New results for the shock wave model

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FF, Husain, Wilson-Ewing, PRD 109 (2024) 084052, *arXiv:2312.02032v2*

FF, *arXiv:2502.03003*

FF, Mehmood, *arXiv:2505.01846v1*

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Motivation

Effective models inspired by LQG are able to describe stellar collapse through effective equations of motion.

A model LQC-based developed in the past years predicts a bounce, and a post-bounce shock wave dynamics.

[Husain, Kelly, Santacruz, Wilson-Ewing, 2022]

QUESTIONS:

- Is the shock wave dynamics unique?
- Is the physical outcome of the model reasonable?
- Are there more realistic alternatives?

Dust collapse in LTB gauge

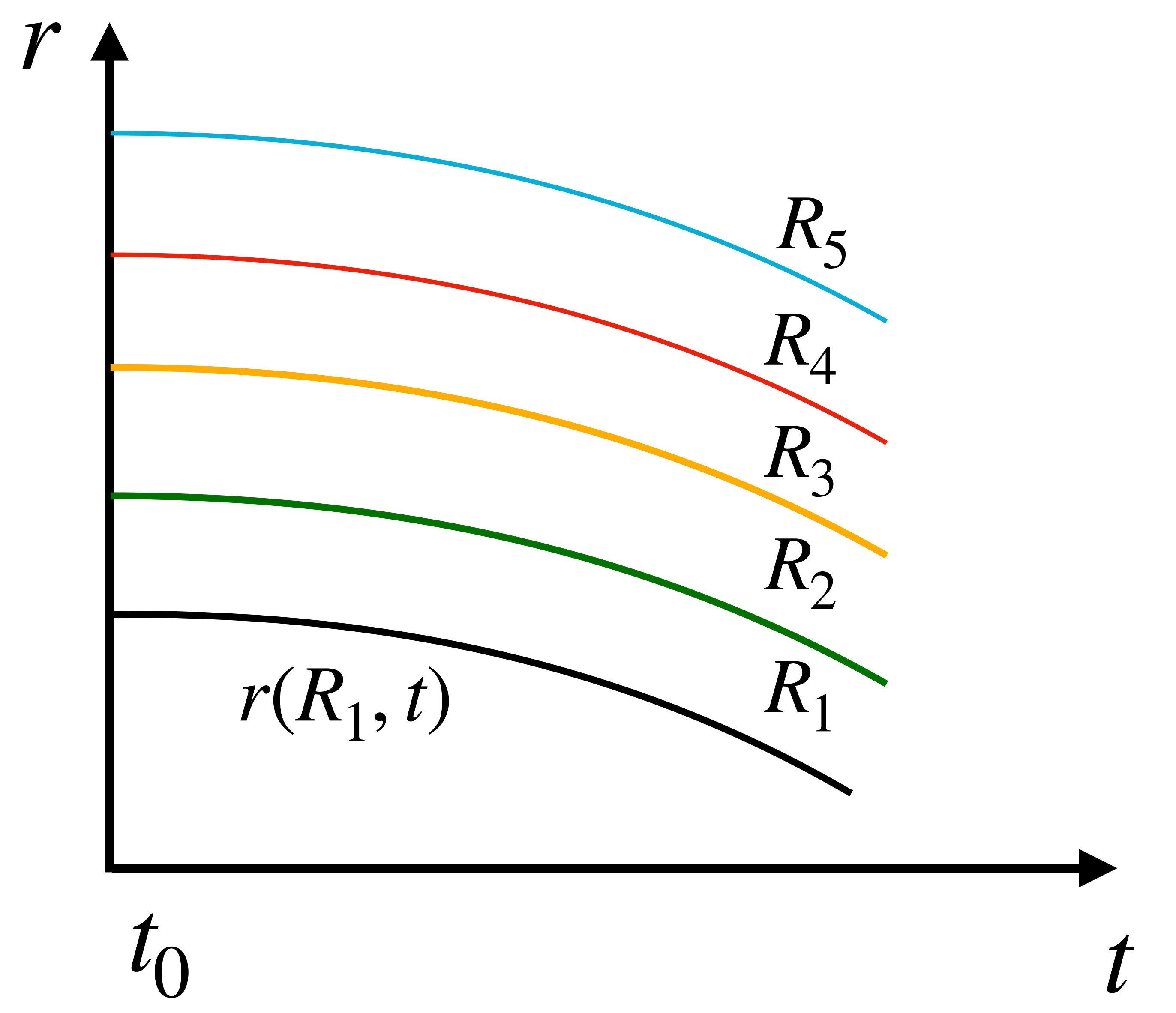
The metric describing generic dust evolution in LTB coordinates reads:

$$\boxed{ds^2 = -\,dt^2 + \frac{\left[\partial_R r(R,t)\right]^2}{1+\varepsilon(R)}dR^2 + r(R,t)^2 d\Omega^2}$$

This metric describes **both** the matter and vacuum region (if exists) of the spacetime.

