# Hawking evaporation in LGG

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FAU'2 WORKSHOP ON QUANTUM GRAVITY ACROSS SCALES 20-22 May 2025









# numerous studies to study the LQG effect on black holes (LQGBH).

# Molivalion



• Black holes and the early universe are among the few probes to study the smoking gun of quantum gravity. The success of LQC in resolving the singularity issue has inspired





- numerous studies to study the LQG effect on black holes (LQGBH).
- Three main interesting open questions
  - Singularity resolution
  - The fate of gravitational collapse
  - The loss paradox

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- numerous studies to study the LQG effect on black holes (LQGBH).
- Three main interesting open questions
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  - The loss paradox
- Yanq'24].

# Molivalion



• Black holes and the early universe are among the few probes to study the smoking gun of quantum gravity. The success of LQC in resolving the singularity issue has inspired

• A covariant model for a spherically symmetric system has been proposed [Alonso-Bardaji, Brizuela'21; Bojowald, Duque'23; Alonso-Bardaji, Brizuela'24; Zhang, Lewandowski, Ma,

















### (1) The Hawking distribution consistently regains.











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(2) The universality of black hole absorption rates is preserved.

Belfaail











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(3) After the evaporation slows, gravitational instability emerges and prevails, precluding thermal stabilization.











(1) The Hawking distribution consistently regains.

(2) The universality of black hole absorption rates is preserved.

(3) After the evaporation slows, gravitational instability emerges and prevails, precluding thermal stabilization.

(4) The mass hierarchy favours a black to white hole transition.





# (1) Framework, vacuum solutions, and scalar field coupling

## (2) The black hole evaporation

(3) Summary and outlook

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# FRAMEWORK, VACUUM SOLUTIONS, AND SCALAR FIELD COUPLING









$$HDA$$

$$\{H_{x}[N_{1}^{x}], H_{x}[N_{2}^{x}]\} = H_{x} \left[ N_{1}^{x} \left( N_{2}^{x} \right)' - N_{2}^{x} \left( N_{1}^{x} \right)' \right]$$

$$\{H[N_{1}], H_{x}[N_{2}^{x}]\} = -H[N_{2}^{x}N_{1}']$$

$$\{H[N_{1}], H[N_{2}]\} = H_{x} \left[ f(E^{I}, K_{I}) \left( N_{1}N_{2}' - N_{2}N_{1}' \right) \right]$$

# Greometry from Phase Space







$$HDA$$

$$\{H_{x}[N_{1}^{x}], H_{x}[N_{2}^{x}]\} = H_{x} \left[ N_{1}^{x} \left( N_{2}^{x} \right)' - N_{2}^{x} \left( N_{1}^{x} \right)' \right]$$

$$\{H[N_{1}], H_{x}[N_{2}^{x}]\} = -H[N_{2}^{x}N_{1}']$$

$$\{H[N_{1}], H[N_{2}]\} = H_{x} \left[ f(E^{I}, K_{I}) \left( N_{1}N_{2}' - N_{2}N_{1}' \right) \right]$$

## Modified structure function

$$f(E^{x}, E^{\varphi}, K_{\varphi}) = \left[ \left( 1 + \left( \frac{\tilde{\lambda} (E^{x})'}{2E^{\varphi}} \right)^{2} \right) \cos \left( \tilde{\lambda} K_{\varphi} \right)^{2} \right] \chi^{2} - \frac{1}{2E^{\varphi}} d\xi$$













## Modified structure function

$$f(E^{x}, E^{\varphi}, K_{\varphi}) = \left[ \left( 1 + \left( \frac{\tilde{\lambda} (E^{x})'}{2E^{\varphi}} \right)^{2} \right) \cos \left( \tilde{\lambda} K_{\varphi} \right)^{2} \right] \chi^{2} - \frac{1}{2E^{\varphi}} d\xi$$













