 \quad \int dg \, p(g) = 1$$

Perfect knowledge: $p(g) = \delta(g - g_0)$

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Random-unitary $\Rightarrow$ necessarily **unital**. Holds for ideal frames with factorized preparation, no B–SE correlation, factorized action $V_{SE}(g) = V_S(g) \otimes V_E(g)$.

Perfect knowledge: $p(g) = \delta(g - g_0)$

UNITALITY WITNESS

If the original dynamics is unital but the transformed reduced dynamics is not unital anymore it can not be a classical misalignment

PART III



Pure dephasing and its operational meaning

Pure-dephasing

- A paradigmatic noise model in which populations are preserved while coherences decay;
- Dominant decoherence model in many experimental platforms;
- Arises naturally in different gravitational decoherence model. ^(1,2)

(1) M. P. Blencowe. “Effective Field Theory Approach to Gravitationally Induced Decoherence”. In: In *Phys. Rev. Lett*

(2) P. M. Bonifacio et al. “Dephasing of a non-relativistic quantum particle due to a conformally fluctuating spacetime”. In: *Classical and Quantum Gravity*

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Pure dephasing dynamics:

$$H = H_S + H_E + H_I$$

$$H_S = \sum_n \lambda_n P_n$$

System energy projectors

$$P_n = |n\rangle\langle n|$$

$$[H_S, H_I] = 0 \quad \longrightarrow$$

$$\text{Controlled evolution: } U_{SE}(t) = \sum_n P_n \otimes U_n(t)$$

U_n : controlled unitary on E

System's populations are constant in time

$$p_n(t) := \text{Tr}[P_n \rho_S(t)] = \text{Tr}[P_n \rho_S(0)] = p_n(0)$$

Only system's coherence affected.

(1) M. P. Blencowe. "Effective Field Theory Approach to Gravitationally Induced Decoherence". In: *Phys. Rev. Lett*

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Population preservation under reduced QRF changes

A' perspective:

Pure dephasing respect to $\{P_m^{(A)}\}_m$

Q



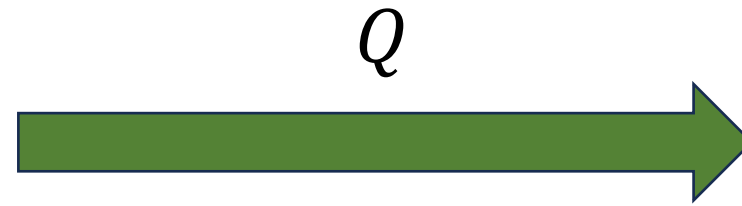
B' perspective:

When is it still only the coherences that dephase?

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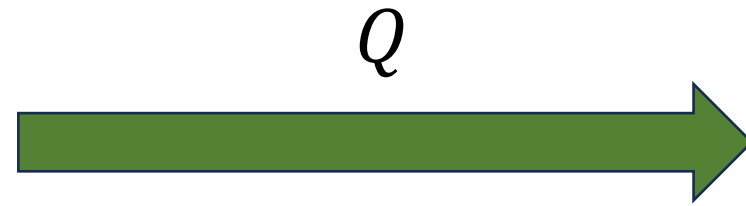
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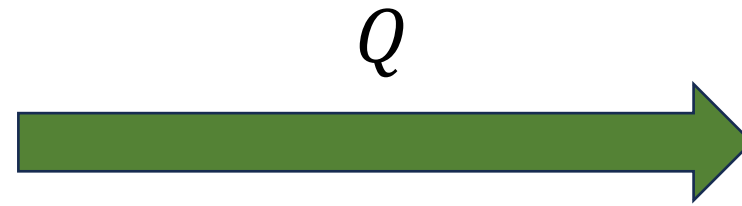
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$$\left[\Pi_{nm}^{(A)}, U_m^{(A)}(t) \right] = 0$$

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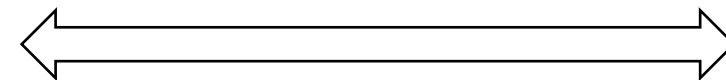
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Theorem:

Populations preserving in B



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Frame-induced decoherence

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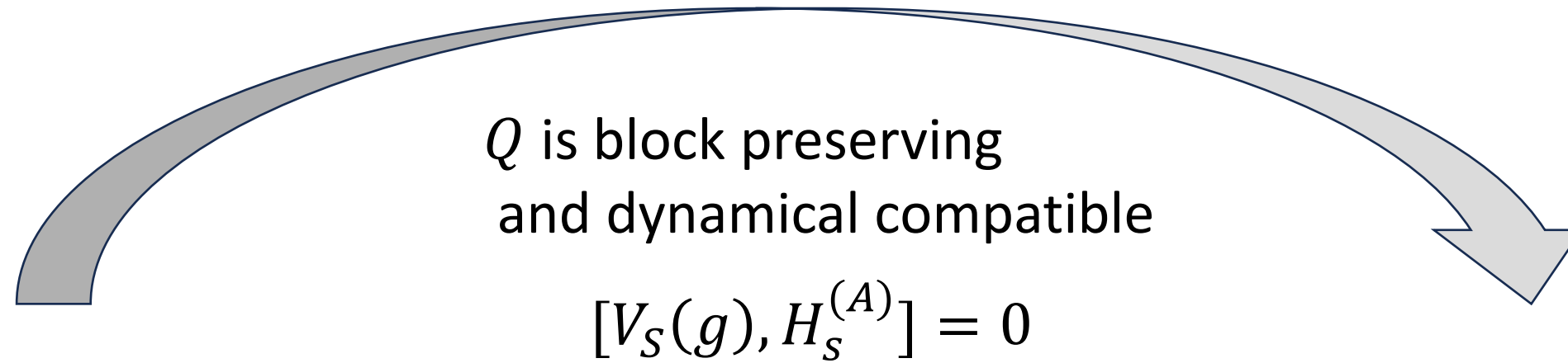
Pure dephasing