Interpretation: we can imagine to divide the spatial part of the manifold in spherical shells parametrized by the radial coordinate R.

The spacetime evolution is described through the evolution of the **areal radius r of the shell R at time t.**



Effective star collapse in LTB coordinates

The LTB effective equations for spherically symmetric dust collapse (marginally bound case $\varepsilon(R) = 0$):

$$\boxed{\left(\frac{\dot{r}}{r}\right)^2 = \frac{2Gm}{r^3}\left(1 - \frac{2\Delta Gm}{r^3}\right),}$$

[Giesel, Liu, Singh, Weigl, 2023]

where Δ is the area gap in LQG: $\Delta \sim l_P^2$ and $m(R)$ is the mass function and is fixed by the initial energy density profile.

The general solution of the EOM:

$$\boxed{r(R,t) = \left[2Gm(R)\right]^{1/3}\left[\frac{9}{4}\left(t - \alpha(R)\right)^2 + \Delta\right]^{1/3}}$$

$\Longrightarrow$ Each shell R should bounce at: $t = \alpha(R)$.

But: LTB equations break down when the solution develops *shell crossing singularities*.

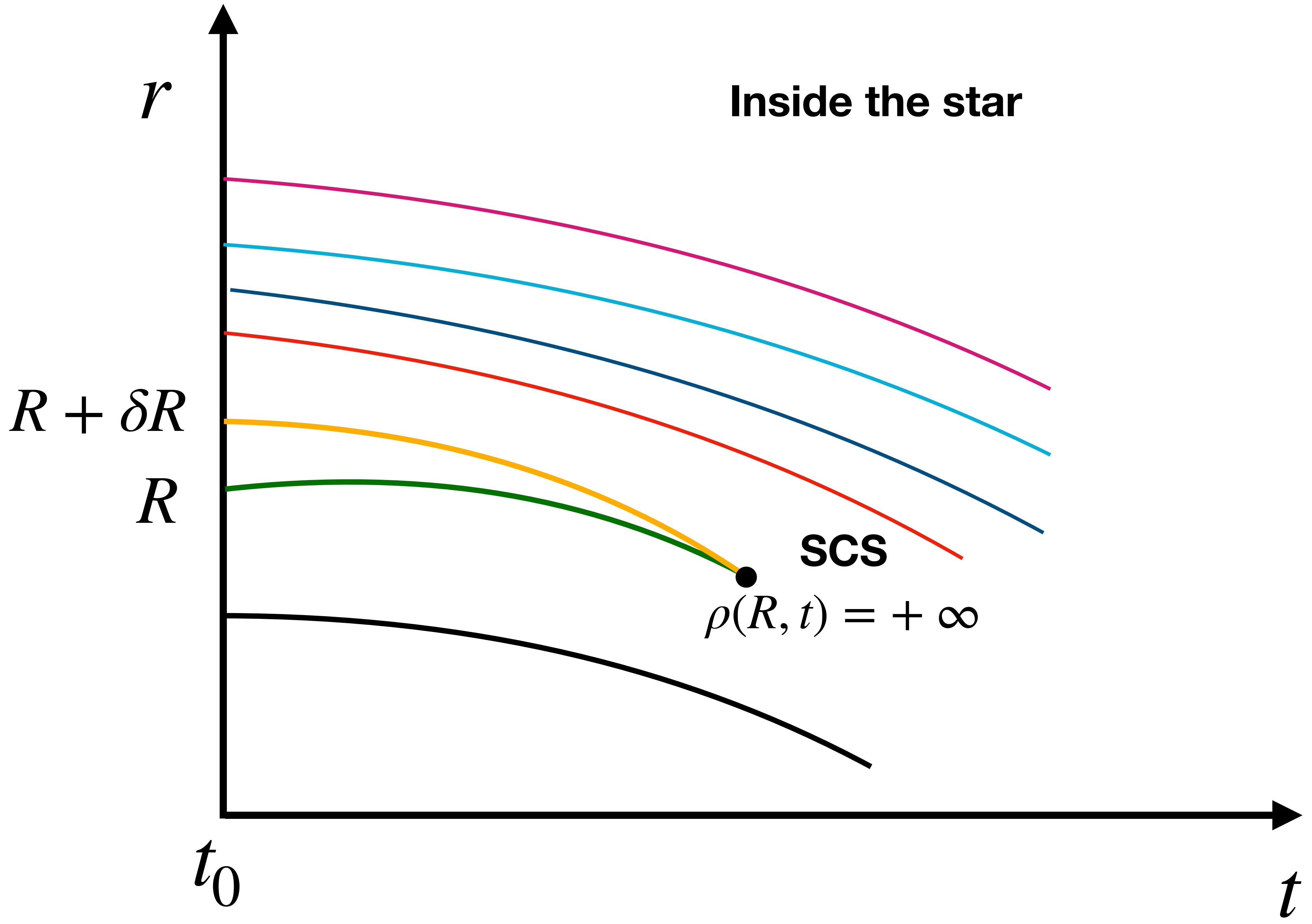
Shell-crossing singularities in generic LTB space-times