### Modified structure function

$$f(E^{x}, E^{\varphi}, K_{\varphi}) = \left[ \left( 1 + \left( \frac{\tilde{\lambda} (E^{x})'}{2E^{\varphi}} \right)^{2} \right) \cos \left( \tilde{\lambda} K_{\varphi} \right)^{2} \right] \chi^{2} - \frac{1}{2E^{\varphi}} d\xi$$

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \frac{1}{f} \left(\mathrm{d}x + N^x \mathrm{d}t\right)^2 + E^x \mathrm{d}\Omega^2$$

### Idrus Husin Belfaqih











![](_page_14_Figure_2.jpeg)

![](_page_14_Figure_4.jpeg)

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + \frac{1}{f} \left(\mathrm{d}x + N^x \mathrm{d}t\right)^2 + E^x \mathrm{d}\Omega^2$$

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![](_page_14_Picture_8.jpeg)

![](_page_15_Picture_0.jpeg)

$$\tilde{H} = a_0 + ((E^x)')^2 a_{xx} + ((E^{\varphi})')^2 a_{\varphi\varphi} + (E^x)'(E^{\varphi})' a_{x\varphi} + (E^{\varphi})' a_{x\varphi} + (E^{\varphi})' a_{y\varphi} + (E^$$

maintained.

![](_page_15_Picture_6.jpeg)

• Starting with the most general ansatz for the Hamiltonian constraint containing up to second-order derivatives and quadratic first-order derivative terms [Alonso-Bardaji and Brizuela'22; Bojowald

 $(E^{x})''a_{2} + (K'_{\varphi})^{2}b_{\varphi\varphi} + (K_{\varphi})''b_{2} + (E^{x})'K'_{\varphi}c_{x\varphi} + (E^{\varphi})'K'_{\varphi}c_{\varphi\varphi} + (E^{\varphi})''c_{2}$ 

![](_page_15_Picture_12.jpeg)

![](_page_16_Picture_0.jpeg)

![](_page_16_Picture_1.jpeg)

$$\tilde{H} = a_0 + ((E^x)')^2 a_{xx} + ((E^{\varphi})')^2 a_{\varphi\varphi} + (E^x)'(E^{\varphi})' a_{x\varphi} + (E^{\varphi})' a_{\varphi\varphi} + (E^{\varphi})' a_{\varphi} + (E^{\varphi})' a_{\varphi}$$

maintained.

![](_page_16_Figure_5.jpeg)

$$\begin{split} & Non-periodic \ phase \ space \\ & \tilde{q}^{xx} = \left( \left( 1 + \lambda^2 \left( \frac{(E^x)'}{2E^{\varphi}} \right)^2 \right) \cos^2 \left( \lambda K_{\varphi} \right) - 2q\lambda^2 \frac{\sin(2\lambda K_{\varphi})}{2\lambda} \right) \chi^2 \frac{E^x}{(E^{\varphi})^2} \end{split}$$

• Starting with the most general ansatz for the Hamiltonian constraint containing up to second-order derivatives and quadratic first-order derivative terms [Alonso-Bardaji and Brizuela'22; Bojowald

 $(E^{x})''a_{2} + (K'_{\varphi})^{2}b_{\varphi\varphi} + (K_{\varphi})''b_{2} + (E^{x})'K'_{\varphi}c_{x\varphi} + (E^{\varphi})'K'_{\varphi}c_{\varphi\varphi} + (E^{\varphi})''c_{2}$ 

![](_page_16_Picture_13.jpeg)

![](_page_17_Picture_0.jpeg)