Decoherence rate: $\gamma_{nm}^{(A)}(t) = -\frac{d}{dt} \ln \rho_{nm}^{(A)}(t)$

No S-B interaction

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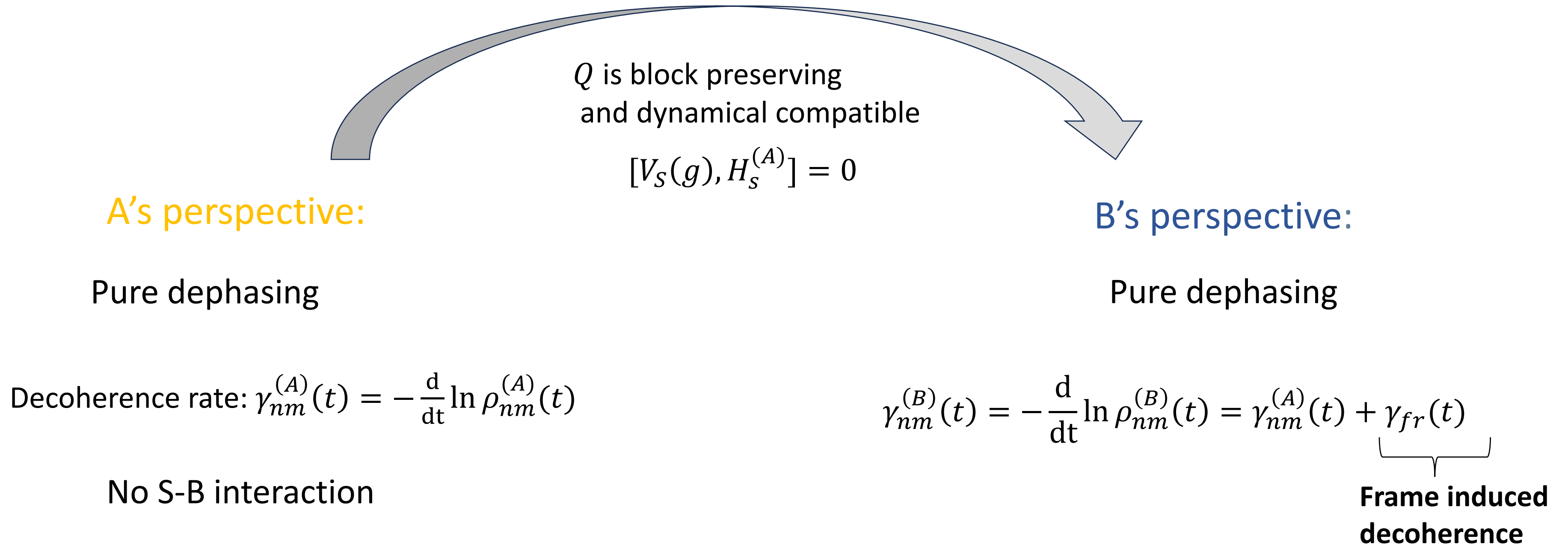
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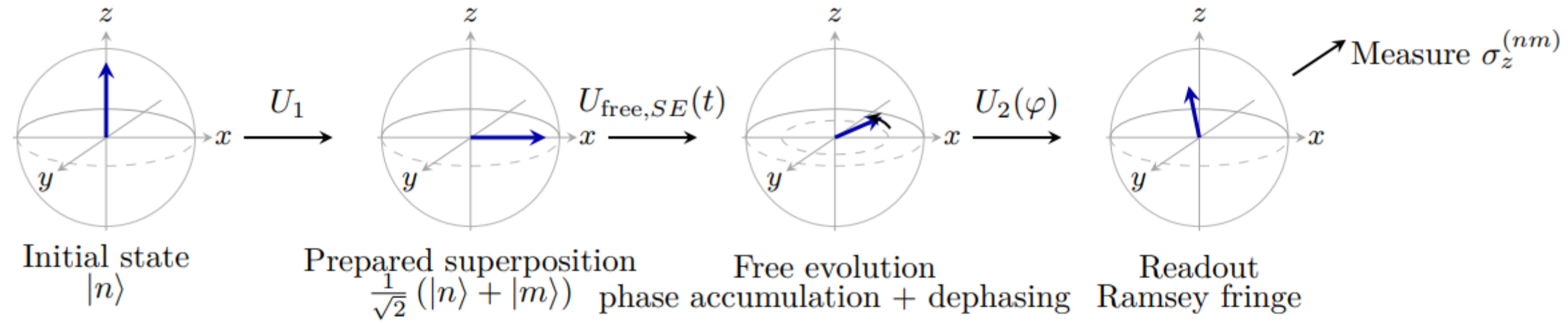
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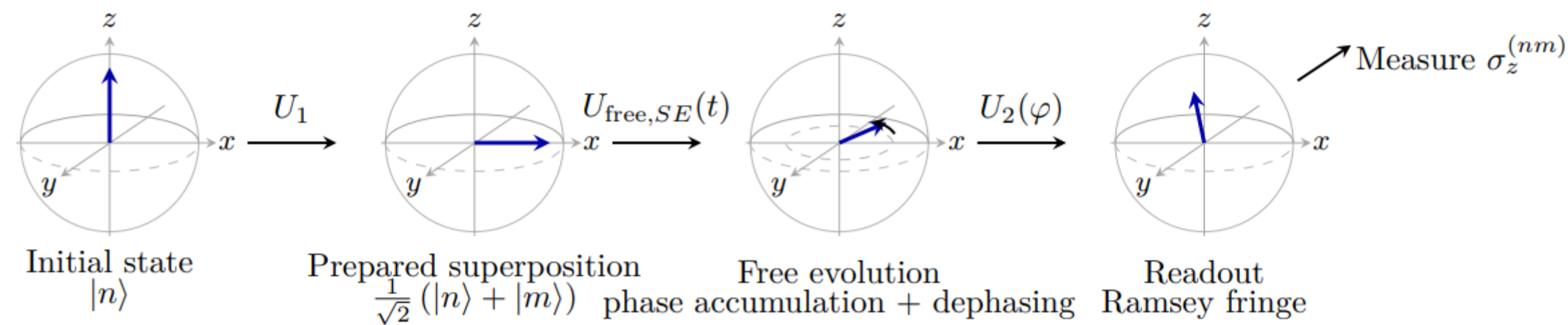


No deformation of the reference profile $\rightarrow$
no frame-induced decoherence ($\gamma_{fr}(t) = 0$).