The energy density is given by:

$$\rho(R,t) = \frac{\partial_R m(R)}{4\pi r^2 \partial_R r(R,t)}$$

If :
- $\partial_R r(R,t) = 0$ for some R at some time t (shell-crossing)
- For the same R, $\partial_R m(R) \neq 0$ (shell-crossing in the matter region)

$$\implies \rho(R,t) = +\infty, \quad R_{\mu\nu}g^{\mu\nu} = +\infty.$$



A shell-crossing singularity (SCS) forms: it is a physical *weak* singularity.

In classical GR, many initial configurations develop SCS, but one can choose suitable initial profiles that don't develop such singularities *[Hellaby, Lake, 1984]*.

Can SCS be avoided in effective dust collapse?

Theorem: *"for the marginally bound case, a shell-crossing singularity will necessary form at some R if the initial energy density profile is non-negative, continuous, of compact support and for which $m(R)$ is not everywhere zero."* *[FF, Husain, Wilson-Ewing, 2023]*

Additionally, if $\partial_R \rho(R, t_0) \leq 0$, the time at which shell-crossing singularities form will be:

[FF, 2025]

$$\boxed{t_{bounce}(R) < t_{SCS}(R) < t_{bounce}(R) + \frac{2}{3}\sqrt{\Delta}}$$

Spacetime dynamics beyond SCS formation

LTB coordinates cannot be used to extend the dynamics beyond SCS formation.