![](_page_17_Picture_1.jpeg)

$$\tilde{H} = a_0 + ((E^x)')^2 a_{xx} + ((E^{\varphi})')^2 a_{\varphi\varphi} + (E^x)'(E^{\varphi})' a_{x\varphi} + (E^x)'' a_2 + (K'_{\varphi})^2 b_{\varphi\varphi} + (K_{\varphi})'' b_2 + (E^x)' K'_{\varphi} c_{x\varphi} + (E^{\varphi})' K'_{\varphi} c_{\varphi\varphi} + (E^{\varphi})'' c_2 + (E^x)' k_{\varphi} c_{x\varphi} + (E^{\varphi})' k_{\varphi} c_{\varphi\varphi} + (E^{\varphi})'' c_2 + (E^x)' k_{\varphi} c_{x\varphi} + (E^{\varphi})' k_{\varphi} c_{\varphi\varphi} + (E^{\varphi})'' c_2 + (E^x)' k_{\varphi} c_{x\varphi} + (E^{\varphi})' k_{\varphi} c_{\varphi\varphi} + (E^{\varphi})' k_{\varphi} c_{\varphi} c_{\varphi} c$$

maintained.

![](_page_17_Figure_5.jpeg)

$$\begin{split} & Non-periodic \ phase \ space \\ & \tilde{q}^{xx} = \left( \left( 1 + \lambda^2 \left( \frac{(E^x)'}{2E^{\varphi}} \right)^2 \right) \cos^2 \left( \lambda K_{\varphi} \right) - 2q\lambda^2 \frac{\sin(2\lambda K_{\varphi})}{2\lambda} \right) \chi^2 \frac{E^x}{(E^{\varphi})^2} \end{split}$$

• Starting with the most general ansatz for the Hamiltonian constraint containing up to second-order derivatives and quadratic first-order derivative terms [Alonso-Bardaji and Brizuela'22; Bojowald

![](_page_17_Figure_10.jpeg)

$$\begin{aligned} \widetilde{q}^{xx} &= \left( \left( 1 + \left( \frac{(E^x)'\widetilde{\lambda}}{2E^{\varphi}} \right)^2 \right) \cos^2\left( \widetilde{\lambda}K_{\varphi} \right) - 2\frac{\lambda}{\widetilde{\lambda}}q\widetilde{\lambda}^2 \frac{\sin\left( 2\widetilde{\lambda}K_{\varphi} \right)}{2\widetilde{\lambda}} \right) \frac{\widetilde{\lambda}^2}{\lambda^2}\chi^2. \end{aligned}$$

![](_page_17_Picture_14.jpeg)

![](_page_17_Picture_15.jpeg)

![](_page_17_Picture_16.jpeg)

![](_page_18_Picture_0.jpeg)

$$\tilde{H} = a_0 + ((E^x)')^2 a_{xx} + ((E^{\varphi})')^2 a_{\varphi\varphi} + (E^x)'(E^{\varphi})' a_{x\varphi} + (E^{\varphi})' a_{x\varphi} + (E^{\varphi})' a_{y\varphi} + (E^$$

maintained.

$$\frac{\text{Non-periodic phase space}}{\tilde{q}^{xx}} = \left( \left( 1 + \lambda^2 \left( \frac{(E^{x})'}{2E^{\varphi}} \right)^2 \right) \cos^2 \left( \lambda K_{\varphi} \right) - 2q\lambda^2 \frac{\sin(2\lambda K_{\varphi})}{2\lambda} \right) \chi^2 \frac{E^x}{(E^{\varphi})^2}.$$

![](_page_18_Picture_7.jpeg)

• Starting with the most general ansatz for the Hamiltonian constraint containing up to second-order derivatives and quadratic first-order derivative terms [Alonso-Bardaji and Brizuela'22; Bojowald

 $E^{x})''a_{2} + (K'_{\varphi})^{2}b_{\varphi\varphi} + (K_{\varphi})''b_{2} + (E^{x})'K'_{\varphi}c_{x\varphi} + (E^{\varphi})'K'_{\varphi}c_{\varphi\varphi} + (E^{\varphi})''c_{2}$ 

![](_page_18_Figure_11.jpeg)

![](_page_18_Picture_14.jpeg)

![](_page_18_Picture_15.jpeg)

![](_page_19_Picture_0.jpeg)

![](_page_19_Picture_1.jpeg)

![](_page_19_Figure_2.jpeg)

![](_page_19_Figure_3.jpeg)

