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Computing particle acceleration in jets using stochastic differential equations

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Physical setting

- Situation: AGN jet with shock
- Mechanisms:
  - Advection with bulk motion
  - Spatial and momentum diffusion
  - Synchrotron losses
  - Diffusive shock (Fermi I) acceleration
  - Second order Fermi acceleration

Fermi I acceleration

= diffusive shock acceleration

- Shock: quasi-instantaneous change in streaming velocity
- Particles crossing shock front gain momentum
- Diffusive: particles may cross the shock front several times
- Injection problem: only already superthermal particles are affected

CR acceleration

Fokker-Planck equation (without momentum diffusion)

$$\frac{\partial f}{\partial t} = -c\beta(z)\frac{\partial f}{\partial z}$$ bulk drift

$$+\left(\frac{c}{3}\frac{\mathrm{d}\beta}{\mathrm{d}z}p\right)\frac{\partial f}{\partial p}$$ First-order Fermi acceleration

$$+\frac{\partial}{\partial z}\left(\kappa(z,p)\frac{\partial f}{\partial z}\right)$$ spatial diffusion

$$+\frac{1}{p^2}\frac{\partial}{\partial p}\left(\frac{2}{3}k_{\mathrm{syn}}(z)p^4 f\right)$$ synchrotron losses

$$+Q$$ source terms

Stochastic DEs

- DEs with a stochastic process term

$$\frac{\mathrm{d}X}{\mathrm{d}t} = a(t, X) + b(t, X)\xi_t$$

- Wiener process $\frac{\mathrm{d}W_t}{\mathrm{d}t} = \xi_t$
  - mean is zero: $\langle W_t \rangle = 0$
  - not autocorrelated: $\langle W_i W_j \rangle = \delta_{ij}$
  - normal distributed with

