Classical Gravity Meets Classical Information

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FAU^2 Workshop: Quantum Gravity Meets Quantum Information

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slides → ebianchi.org/s/Erlangen-2026.pdf

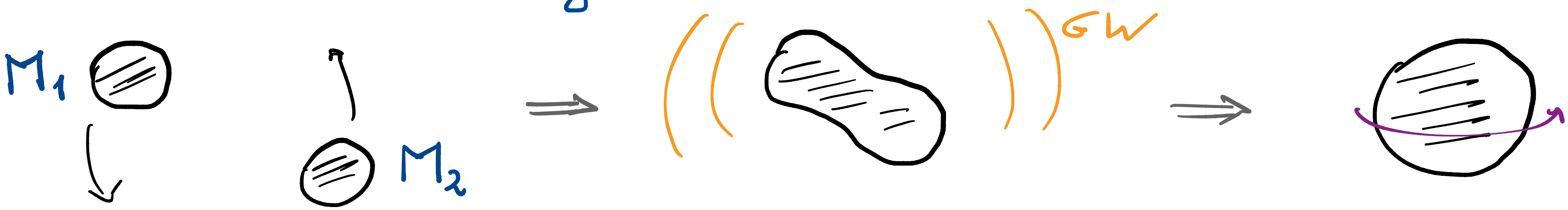
Plan:

1. A surprising observation:
   Entropy in Black Hole Mergers ◆

2. Principles

◆ [2601.22388], PRL (2026)
with: Monica Rincon-Ramirez, Nathan K. Johnson-McDaniel, Ish Gupta, Vaishak Prasad, and B.S. Sathyaprakash

■ Black Hole Merger:



$M_{tot} \equiv M_1 + M_2$, $J_0 =$ orbital angular momentum $\qquad M_f$, J_f

■ Numerical Relativity:

Scheel et al, PRD (2009), [0810.1767]
SXS, CQG (2025), [2505.13378]

For equal mass, non-spinning BHs initially in a quasi-circular orbit $\Rightarrow$

$$\boxed{\begin{aligned} \epsilon_f &= 1 - \frac{M_f}{M_{tot}} = (4.838 \pm 0.002)\,\% \\ \chi_f &= \frac{J_f}{M_f^2} = 0.68646 \pm 0.00004 \end{aligned}}$$

* Why these values of ϵ_f and χ_f ? Thermodynamic principles in GR ?

◆ A surprising observation: [2601.22388], PRL (2026)

Max Entropy + Flux Balance Laws (approx.) $\Rightarrow$ $\left\{ \begin{array}{l} \text{Values of } \epsilon_f, \chi_f \\ \text{within a few \%} \end{array} \right.$

LIGO-Virgo-Kagra: Routinely Observed Events

GWTC-1.0 [1811.12907]

Event	$m_1/\mathrm{M}_\odot$	$m_2/\mathrm{M}_\odot$	$\mathcal{M}/\mathrm{M}_\odot$	χ_{eff}	$M_f/\mathrm{M}_\odot$	a_f	$E_{\mathrm{rad}}/(\mathrm{M}_\odot c^2)$	$\ell_{\mathrm{peak}}/(\mathrm{erg\,s^{-1}})$	d_L/Mpc	z	$\Delta\Omega/\mathrm{deg}^2$
GW150914	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	$28.6^{+1.6}_{-1.5}$	$-0.01^{+0.12}_{-0.13}$	$63.1^{+3.3}_{-3.0}$	$0.69^{+0.05}_{-0.04}$	$3.1^{+0.4}_{-0.4}$	$3.6^{+0.4}_{-0.4} \times 10^{56}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$	179

$\to P \sim 10^{49}\ W$

cf: $\frac{c^5}{G} \simeq 10^{52}\ W$

GWTC-5.0 [2605.27225]