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{x}\right)\frac{\mathrm{d}t^2}{\alpha^2\chi^2} + \frac{\mathrm{d}x^2}{\chi^2\left(1 - \frac{2M}{x}\right)\left(1 + \lambda^2(x)\left(1 - \frac{2M}{x}\right)\right)} + x^2\mathrm{d}\Omega^2$$

![](_page_19_Picture_9.jpeg)

![](_page_19_Picture_10.jpeg)

![](_page_19_Picture_11.jpeg)

![](_page_20_Picture_0.jpeg)

![](_page_20_Picture_1.jpeg)

![](_page_20_Figure_2.jpeg)

![](_page_20_Figure_3.jpeg)

![](_page_20_Figure_4.jpeg)

1. <u>Reflection symmetry surface</u>

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{x}\right)\frac{\mathrm{d}t^2}{\alpha^2\chi^2} + \frac{\mathrm{d}x^2}{\chi^2\left(1 - \frac{2M}{x}\right)\left(1 + \lambda^2(x)\left(1 - \frac{2M}{x}\right)\right)} + x^2\mathrm{d}\Omega^2$$

$$1 + \lambda^2 \left( 1 - \frac{2M}{\sqrt{E^x}} - \frac{\Lambda}{3} E^x \right) = 0$$

![](_page_20_Picture_12.jpeg)

![](_page_20_Picture_13.jpeg)

![](_page_20_Picture_14.jpeg)

![](_page_21_Picture_0.jpeg)

![](_page_21_Picture_1.jpeg)

![](_page_21_Figure_2.jpeg)

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{x}\right)\frac{\mathrm{d}t^2}{\alpha^2\chi^2} + \frac{\mathrm{d}x^2}{\chi^2\left(1 - \frac{2M}{x}\right)\left(1 + \lambda^2(x)\left(1 - \frac{2M}{x}\right)\right)} + x^2\mathrm{d}\Omega^2$$

$$1 + \lambda^2 \left( 1 - \frac{2M}{\sqrt{E^x}} - \frac{\Lambda}{3} E^x \right) = 0$$

![](_page_21_Picture_11.jpeg)

![](_page_21_Picture_12.jpeg)

![](_page_21_Picture_13.jpeg)

![](_page_22_Picture_0.jpeg)

![](_page_22_Picture_1.jpeg)

![](_page_22_Figure_2.jpeg)

# Global structure

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{x}\right)\frac{\mathrm{d}t^2}{\alpha^2\chi^2} + \frac{\mathrm{d}x^2}{\chi^2\left(1 - \frac{2M}{x}\right)\left(1 + \lambda^2(x)\left(1 - \frac{2M}{x}\right)\right)} + x^2\mathrm{d}\Omega^2$$

$$1 + \lambda^2 \left( 1 - \frac{2M}{\sqrt{E^x}} - \frac{\Lambda}{3} E^x \right) = 0$$

$$\mathrm{d}s^2 \approx -\frac{\mathrm{d}t^2}{\alpha^2 \chi(\infty)^2} + \left(1 + \lambda(\infty)^2\right)^{-1} \frac{\mathrm{d}x^2}{\chi(\infty)^2} + x^2 \mathrm{d}\Omega^2 \,.$$

Requiring asymptotic flatness implies  $\alpha = \chi(\infty)^{-1} \chi(\infty) = 1/\sqrt{1 + \lambda(\infty)^2}$ .