    $\mathrm{Var}\left(W_{t+\Delta t} - W_t\right) = \Delta t$

- Nowhere continuously differentiable

SDE to FPE

$$X_t = \int_{t_0}^{t} a(X_s)\mathrm{d}s + \int_{t_0}^{t} b(X_s)\mathrm{d}W_s$$

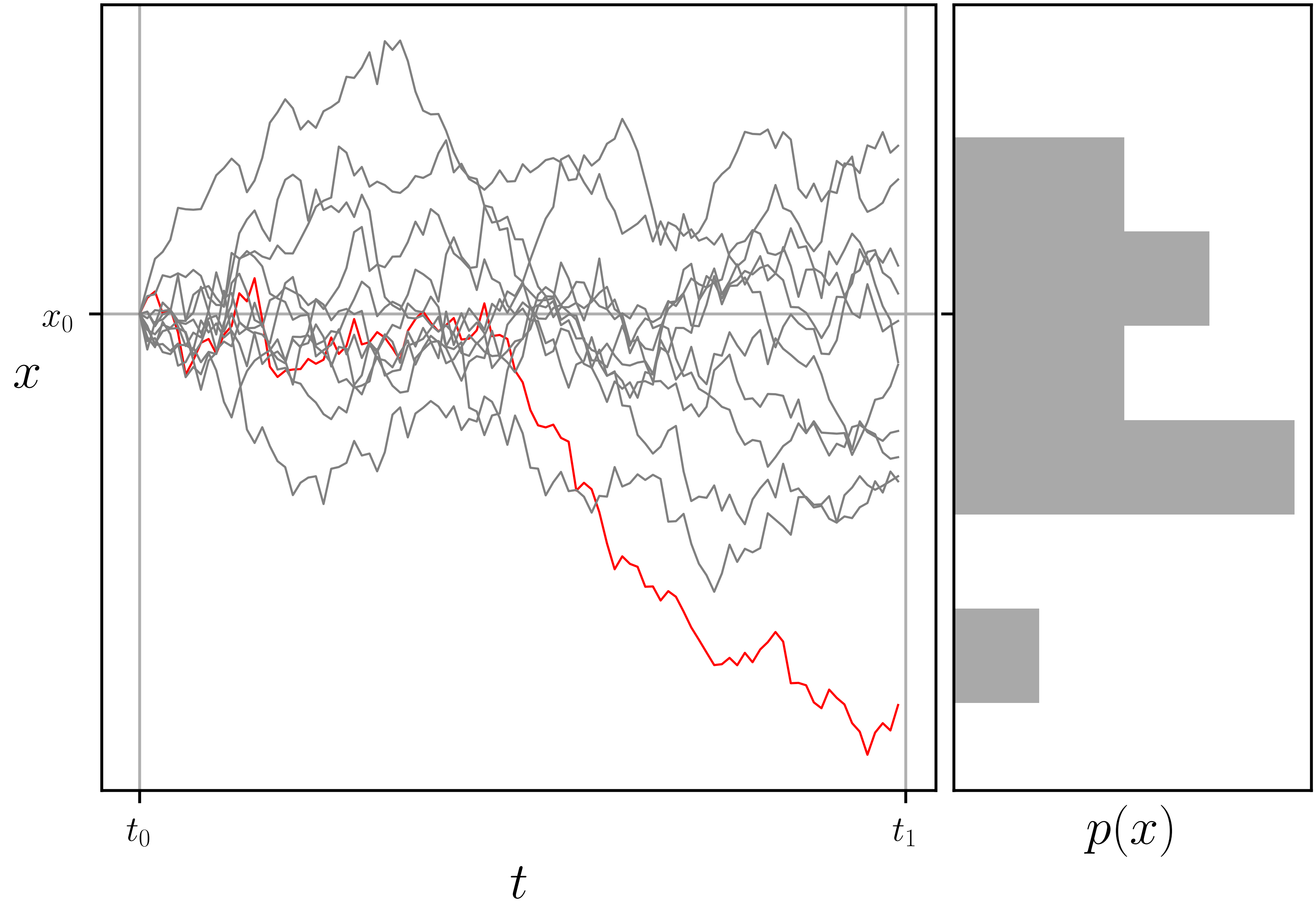
- $X_0 ... X_t$ is a Markov chain and therefore has a transition probability

$$p(t_1, X_1; t_2, X_2)$$

- This transition probability follows the FPE

$$\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}(ap) - \frac{\partial^2}{\partial x^2}(b^2 p) = 0$$

Simple diffusion



Multiple dimensions

- Extending to multiple dimensions is straightforward
- System of coupled SDEs

$$\mathrm{d}X_{i,t} = a_i(\vec{X}_t)\mathrm{d}t + \sum_j b_{ij}(\vec{X}_t)\mathrm{d}W_{i,t}$$

- Corresponding FPE

$$\frac{\partial p}{\partial t} + \sum_i \frac{\partial}{\partial x}(a_i p) - \sum_{ij} \frac{\partial^2}{\partial x_i \partial x_j}\left((b \cdot b^T)_{ij} p\right) = 0$$

Solving SDEs

- Approximating using an Euler-Mayurama-scheme

$$\vec{X}(t+\Delta t) = \vec{X}(t) + \vec{a}(\vec{X}(t))\Delta t + \hat{B}(\vec{X}(t))W_t$$