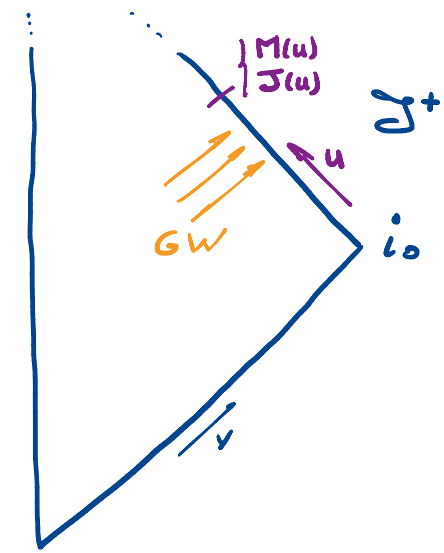
Candidate	M $[M_\odot]$	$\mathcal{M}$ $[M_\odot]$	m_1 $[M_\odot]$	m_2 $[M_\odot]$	χ_{eff}	D_L [Gpc]	z	M_f $[M_\odot]$	χ_f	$\Delta\Omega$ $[\mathrm{deg}^2]$	SNR
GW250114_082203	$66.0^{+1.1}_{-1.2}$	$28.71^{+0.50}_{-0.51}$	$33.76^{+1.28}_{-0.90}$	$32.26^{+0.95}_{-1.47}$	$-0.03^{+0.04}_{-0.05}$	$0.40^{+0.08}_{-0.07}$	$0.09^{+0.02}_{-0.01}$	$62.9^{+1.0}_{-1.1}$	$0.68^{+0.01}_{-0.01}$	42	$76.9^{+0.0}_{-0.1}$
⋮											
GW240618_071627	105^{+30}_{-24}	43^{+13}_{-10}	65^{+24}_{-19}	40^{+17}_{-15}	$0.00^{+0.30}_{-0.32}$	$5.5^{+5.1}_{-2.9}$	$0.84^{+0.59}_{-0.39}$	101^{+28}_{-23}	$0.68^{+0.13}_{-0.17}$	5400	$8.1^{+0.4}_{-0.6}$
GW240621_195059	$66.0^{+4.6}_{-2.8}$	$28.4^{+2.2}_{-1.4}$	$36.8^{+4.4}_{-3.7}$	$29.7^{+4.1}_{-5.2}$	$-0.00^{+0.10}_{-0.10}$	$1.21^{+0.26}_{-0.43}$	$0.23^{+0.04}_{-0.08}$	$62.9^{+4.2}_{-2.6}$	$0.68^{+0.04}_{-0.04}$	32	$27.6^{+0.1}_{-0.2}$
GW240621_200935	73^{+17}_{-12}	$31.1^{+7.6}_{-5.3}$	$41.6^{+11.6}_{-8.3}$	$31.6^{+9.8}_{-9.5}$	$-0.09^{+0.23}_{-0.28}$	$4.8^{+3.0}_{-2.4}$	$0.75^{+0.36}_{-0.32}$	70^{+16}_{-11}	$0.67^{+0.10}_{-0.12}$	1600	$8.9^{+0.3}_{-0.5}$
GW240621_214041	76^{+24}_{-16}	$32.1^{+10.3}_{-6.7}$	45^{+17}_{-11}	32^{+13}_{-11}	$0.00^{+0.26}_{-0.28}$	$6.8^{+4.8}_{-3.4}$	$1.00^{+0.54}_{-0.44}$	73^{+22}_{-15}	$0.69^{+0.11}_{-0.13}$	3200	$7.5^{+0.4}_{-0.6}$

GW250114:

$$\begin{cases} \epsilon_f = 1 - \dfrac{M_f}{M_{tot}} \simeq (4.7 \pm 0.1)\ \% \\ \chi_f = \dfrac{J_f}{M_f^2} \simeq 0.68 \pm 0.01 \end{cases}$$

1 Max Entropy Conjecture in BH Mergers

- Ingredients at Future Null Infinity $\mathscr{I}^+$:



i) Mass $M(u)$ and Ang. Mom. $J(u)$

$\rightsquigarrow$ relational $\boxed{M(J)}$ with J as "time"

ii) GW flux $F_E(u) = -\frac{dM}{du}$, $F_J(u) = -\frac{dJ}{du}$

with balance law $\boxed{\frac{dM}{dJ}} = \frac{F_E(u)}{F_J(u)} \approx \Omega_{orb}(u)$

for quasi-circular orbits $\rightsquigarrow$ Post-Newtonian Approx. (PN)