![](_page_22_Picture_13.jpeg)

![](_page_22_Picture_14.jpeg)

![](_page_22_Picture_15.jpeg)

![](_page_23_Picture_0.jpeg)

![](_page_23_Picture_1.jpeg)

![](_page_23_Figure_2.jpeg)

# Global structure

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{x}\right)\frac{\mathrm{d}t^2}{\alpha^2\chi^2} + \frac{\mathrm{d}x^2}{\chi^2\left(1 - \frac{2M}{x}\right)\left(1 + \lambda^2(x)\left(1 - \frac{2M}{x}\right)\right)} + x^2\mathrm{d}\Omega^2$$

$$1 + \lambda^2 \left( 1 - \frac{2M}{\sqrt{E^x}} - \frac{\Lambda}{3} E^x \right) = 0$$

$$\mathrm{d}s^2 \approx -\frac{\mathrm{d}t^2}{\alpha^2 \chi(\infty)^2} + \left(1 + \lambda(\infty)^2\right)^{-1} \frac{\mathrm{d}x^2}{\chi(\infty)^2} + x^2 \mathrm{d}\Omega^2 \,.$$

Requiring asymptotic flatness implies  $\alpha = \chi(\infty)^{-1} \chi(\infty) = 1/\sqrt{1 + \lambda(\infty)^2}$ .

Recovering the flat time attained by  $\chi^2 = \alpha^{-2} \equiv \chi_0^2 = \left(1 + \lambda_\infty^2\right)^{-1}$ 

$$ds^{2} = -dt^{2} + \frac{1}{\chi_{0}^{2} \left(1 + \lambda^{2}(x)\right)} dx^{2} + x^{2} d\Omega^{2}$$

![](_page_23_Picture_17.jpeg)

![](_page_23_Picture_18.jpeg)

![](_page_23_Picture_19.jpeg)

![](_page_24_Picture_0.jpeg)

![](_page_24_Picture_1.jpeg)

![](_page_24_Figure_2.jpeg)

Large x effect

![](_page_24_Figure_4.jpeg)

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![](_page_24_Picture_7.jpeg)

![](_page_24_Picture_10.jpeg)

![](_page_25_Picture_0.jpeg)

![](_page_25_Picture_1.jpeg)

![](_page_25_Figure_2.jpeg)

Large x effect

![](_page_25_Figure_4.jpeg)

The University of Edinburgh

# Global structure

![](_page_25_Picture_7.jpeg)

![](_page_25_Figure_8.jpeg)

No IR effect

![](_page_26_Picture_0.jpeg)

![](_page_26_Picture_1.jpeg)

Hamiltonian constraint of the gravitational degrees of freedom with scalar matter

 $\tilde{H} = \tilde{H}_{\text{grav}} + \tilde{H}_{\phi}$ 

The covariance allowed for a variety of coupling schemes [Bojowald and Duque'24]. Two interesting scenarios:

**O** Minimally coupled scalar field (low-curvature)

$$\tilde{H} = E^x \sqrt{\tilde{q}_{xx}} \left[ \frac{1}{2} \right]$$

**O** Non-minimally coupled scalar (high-curvature)

$$\tilde{H} = \chi \frac{\sqrt{E^{x}}}{2} \left( \frac{P_{\phi}^{2}}{E^{\varphi} E^{x}} \left( 1 + \lambda^{2} (E^{x}) \left( \frac{(E^{x})^{'}}{2E^{\varphi}} \right)^{2} \right) \cos^{2}(\lambda K_{\varphi}) + \frac{E^{x}}{E^{\varphi}} \left( \phi^{\prime} \right)^{2} + 2E^{\varphi} V(\phi) \right) \right)$$

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$$\frac{P_{\phi}^2}{(E^x)^2 \tilde{q}_{xx}} + \frac{(\phi')^2}{2\tilde{q}_{xx}} + V(\phi)$$

![](_page_26_Picture_13.jpeg)

![](_page_27_Picture_0.jpeg)

# THE BLACK HOLE EVAPORATION

![](_page_27_Picture_2.jpeg)

![](_page_28_Picture_0.jpeg)

![](_page_28_Picture_1.jpeg)

<sup>o</sup> Near-horizon  $x = 2M\left(1 + \left(\chi_0\zeta/4M\right)^2\right)$ , the line-element can be cast into Rindler line-element

 $ds^{2} = -\zeta^{2}d\tau^{2} + d\zeta^{2}$  Near horizon temperature  $T = \frac{\chi_{0}}{4\pi} \frac{1}{\sqrt{2Mx(1 - 2M/x)}}$ 