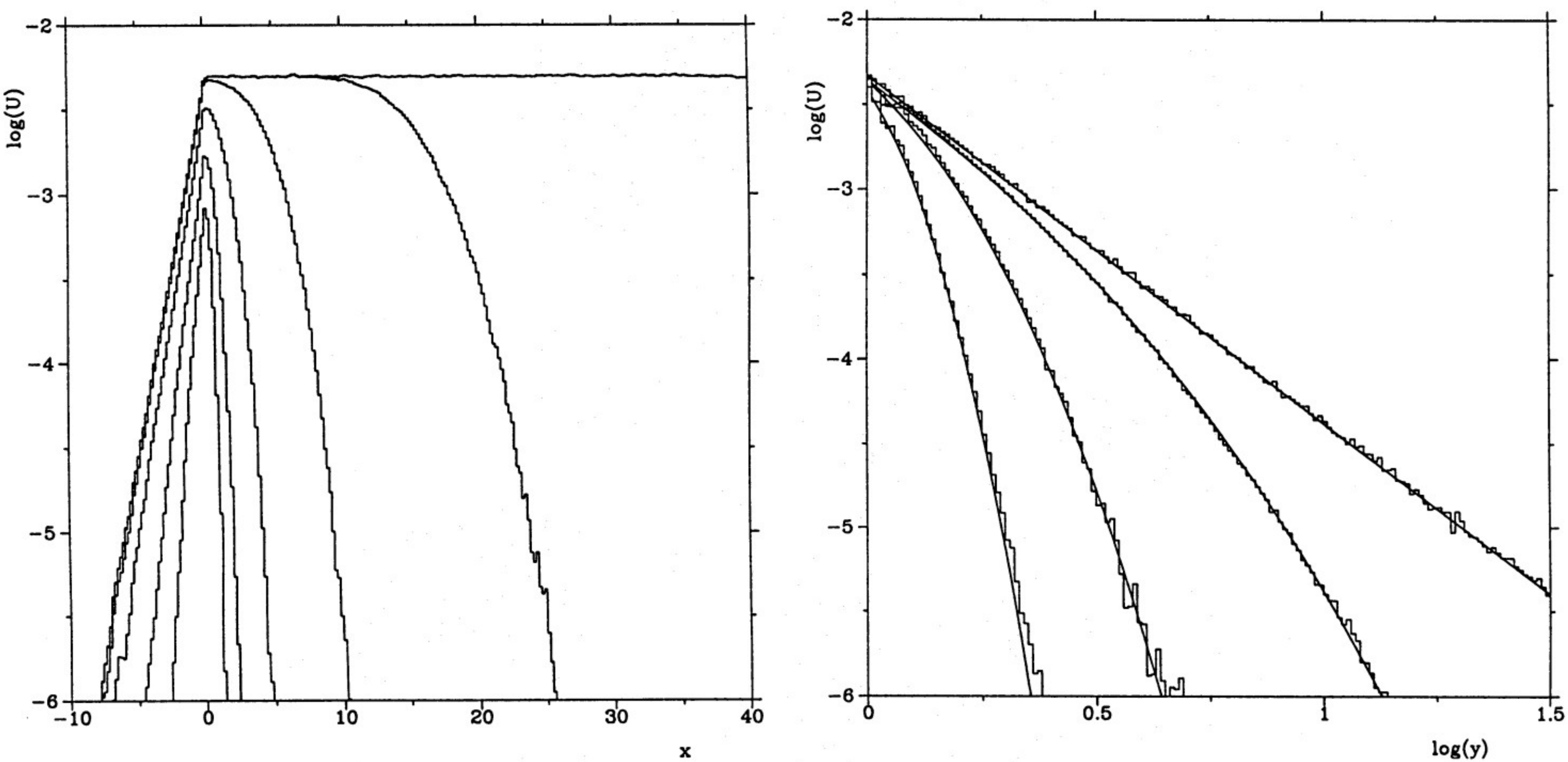
- Boundary conditions are checked at every iteration
- Density of $\vec{X}(T)$ gives the solution

Why SDEs?

- Simple implementation
- Threading is straightforward
- Efficient by dynamically choosing the timestep
- Numerical stability