Two Ramsey scenarios you must not conflate



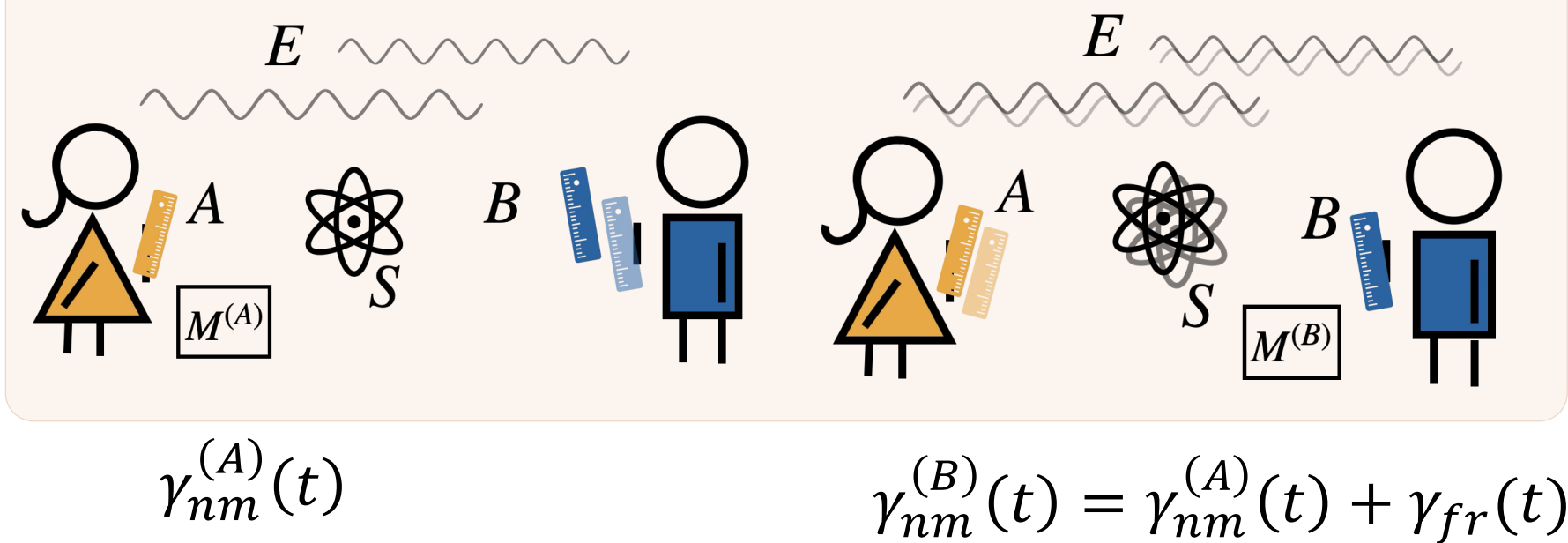
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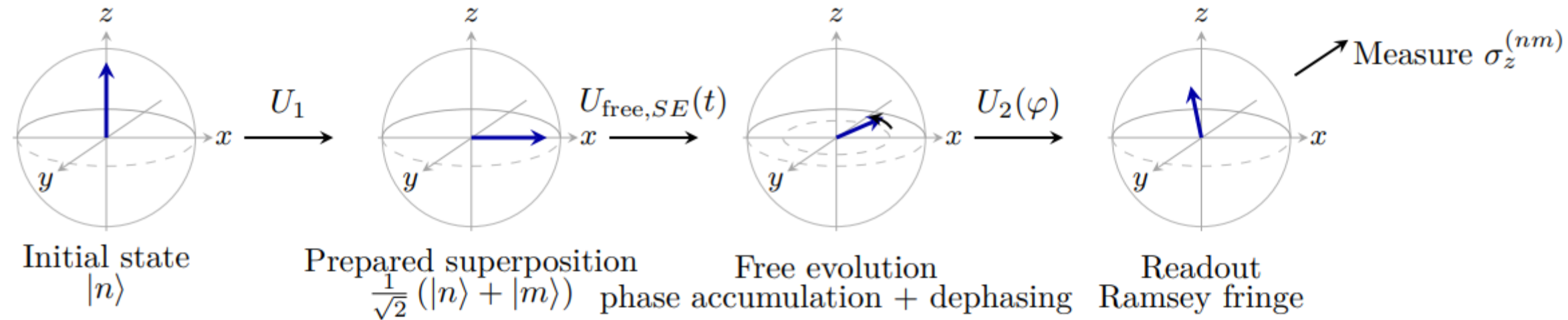
SCENARIO 1 – LOCAL RAMSEY