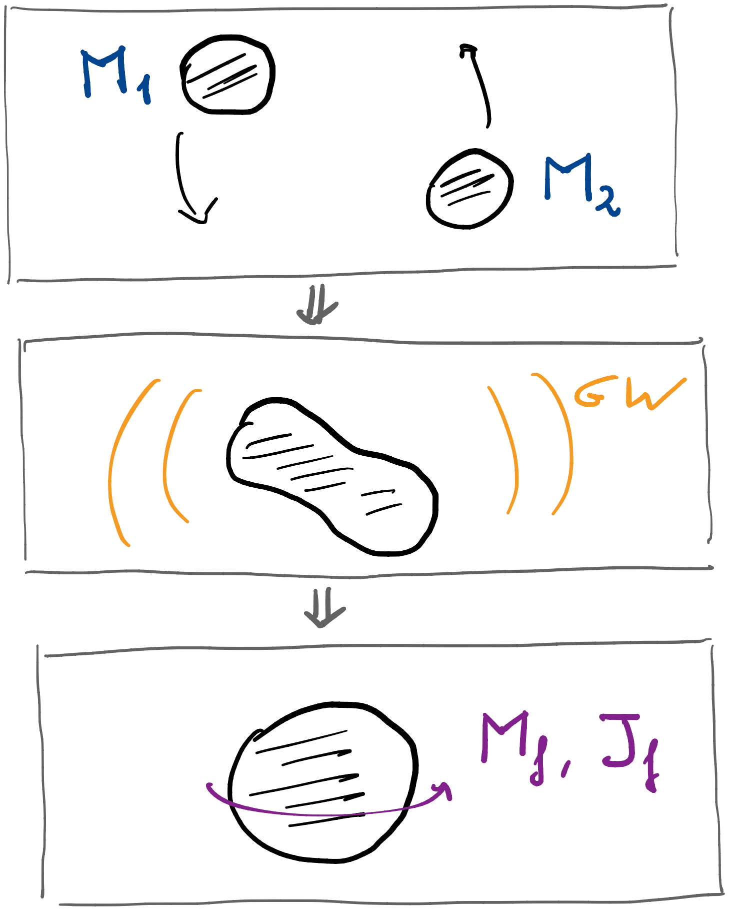
iii) Entropy: ad hoc Kerr

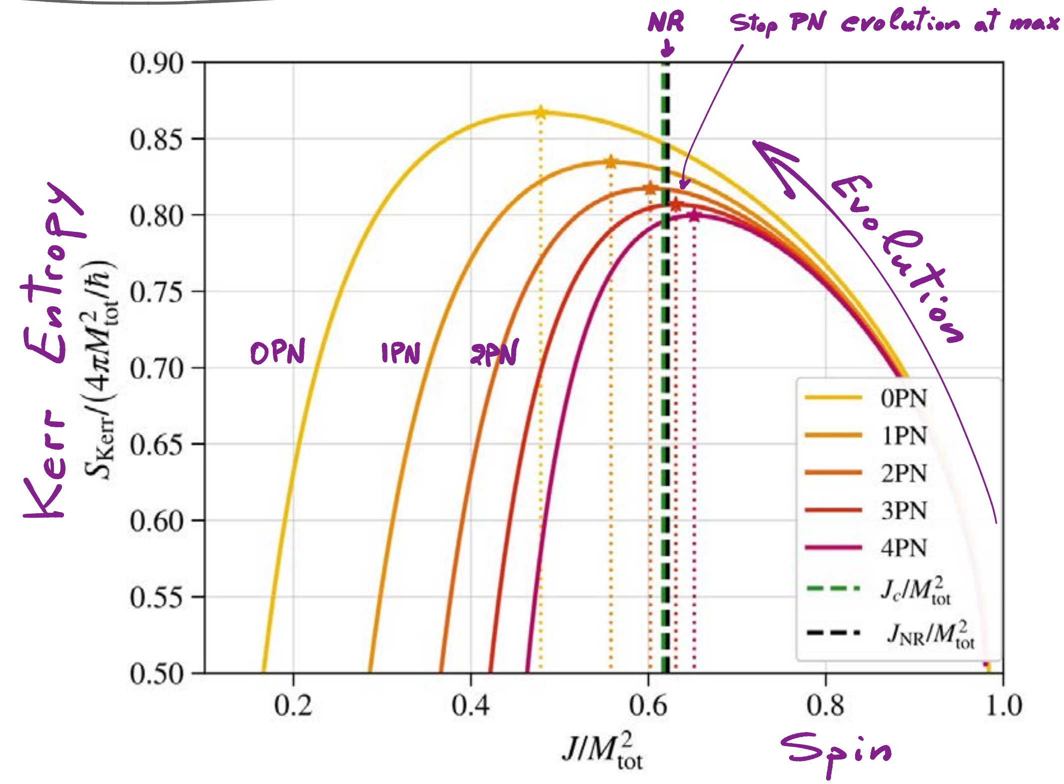
$$S(M,J) = \frac{A_{Kerr}(M,J)}{4\hbar} = \frac{8\pi\left(1+\sqrt{1-\frac{J^2}{M^4}}\right)M^2}{4\hbar}$$