![](_page_28_Picture_6.jpeg)

![](_page_28_Picture_9.jpeg)

![](_page_28_Picture_10.jpeg)

![](_page_29_Picture_0.jpeg)

![](_page_29_Picture_1.jpeg)

<sup>o</sup> Near-horizon  $x = 2M\left(1 + \left(\chi_0\zeta/4M\right)^2\right)$ , the line-element can be cast into Rindler line-element

$$ds^2 = -\zeta^2 d\tau^2 + d\zeta^2$$
 Near horiz

<sup>o</sup> The temperature at arbitrary  $\bar{x} > x$ 

zon temperature  $T = \frac{\chi_0}{4\pi} \frac{1}{\sqrt{2Mx(1 - 2M/x)}}$ 

$$T(\bar{x}) = \frac{\chi_0}{8\pi M} \left(1 - \frac{2M}{\bar{x}}\right)^{-1/2}$$

![](_page_29_Picture_9.jpeg)

![](_page_29_Picture_12.jpeg)

![](_page_29_Picture_13.jpeg)

![](_page_29_Picture_14.jpeg)

![](_page_30_Picture_0.jpeg)

<sup>o</sup> Near-horizon 
$$x = 2M\left(1 + \left(\chi_0\zeta/4M\right)^2\right)$$
, the line-element

$$ds^2 = -\zeta^2 d\tau^2 + d\zeta^2$$
 Near horiz

<sup>o</sup> The temperature at arbitrary  $\bar{x} > x$ 

 $T(\bar{x}) = \frac{7}{8}$ 

• The temperature measured by the asymptotic observer

 $T(\infty) =$ 

![](_page_30_Picture_9.jpeg)

it can be cast into Rindler line-element

zon temperature  $T = \frac{\chi_0}{4\pi} \frac{1}{\sqrt{2Mx(1 - 2M/x)}}$ 

$$\frac{\chi_0}{\pi M} \left(1 - \frac{2M}{\bar{x}}\right)^{-1/2}$$

$$\frac{1}{8\pi M}\chi_0 = \chi_0 T_{\rm H}$$

Hawking temperature recovered in the scaledependent holonomy

![](_page_30_Picture_17.jpeg)

![](_page_30_Picture_20.jpeg)

![](_page_30_Picture_21.jpeg)

![](_page_31_Picture_0.jpeg)

### <u>A geometric approach</u>

Brown-York quasi-local energy

 $E_{BY}(x) = x\chi_0 \left( \sqrt{1 + \lambda^2} - \sqrt{1 - \frac{2M}{x}} \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)} \right)$ 

# Black hole entropy

![](_page_31_Picture_7.jpeg)

![](_page_32_Picture_0.jpeg)

### <u>A geometric approach</u>

Brown-York quasi-local energy

$$E_{BY}(x) = x\chi_0 \left( \sqrt{1 + \lambda^2} - \sqrt{1 - \frac{2M}{x}} \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)} \right)$$

$$S(x) = \frac{8\pi x^2}{15\lambda^4} \left[ \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)} \left(3 + \lambda^2 \left(1 + \frac{3M}{x}\right) - 2\lambda^4 \left(1 + \frac{M}{x} - \frac{6M^2}{x^2}\right) \right) - \sqrt{1 + \lambda^2} \left(3 + \lambda^2 - 2\lambda^4\right) \right]$$

![](_page_32_Picture_9.jpeg)

![](_page_33_Picture_0.jpeg)

### A geometric approach

Brown-York quasi-local energy

 $E_{BY}(x) = x\chi_0 \left(\sqrt{1+\lambda}\right)$ 

$$S(x) = \frac{8\pi x^2}{15\lambda^4} \left[ \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)} \left(3 + \lambda^2 \left(1 + \frac{3M}{x}\right) - 2\lambda^4 \left(1 + \frac{M}{x} - \frac{6M^2}{x^2}\right) \right) - \sqrt{1 + \lambda^2} \left(3 + \lambda^2 - 2\lambda^4\right) \right]$$