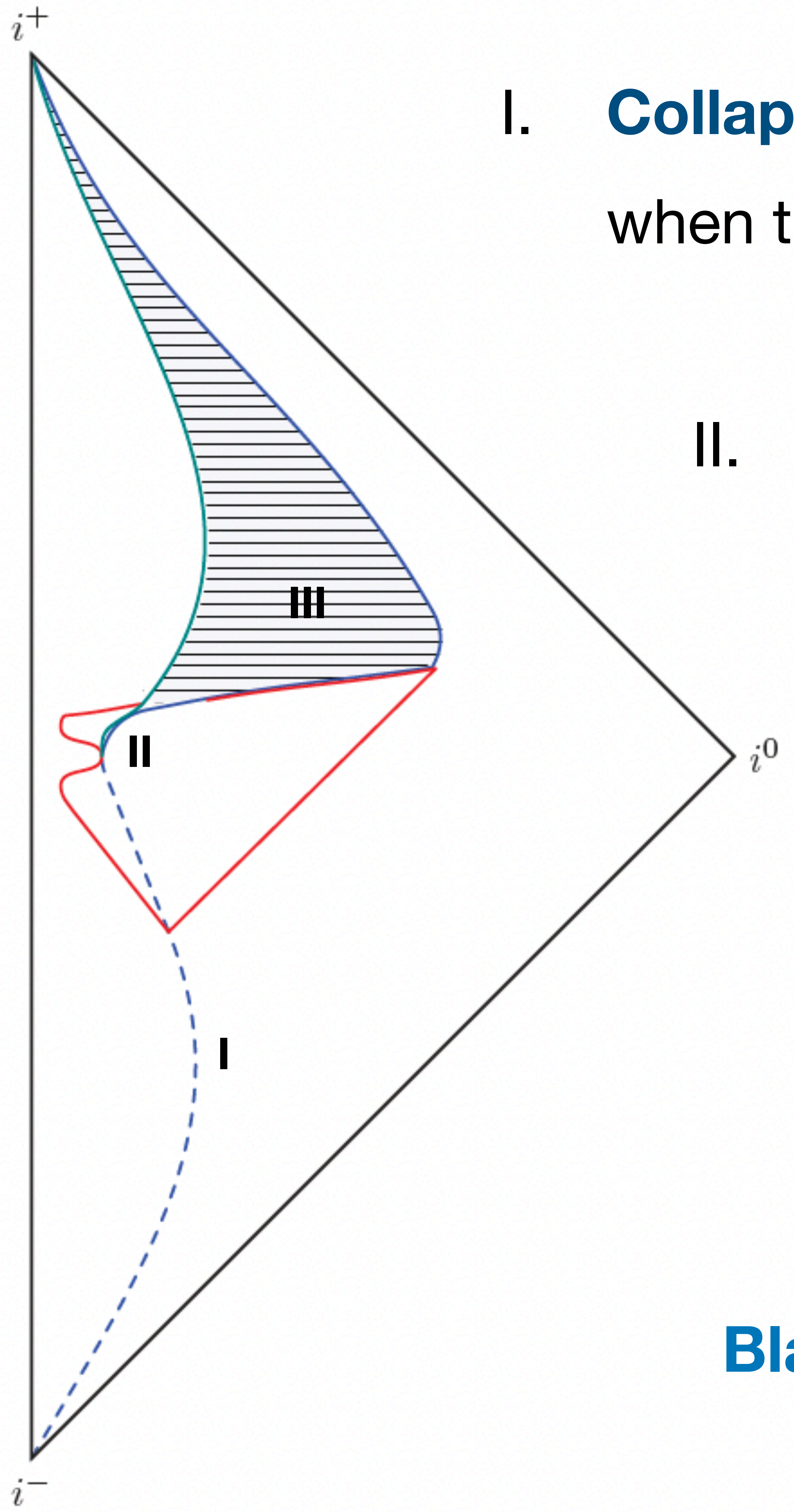
However:

LTB Coordinates		**PG Coordinates**
Shell-crossing singularity in decoupled ODEs	$=$	Characteristic crossing in the PDE/ discontinuity in the extrinsic curvature

[FF, Husain, Wilson-Ewing, 2023]

The dynamics beyond characteristic crossing in a PDE can be studied by using the integral form of the equations (this is commonly done to study shockwaves in fluid dynamics, but also for SCS dynamical extension in classical GR *[Nolan, 2003]*).

Penrose diagram and black hole life-time



I. **Collapse phase**: the energy density of the star progressively increases and its volume decreases; when the horizon forms the star becomes as a black hole.

II. **Bouncing phase**: when the energy density of the core becomes planckian, the shells of the core bounce and crush the collapsing shells of the tail.

$\rightarrow$ A *shell-crossing singularity* forms, together with a *discontinuity* in the gravitational field.

III. **Shockwave phase**: the whole matter content of the star rapidly concentrates in a thin shell moving outward together with the discontinuity. When the shell reaches the outer horizon, the black hole disappears.

Black hole life-time: $$\boxed{T \sim \frac{8\pi M^2}{3m_p}}$$ *[Husain, Kelly, Santacruz, Wilson-Ewing, 2022]*

But . . .

The spacetime extension beyond shell-crossing singularity through weak solutions suffer of two important problems:

- Weak solutions, differently from strong solutions, suffer of non-uniqueness.
- Shocks dynamics in many weak solutions for stellar collapse produce superluminal behavior.

Non-uniqueness

A PDE written as a conservation law or balance law, takes the form:

$$\partial_t b(r,t) + \partial_r (m(b,r)) = f(b,r)$$

Where $f(b,r)$ does not contain spatial derivatives of the field. When the integral equation develops a shock, its velocity is:

$$\boxed{\frac{dL}{dt} = \frac{[m(b,r)]^+_-}{[b]^+_-}}$$

If we multiply the original PDE by functions of the field, and rewrite in the form of a conservation or balance law, we get:

$$\frac{dL}{dt} = \frac{[f(v(b),r)]^+_-}{[v(b)]^+_-} \neq \frac{[m(b,r)]^+_-}{[b]^+_-} \longrightarrow \text{Non-uniqueness}$$