- Max-Entropy Conjecture for Black Hole Mergers

Final Kerr BH $\begin{cases} M_f \\ J_f \end{cases}$ such that $\boxed{\frac{d}{dJ} S(M(J), J) = 0}$

◆ Max-Entropy Conjecture: Evidence using PN approx ($M_1 = M_2$)





Assuming equal-mass binary $M_1 = M_2$
initially circular orbit, non-spinning BHs

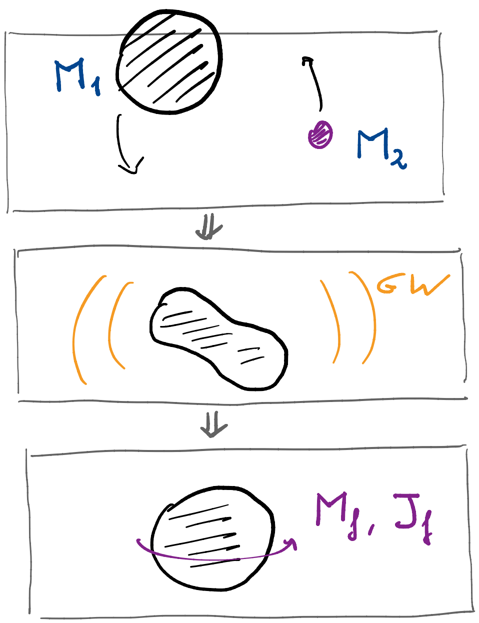
def: $\chi_f = \dfrac{J_f}{M_f^{\,2}}$

$\epsilon_f = 1 - \dfrac{M_f}{M_{tot}}$

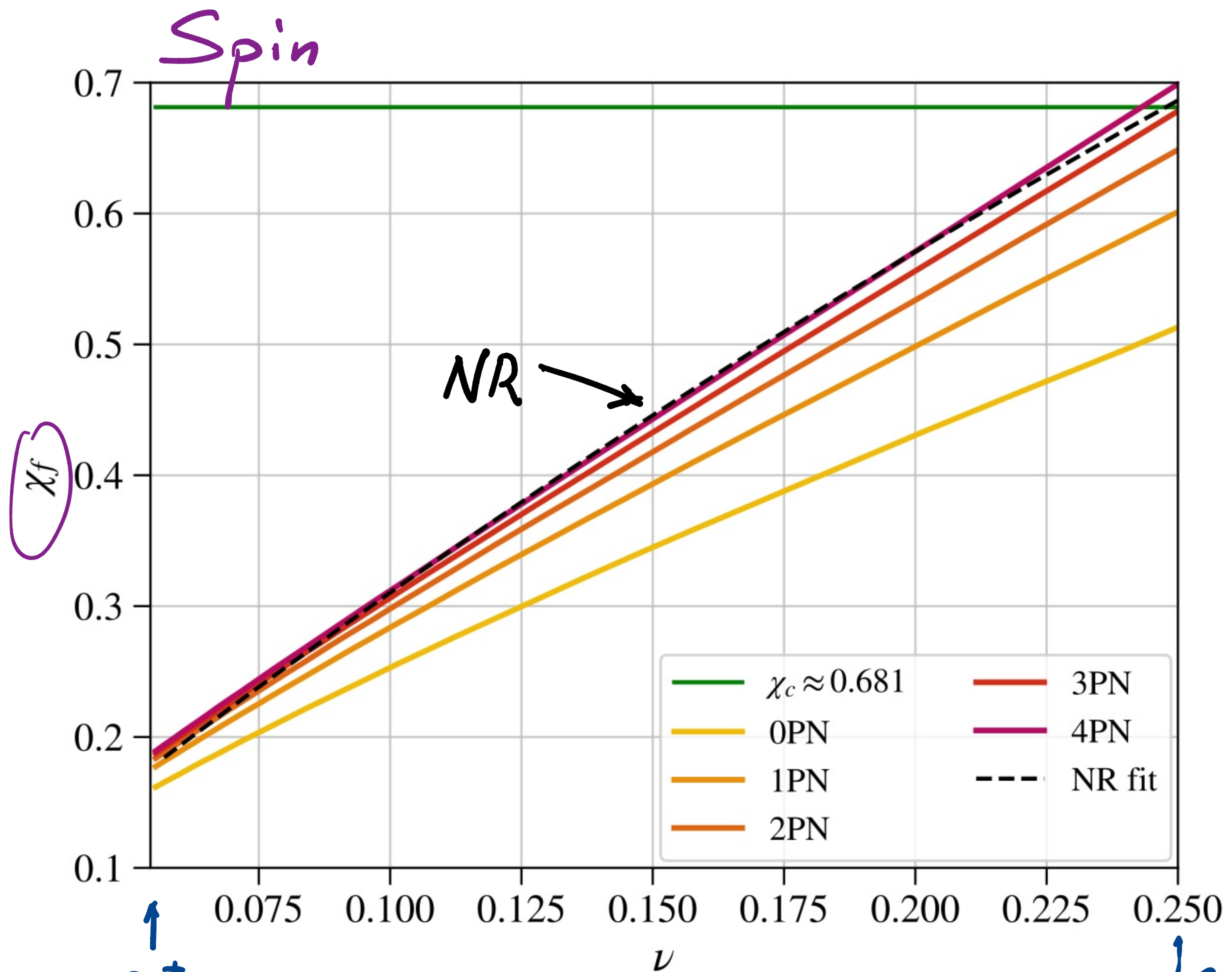
	0PN	1PN	2PN	3PN	4PN	NR
χ_f	0.513	0.601	0.649	0.678	0.699	0.686
ϵ_f	3.4%	3.6%	3.6%	3.5%	3.4%	
$\frac{M_f}{M_{\rm tot}}$	0.966	0.963	0.964	0.965	0.966	0.952
$\chi_{\rm MECO}$	–	0.674	0.811	0.817	0.828	