Important points:

 $S(x) \xrightarrow[\lambda \to 0]{\chi_0 \to 1} \pi (2M)^2 = \frac{A_{\rm H}}{4} = S_{\rm BH}$ Classical limit:

![](_page_33_Picture_11.jpeg)

$$\overline{\lambda^2} - \sqrt{1 - \frac{2M}{x}} \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)}$$

![](_page_34_Picture_0.jpeg)

### A geometric approach

Brown-York quasi-local energy

 $E_{BY}(x) = x\chi_0 \left(\sqrt{1 + \lambda^2}\right)$ 

$$S(x) = \frac{8\pi x^2}{15\lambda^4} \left[ \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)} \left(3 + \lambda^2 \left(1 + \frac{3M}{x}\right) - 2\lambda^4 \left(1 + \frac{M}{x} - \frac{6M^2}{x^2}\right) \right) - \sqrt{1 + \lambda^2} \left(3 + \lambda^2 - 2\lambda^4\right) \right]$$

Important points:

Classical limit:

$$S(x) \xrightarrow{\chi_0 \to 1}_{\lambda \to 0} \pi (2M)^2 = \frac{A_{\rm H}}{4} = S_{\rm BH}$$
$$S_{\infty} = \frac{1 + 2\lambda_{\infty}^2}{-----} S_{\rm BH}$$

 $\sqrt{1+\lambda_{\infty}^2}$ 

ii. Asymptotic limit:

![](_page_34_Picture_13.jpeg)

$$\overline{\lambda^2} - \sqrt{1 - \frac{2M}{x}} \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)}$$

![](_page_35_Picture_0.jpeg)

### A geometric approach

Brown-York quasi-local energy

 $E_{BY}(x) = x\chi_0 \left(\sqrt{1 + \lambda^2}\right)$ 

$$S(x) = \frac{8\pi x^2}{15\lambda^4} \left[ \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)} \left(3 + \lambda^2 \left(1 + \frac{3M}{x}\right) - 2\lambda^4 \left(1 + \frac{M}{x} - \frac{6M^2}{x^2}\right) \right) - \sqrt{1 + \lambda^2} \left(3 + \lambda^2 - 2\lambda^4\right) \right]$$

Important points:

- A  $v \rightarrow 1$ Classical limit:
- ii. Asymptotic limit:

$$S(x) \xrightarrow{\chi_0 \to 1}_{\lambda \to 0} \pi (2M)^2 = \frac{\pi_{\rm H}}{4} = S_{\rm BH}$$
$$1 \pm 2\lambda^2$$

$$S_{\infty} = \frac{1 + 2\lambda_{\infty}^2}{\sqrt{1 + \lambda_{\infty}^2}} S_{\rm BH}$$

$$S(2M) = \left(1 + \frac{\lambda_{\rm H}^2}{2} + O\left(\lambda_{\rm H}^4\right)\right) S_{\rm BH}$$

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![](_page_35_Picture_15.jpeg)

$$\overline{\lambda^2} - \sqrt{1 - \frac{2M}{x}} \sqrt{1 + \lambda^2 \left(1 - \frac{2M}{x}\right)}$$












# Parikh-Milczek formalism









# Parikh-Milczek formalism









# Parikh-Wilczek formalism





## **Important**: The ADM mass $M_{\rm ADM}$ is held constant throughout the process

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<sup>O</sup> As the particle carries away the energy  $\omega$  reducing the black hole mass parameter  $M-\omega$ .