Non-uniqueness of the shockwave dynamics

For the class of different integral forms of the equations considered, black holes life-times differ significantly:

$$T_{BH}(R_S|n) \sim \frac{2\pi R_S^{n+2}}{3\gamma^{n+1}2^n\Delta^{\frac{n+1}{2}}}\binom{n}{n/2} \qquad \textit{for} \quad n \geq 0 \quad \textit{even}$$

$$T_{BH}(R_S|n) \sim \frac{R_S^{n+2}\Gamma\left[\frac{-n}{2}\right]\Gamma\left[\frac{n+1}{2}\right]}{3\gamma^{n+1}2^{n-2}\Delta^{\frac{n+1}{2}}\sqrt{\pi}} \qquad \textit{for} \quad n > 0 \quad \textit{odd}$$

[FF, 2025]

“Continuous” weak solution

The most reasonable way to solve the first problem is to consider a weak solution that has an induced continuous metric at SCS/shock. *[Husain, Mehmood, 2025; Liu, Qu, 2025]*

This assumption allows to get a well-defined shock velocity, which helps to face the superluminal issue.

The induced metric at the shock, as seen from the two sides, is:

$$ds_{\pm}^2 = -\left(1 - (\dot{L} + N_{\pm}^r)^2\right) dt^2 + L(t)^2 d\Omega^2$$

And requiring non-trivial continuity leads to:

$$\boxed{\frac{dL(t)}{dt} = -\frac{N_+^r + N_-^r}{2}}$$

Then, one constructs the integral form of the equation that gives the previous as shock velocity.

Collapsing classical thin shell

The induced metric on the shell surface, using the velocity derived before, is:

$$\boxed{ds^2_\Sigma = -\left(1 - \frac{R_S}{4L(t)}\right) dt^2 + L(t)^2 d\Omega^2}$$

When the shock reaches $L(t) = R_S/4$, the induced metric changes signature:

- Only a dust particle can stay on the shock for $L > R_S/4$ (good).
- Only a null particle can stay on the shock for $L = R_S/4$ (bad).
- Only a spacelike particle can stay on the shock for $L < R_S/4$ (very bad).

$\longrightarrow$ unphysical dynamics

Where did the construction go wrong?

We can relax the continuity of Painlevé-Gullstrand time at the shock *[FF, Mehmood, 2025]:*

$$ds^2 = \left[-1 + \left(\frac{dL}{dt^+} + N^r_+\right)^2\right] dt_+^2 + L^2 d\Omega^2$$

$$ds^2 = \left[-1 + \left(\frac{dL}{dt^-} + N^r_-\right)^2\right] dt_-^2 + L^2 d\Omega^2$$

Two different time dilations

Two different PG-time flowings

This is physically motivated: two observers in different spacetimes measure different time dilations at the shock, and consequently their times flow differently.

If then we require then signature preservation $\longrightarrow$ First Israel junction condition

Thin shell dynamics: Israel vs weak solution

By imposing the relation between the stress tensor at the shell and the jump in the extrinsic curvature, we get the Israel dynamics.

WEAK DYNAMICS

The induced metric is always continuous.
(solves the non-uniqueness problem)

The induced metric changes signature.
(does not solve the superluminal problem)

The PG time is forced to be continuous at the shock by construction.

ISRAEL DYNAMICS

The induced metric is always continuous.
(solves the non-uniqueness problem)

The induced metric is always timelike.
(solves the superluminal problem)

The PG time becomes discontinuous at the shock.

[FF, Mehmood, 2025]

⟶ The Israel approach provides the most reasonable dynamics for classical thin shell collapse.

What about the other weak solutions?

When a SCS forms (in the classical or effective theory):

Shell-crossing singularity $\longrightarrow$ discontinuity in the mass function $\longrightarrow$ different PG time flowing

Since the integral approach assumes implicitly continuity of the PG time at the shock, it is not reliable also for:

- any initial profile that develops SCS within classical GR.
- effective theories that admit SCS.
- any kind of integral equations, which have continuous induced metric, or not.

A formulation of the shock dynamics through the Israel approach is needed.

Conclusions

Effective stellar collapse inspired by LQG/LQC is characterized by:

- Bounce of the stellar core, when the energy density becomes planckian.
- Formation of shell-crossing singularities.

The dynamics beyond SCS can be extended through weak solutions, but:

- Non-uniqueness of the weak solutions.
- Superluminal behavior of the shock.