χ_{MECO} from x_* s.t. $\dfrac{dM(x)}{dx}\Big|_{x_*} = 0$ (Minimal Energy Circular Orbit)

◆ Max-Entropy Conjecture: Evidence using PN approx $\boxed{M_1 \neq M_2}$



def: $\chi_f = \frac{J_f}{M_f^2}$



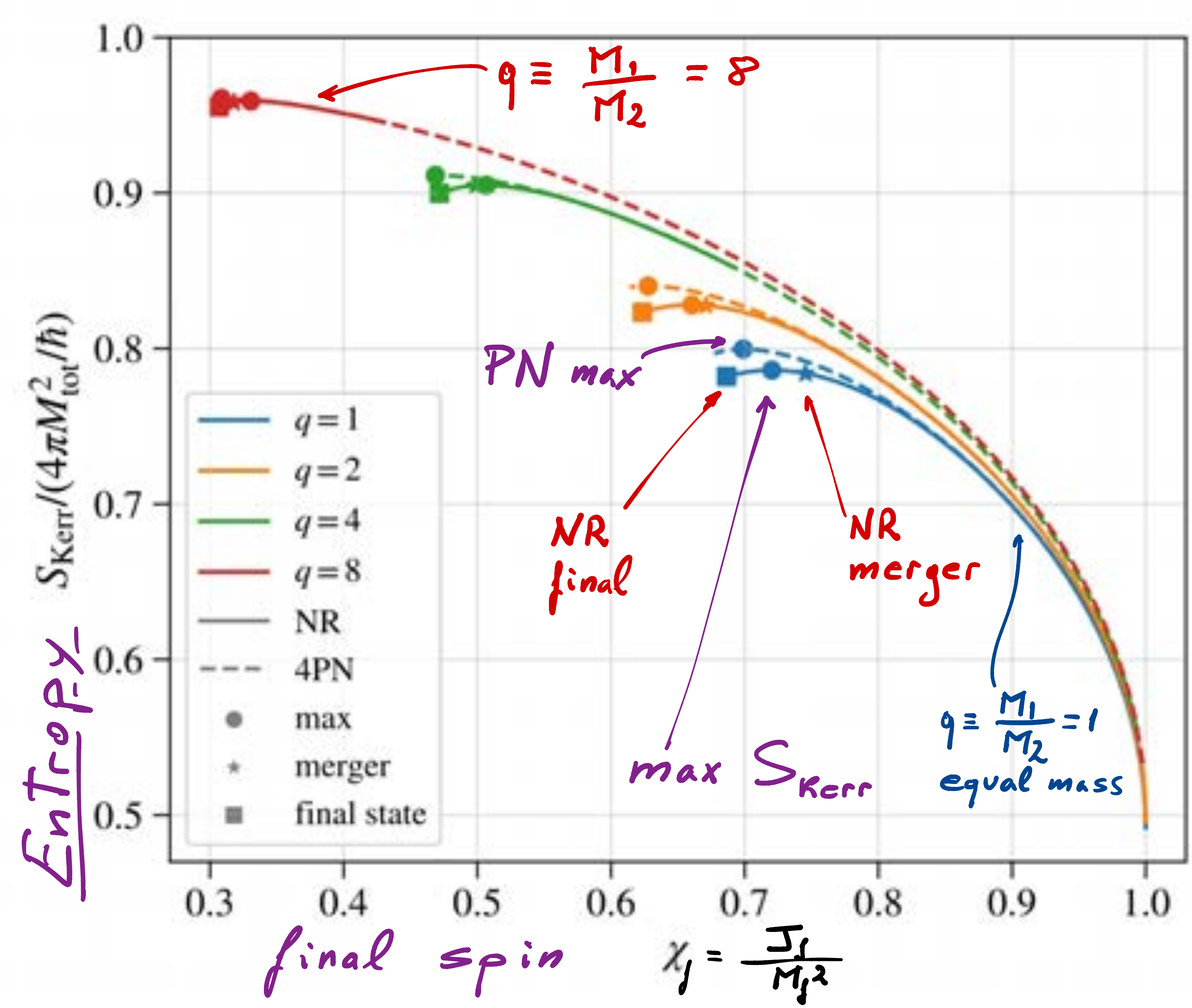
extreme ratio $M_1 \gg M_2$

equal mass $M_1 = M_2$

$$\nu = \frac{M_1 M_2}{(M_1 + M_2)^2}$$

◆ Max-Entropy Conjecture: Evidence from Numerical Relativity (v3.0.0 of SXS)

SXS, CQG(2025), [2505.13378]



Plan:

$\boxed{1}$ Max-Entropy in Black Hole Mergers

* Note:
  - not one of the laws of BH mechanics [Bardeen-Carter-Hawking (1973)]
  - "addendum" to the 2nd law
  - no $\hbar$

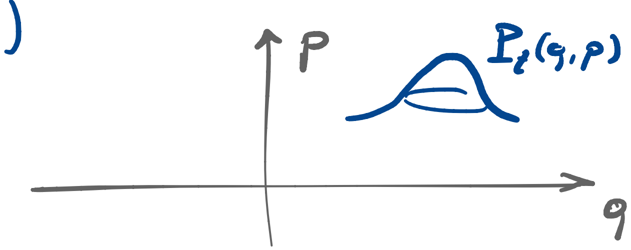
$\boxed{2}$ Principles:

What is the entropy of the classical gravitational field?

2 The Postulates of Classical Mechanics

I. **States**: prob. distrib. $\boxed{P(q_i, p_i)}$ in phase space Γ

II. **Observables**: function $O(q_i, p_i)$



III. **Dynamics**: $\frac{\partial}{\partial t} P_t = \{H, P_t\}$

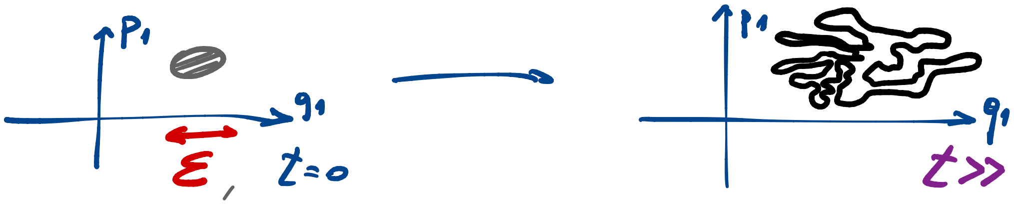
IV. **Measurement**: Given a state P and an observable O, the outcomes λ of the measurement of O (the clicks) are randomly distributed according to the marginal PDF

$$P_O(\lambda) = \int_\Gamma \delta(O(q_i,p_i) - \lambda)\, P(q_i,p_i) \prod_i dq_i\, dp_i \, .$$

V. **Preparation**: The measurement of O with post-sel. λ prepares the state $\tilde{P}(q_i,p_i) = \frac{1}{N} \delta(O(q_i,p_i) - \lambda)\, P(q_i,p_i)$.