Each observer probes against their **own phase standard** . Inferred rates can differ by the frame factor:

$$\mathcal{V}_{nm}^{(B)}(t) = |F_{nm}(t)| \mathcal{V}_{nm}^{(A)}(t)$$



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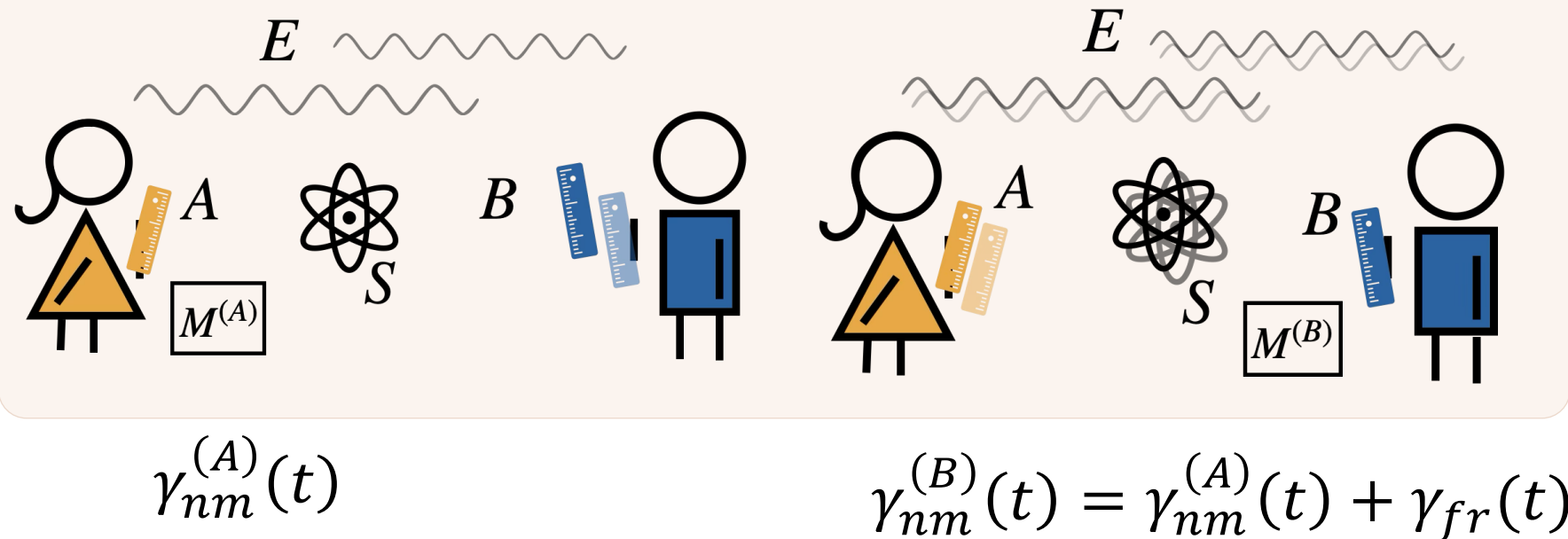


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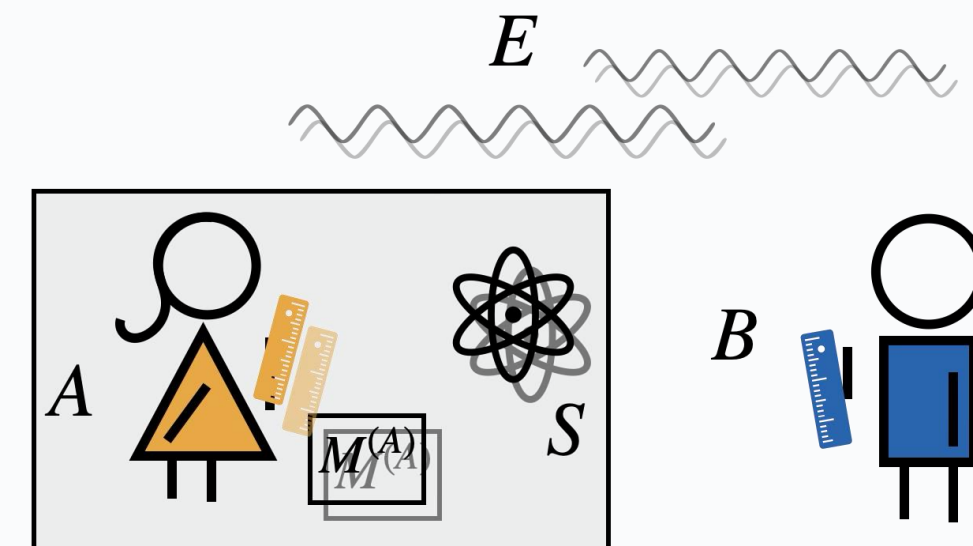
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SCENARIO 2 – SAME INTERFEROMETER