**Important**: The ADM mass  $M_{\rm ADM}$  is held constant throughout the process

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**Important**: The ADM mass  $M_{\rm ADM}$  is held constant throughout the process

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<sup>O</sup> As the particle carries away the energy  $\omega$  reducing the black hole mass parameter

<sup>o</sup> The tunnelling amplitude is defined as  $\Gamma \simeq \exp(-\text{Im}S)$ , where:















**Important**: The ADM mass  $M_{\rm ADM}$  is held constant throughout the process

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<sup>O</sup> As the particle carries away the energy  $\omega$  reducing the black hole mass parameter

<sup>o</sup> The tunnelling amplitude is defined as  $\Gamma \simeq \exp(-\text{Im}S)$ , where:

$$S = \int_{M}^{M-\omega} \left(-d\omega'\right) \int_{2(M-\omega)-\epsilon}^{2(M-\omega)+\epsilon} \frac{dx}{\left(1-\sqrt{\frac{2(M-\omega')}{x}}\right)}$$













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The ADM mass  $M_{\rm ADM}$  is held constant throughout the process

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• The black hole entropy measured by an asymptotic observer can be read from the tunnelling rate  $\Gamma \simeq \exp\left(\Delta S_{\rm BH}\right)$  which leads to

 $S_{\rm BH} = 4\pi M^2$ 































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## The point-splitting momentum energy tensor is given by [Fabbri, Navarro-Salas, Olmo'04; Agullo, Navarro-Salas, Olmo,





$$\langle 0_x \left| N_{l\Omega} \right| 0_x \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} \mathrm{d}y^{-} \int_{-\infty}^{\infty} \mathrm{d}y^{'-} e^{-i\Omega(y^{-}-y^{'-})} \langle 0_x \left| : \partial_{y^{-}} \phi_R(y^{-}) \partial_{y^{'-}} \phi_R(y^{'-}) : \left| 0_x \right\rangle$$

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to the preservation of the location of the horizon, a typical logarithmic relation  $u(v) = v_0 - \kappa^{-1} \ln \kappa (v_0 - v)$ 

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The (right-moving) scalar field  $\phi_{\rm R}(y^-)$  expanded in terms of the positive ( $e^{-i\omega y^-}$ ) and negative ( $e^{i\omega y^-}$ ) frequencies. Due





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**Grey-body** factor











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## $^{\rm O}$ Low-frequency modes $\omega \ll T_\infty$ $\longrightarrow$ $M\omega \ll 1$ [Page'76; Unruh'76; Harmark, Natario, Schiappa'10, etc]

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• The radial propagation reduces to the Schrödinger equation

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$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x_*^2} + \omega^2\right] \left(x\Phi(x)\right) = 0$$







The approximation machinery is analysed by dividing the space into:

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- Region III

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$$T_l(\omega) = \left| \frac{J_{\mathcal{I}^+}}{J_{H^-}} \right|$$



$$\lambda(x) = \sqrt{\Delta/x^2}$$









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- The holonomy correction reduces the absorption rates.
- <sup>o</sup> The classical limit is recovered when  $\Delta \rightarrow 0$ .







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[Parikh & Pereira'24]

2. Thermal stabilization  $\rightarrow M_{\rm r} \approx 0.15 \sqrt{\Delta}$ 

[IHB, Bojowald, Brahma, & Duque'25]

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## Black to while hole transition



## Thermal stabilization is preceded by gravitational destabilization $\rightarrow$ The repulsive QG effect

$$M_f \approx 3.8\sqrt{\Delta}$$





## CONCLUSION





## Summary

- <sup>O</sup> Consistent (anomaly-free and covariance) vacuum solution for arbitrary holonomy function  $\lambda(x)$ .
- We regained the thermal Hasking distribution and universality of black hole absorption rates to leading order.
- <sup>o</sup> The Hawking temperature is recovered only under the condition that  $\lambda(x)$  decreases monotonically.
- The Brown-York quasi-local formalism and the tunnelling approach yield consistent results for black hole entropy. • The black hole evaporation process slows down due to the holonomy correction. The gravitational instability kicks in before reaching a remnant stage, favouring the black to white hole transition according to the hierarchy  $M_{\rm f} > M_{\rm c} > M_{\rm r}$ . In progress (coming soon)
- Include backreaction effects consistently and more quantitatively.
- Deformed Minkowski spacetime. Maximal acceleration