* Remarks:

- We do not confuse states, observables and clicks
- Pure states are pathological in classical mechanics



$t \gg \lambda^{-1}$ = Lyapunov time

the two limits do not commute $\begin{cases} \varepsilon \to 0 \\ t \to \infty \end{cases}$

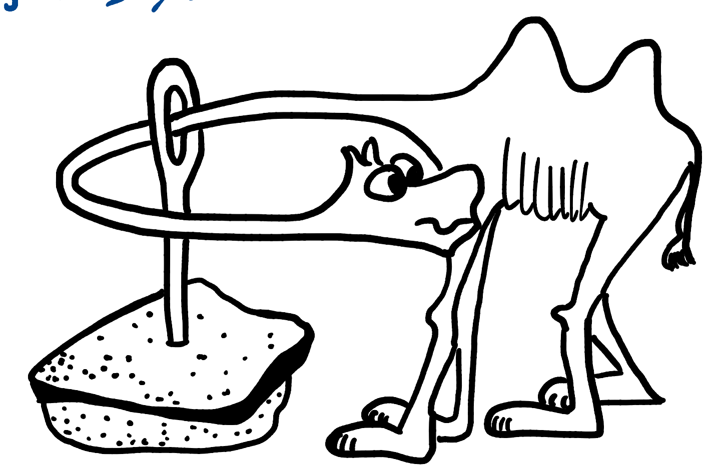
- Gromov's Thm (1985):

$$\left| \begin{array}{l} P_0(q_i, p_i) \quad \text{such that} \\ \Delta q_i \; \Delta p_i \Big|_{t=0} \geq \hbar \quad \forall i \end{array} \right. \implies \left| \begin{array}{l} P_t(q_i, p_i) \quad \forall t \\ \Delta q_j(t) \; \Delta p_j(t) \geq \hbar \end{array} \right.$$

also known as

the principle of

the symplectic camel



[Ian Stewart, Nature (1987)]

Entropy of a classical system

- Subsystem = Subalgebra of observables $\mathcal{A}_A \subseteq \mathcal{A}$
- State of the subsystem → marginal PDF $P_A = P|_{\mathcal{A}_A}$
- Entropy of the subsystem

$$\boxed{S_A = \int -P_A \log\left(\frac{P_A}{P_A^{(max)}}\right) d\Gamma_A}$$

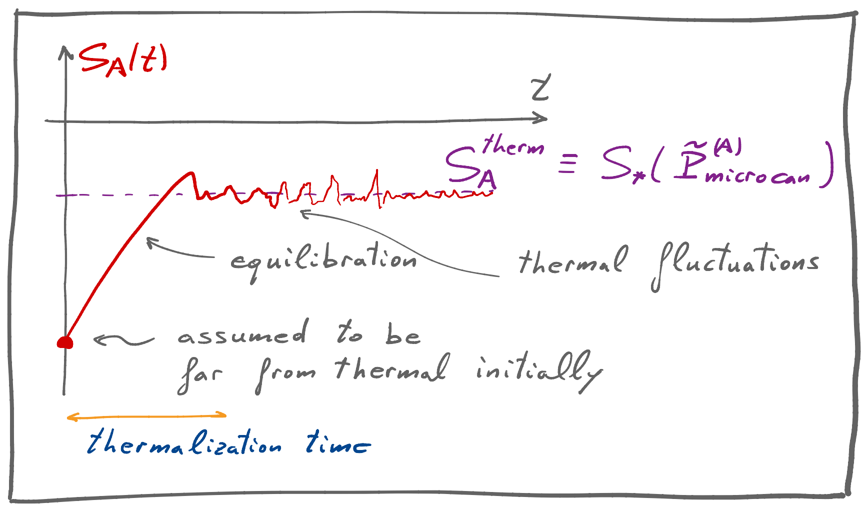
Stein's Lemma
$\mathrm{Prob}_N \sim e^{-D_{KL}(P \| P^{(ref)})}$

→ measure of statistical distinguishability of P_A from $P_A^{(max)}$

* Remark:

the entropy of the system is constant,

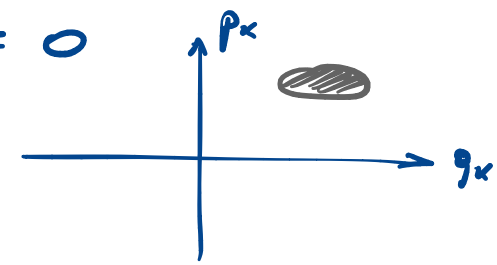
but a subsystem can thermalize



Entropy of the Classical Electromagnetic Field

- Initial State $P_0[\vec{A},\vec{E}]$, $\{\vec{\nabla}\cdot\vec{E}, P\} = 0$

  with dispersions $\Delta q_k \, \Delta p_k \geq \hbar$



- Cavity with speck of coal $\rightsquigarrow$ equipartition of energy

  cannot violate the symplectic camel ineq.