Re-describe one physical experiment in another frame.

Transform **preparation and measurement** together $\Rightarrow$ statistics are **frame-invariant** .



PART IV

IV

Case study: gravity-motivated dephasing

Energy dephasing due to a weak gravitational bath

- System S: harmonic oscillator of frequency Ω ,
$$H_S = \hbar\Omega(N_S + \frac{1}{2})$$
- An environment E modelling a gravitational bath;
- Energy coupling $H_{SE} = H_S \otimes B_E$ motivated by matter-graviton interaction $\propto \int d^4x T_{\mu\nu} h^{\mu\nu}$;
- Since $[H_S, H_{SE}] = 0$, pure dephasing dynamics

LAB-FRAME DECOHERENCE RATE

$$\Gamma_{nm}^{\text{grav}}(t) = \Omega^2 (n - m)^2 \chi_{\text{grav}}(t)$$

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BORN-MARKOV BENCHMARK (1)

$$\Gamma_{\text{grav}}^{\text{BM}}(\Delta E) = \frac{k_B T_g}{\hbar} \left(\frac{\Delta E}{E_P} \right)^2$$

T_g : effective temperature
of the thermal graviton environment

$$E_P = \sqrt{\frac{\hbar c^5}{G}} \text{ Planck energy}$$

(1) M. P. Blencowe. “Effective Field Theory Approach to Gravitationally Induced Decoherence”. In: In *Phys. Rev. Lett*

A U(1) phase reference that spreads on the circle

$$V_S(\theta) = e^{-i\theta N_S} \quad [V_S(\theta), H_S] = 0$$

GAUSSIAN FRAME FACTOR

$$|F_{nm}(t)| = \exp\left[-\frac{(n-m)^2}{2} \sigma_\theta^2(t)\right]$$

DIFFUSIVE – MARKOVIAN

$$\sigma_\theta^2(t) = \sigma_\theta^2(0) + \Gamma_B t$$

BALLISTIC – SHOT-TO-SHOT

$$\sigma_\theta^2(t) = \sigma_\theta^2(0) + \kappa_B t^2$$

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Same $(n - m)^2$ scaling — two different origins

$$\gamma_{nm}^{(B)}(t) = \underbrace{(n - m)^2 \Omega^2 \dot{\chi}_{\text{grav}}(t)}_{\text{gravity-induced}} + \underbrace{\frac{(n-m)^2}{2} \dot{\sigma}_\theta^2(t)}_{\text{reference-induced}}$$

THE CAUTIONARY TALE

A quadratic-in-gap visibility decay is **not** a unique signature of gravitational decoherence — a degrading phase reference can **mimic, mask, or renormalize** it.

Summary

Reduced QRF channels:

- Describe the effective transformation of the accessible system after a QRF change;
- Provide the natural framework to compare open-system dynamics across reference frames.

Structure preservation

- Block-preservation identifies which population–coherence structures survive a QRF change;
- For pure dephasing, **block preservation + dynamical compatibility** are necessary and sufficient to preserve populations.

Operational implications:

- The observed decoherence rate splits into: an **environmental** contribution and a **reference-induced** contribution;
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- A degrading quantum reference can mimic signatures usually attributed to gravitational decoherence.

Outlook

- Beyond pure dephasing: general open-system dynamics;
- Non-ideal quantum reference frames as a resource?
- Open quantum systems in relativistic quantum reference frames.

Thank you!

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