Solution:

The shock dynamics should be studied through the Israel formalism, that:

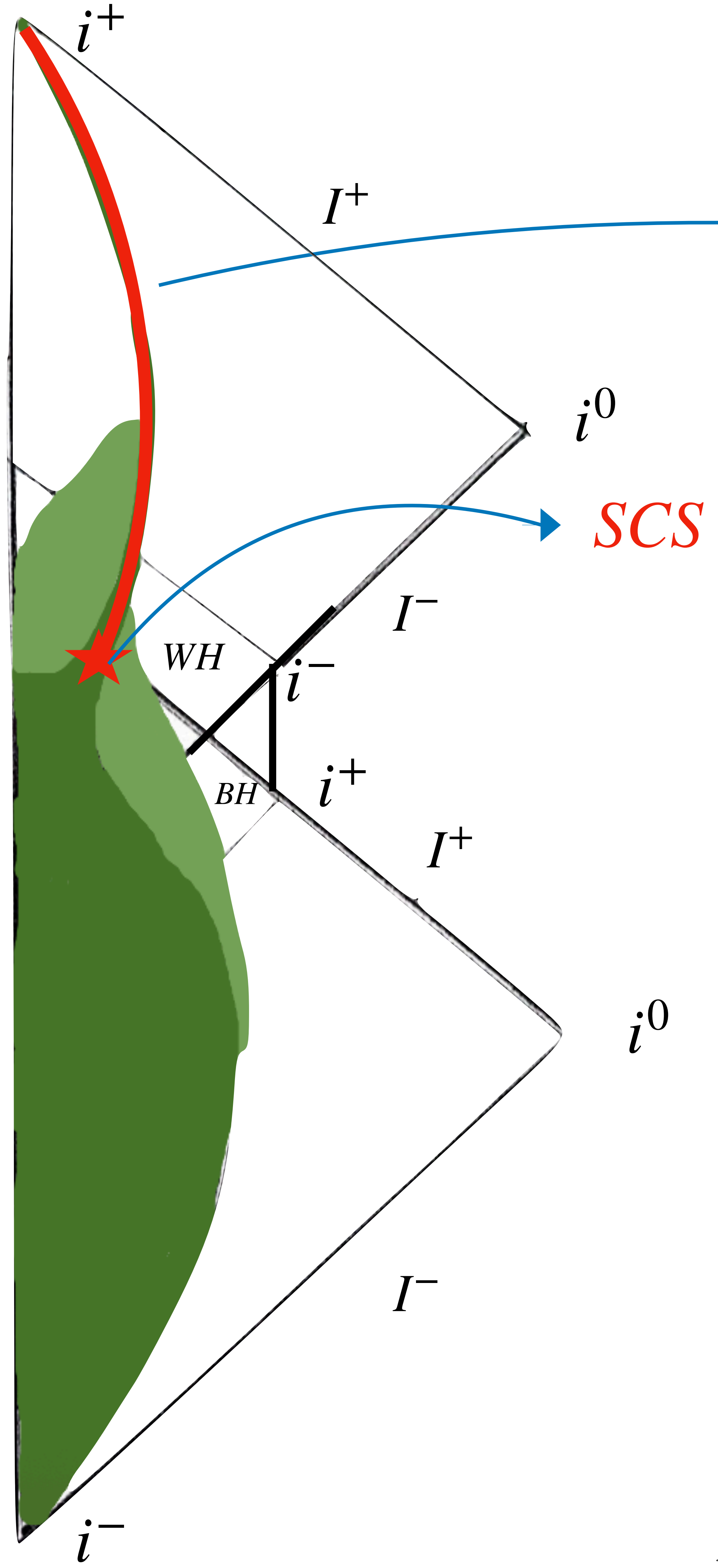
- Guarantees subluminal behavior for a shock (in the dust case).
- Guarantees continuity of the induced metric at the shock.
- Allows the Painlevè-Gullstrand time to be discontinuous at the shock.

Future direction

- Implementing the Israel junction conditions dynamically, to study them systematically in cases where the shocks are developed during the dynamics.

Thanks for your attention!

Backup slide 2



$$\frac{dL}{d\tau} \sim \sqrt{\frac{R_S}{2L}\left(\frac{R_S}{8L}+1\right)}$$

The dynamics:

- is always timelike.
- holds qualitatively for any mass M.
- holds qualitatively (in the post-bounce phase) for any initial profile, included the effective thin shell collapse.