- Equilibrium $P_\beta[\vec{A},\vec{E}] = \prod_k P_\beta(q_k,p_k)$

  with
$$P_\beta(q_k,p_k) = \begin{cases} \frac{1}{Z}\, e^{-\beta\omega_k A(q_k,p_k)} & \hbar\omega_k \leq k_B T \\ \frac{1}{N}\, e^{-\frac{\omega_k A(q_k,p_k)}{\omega_k \hbar}} & \hbar\omega_k > k_B T \end{cases}$$

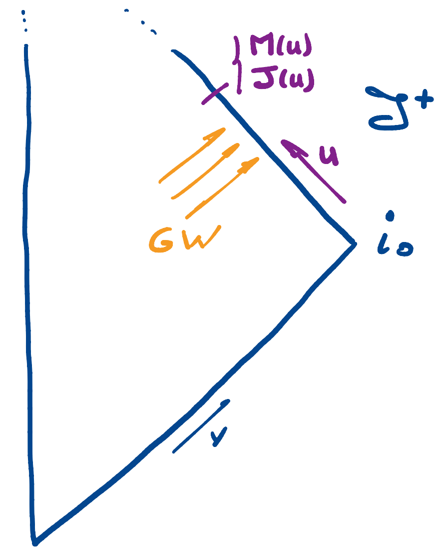
- Entropy $\boxed{S(P_\beta) = \frac{4}{9}\,\frac{V T^3}{\hbar^3}}$

* the symplectic camel saves equipartition from the UV catastrophe

* Planck's 1900 discovery: $\hbar \rightsquigarrow$ universal $\hbar$

Entropy of the classical gravitational field

- State $\mathcal{P}[h_{ab}, \pi^{ab}]$ with $\{\mathcal{H}, \mathcal{P}\} = 0$, $\{V^a, \mathcal{P}\} = 0$
- Subsystem $\longrightarrow$ subalgebra of observables



* For BH mergers :

$$\mathcal{A}_+ = \left\{ \begin{array}{l} \text{Bondi aspects } m(u,\theta,\phi),\ N^A(u,\theta,\phi) \\ \text{and Bondi News } C^{AB}(u,\theta,\phi) \end{array} \right\}$$

- Entropy $S_+(u) = \int -\mathcal{P}_+ \log\left(\dfrac{\mathcal{P}_+}{\mathcal{P}_+^{(ref)}}\right) d\Gamma_+$

- Conjecture: for a state $\mathcal{P}_0$ peaked with uncertainty $\not{a}$

  in amplitude and phase of QNM perturbations of Kerr,

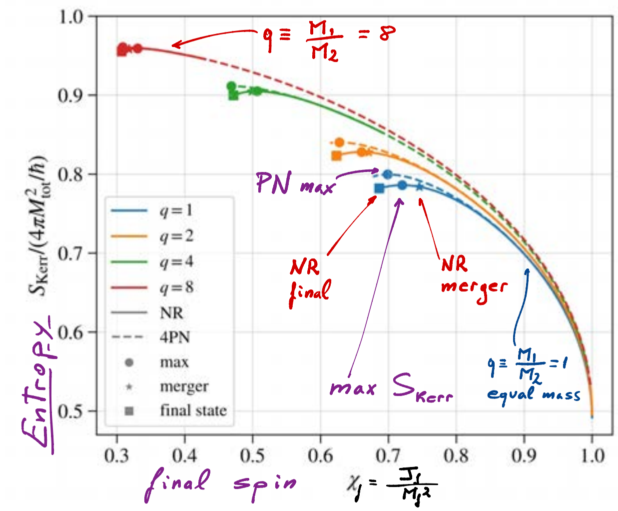
  the entropy is $S_+ = 2\pi \dfrac{A_{Kerr}(M, J)}{8\pi G\ \not{a}}$

* Bekenstein & Hawking (1974) : universal in-vacuum uncertainty $\hbar$.

Summary:

Classical Gravity Meets Classical Information

$\boxed{1}$ Final Mass & Spin in BH mergers from a Max-Entropy principle



$\boxed{2}$ Entropy in Classical GR