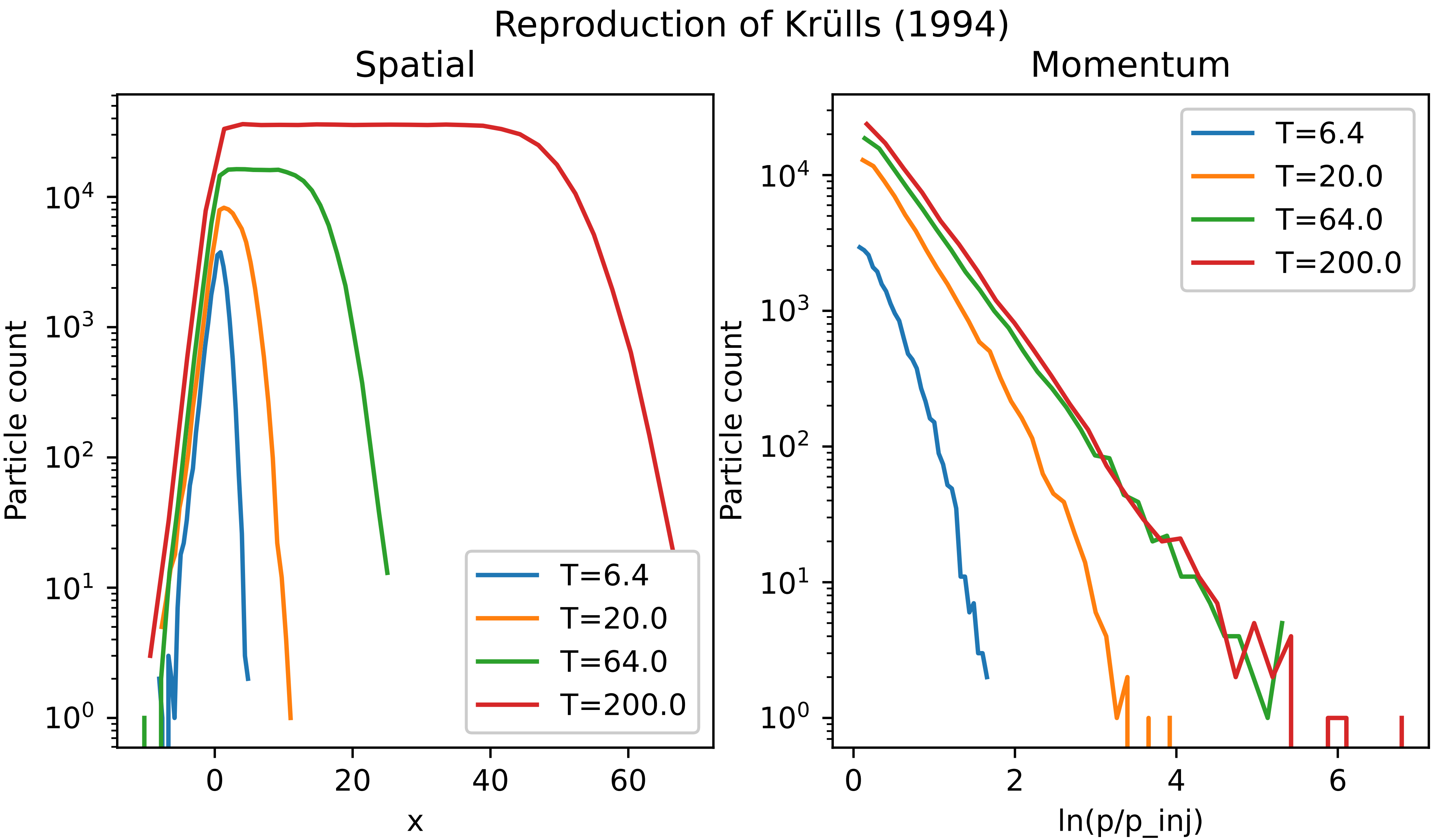
Current status

- Reproducing results from Kruells 94



From: Krülls and Achterberg (1994)

Current status



Plans

- Reproducing results for 2nd order Fermi acceleration (neglected so far)
- Comparison with analytical solutions for special cases
- Look into different injection mechanisms: stochastic injection
- Parameter studies for different ratios between loss time scales and adv./diff. time scales
- Synthetic light curves

Summary

- Modeling CR transport gives rise to a Fokker-Planck equation
- Solving is efficiently possible by finding the corresponding equivalent SDE
- Target: study injection models