There & back again — overlapping dofs in quantization & mereology

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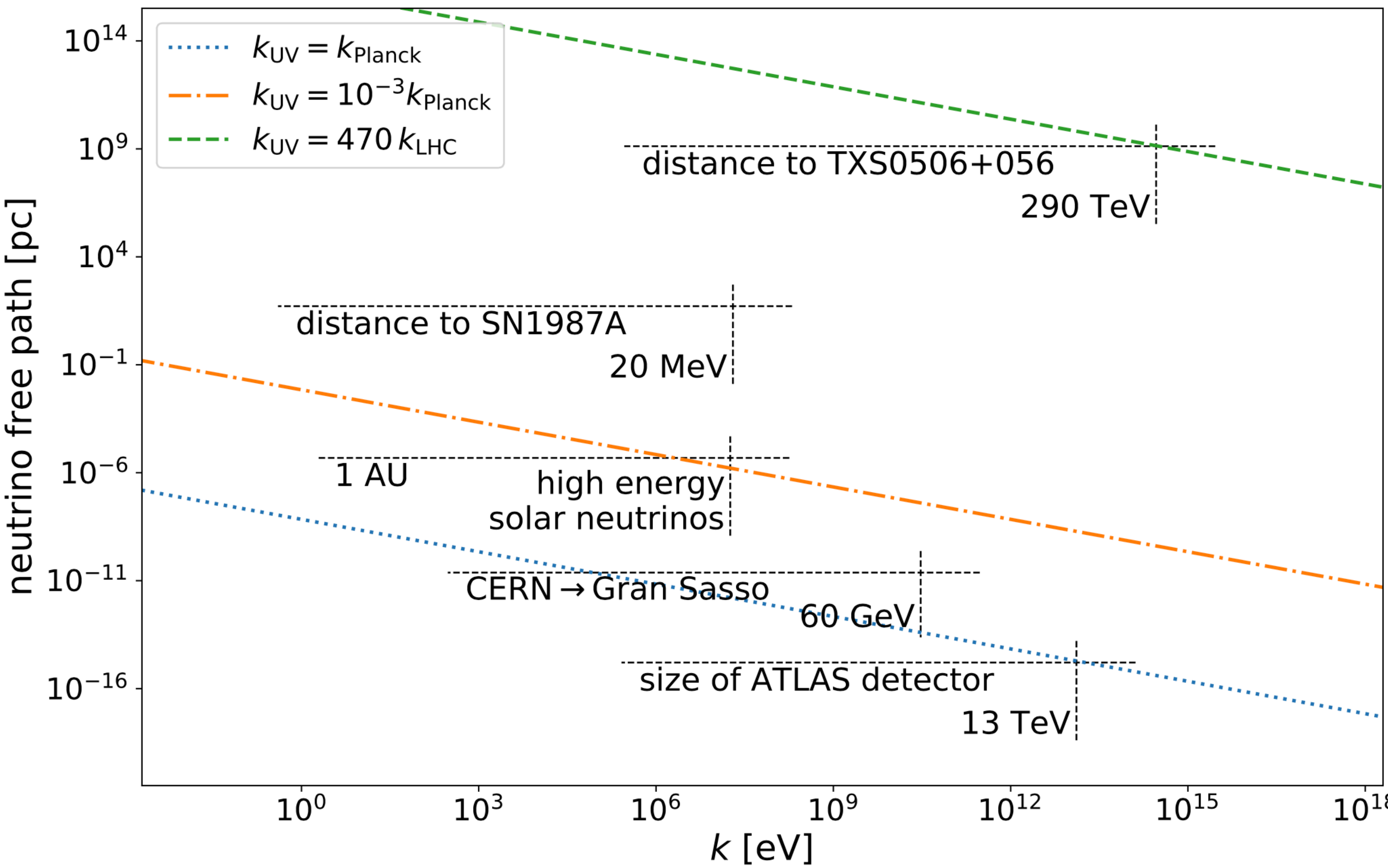
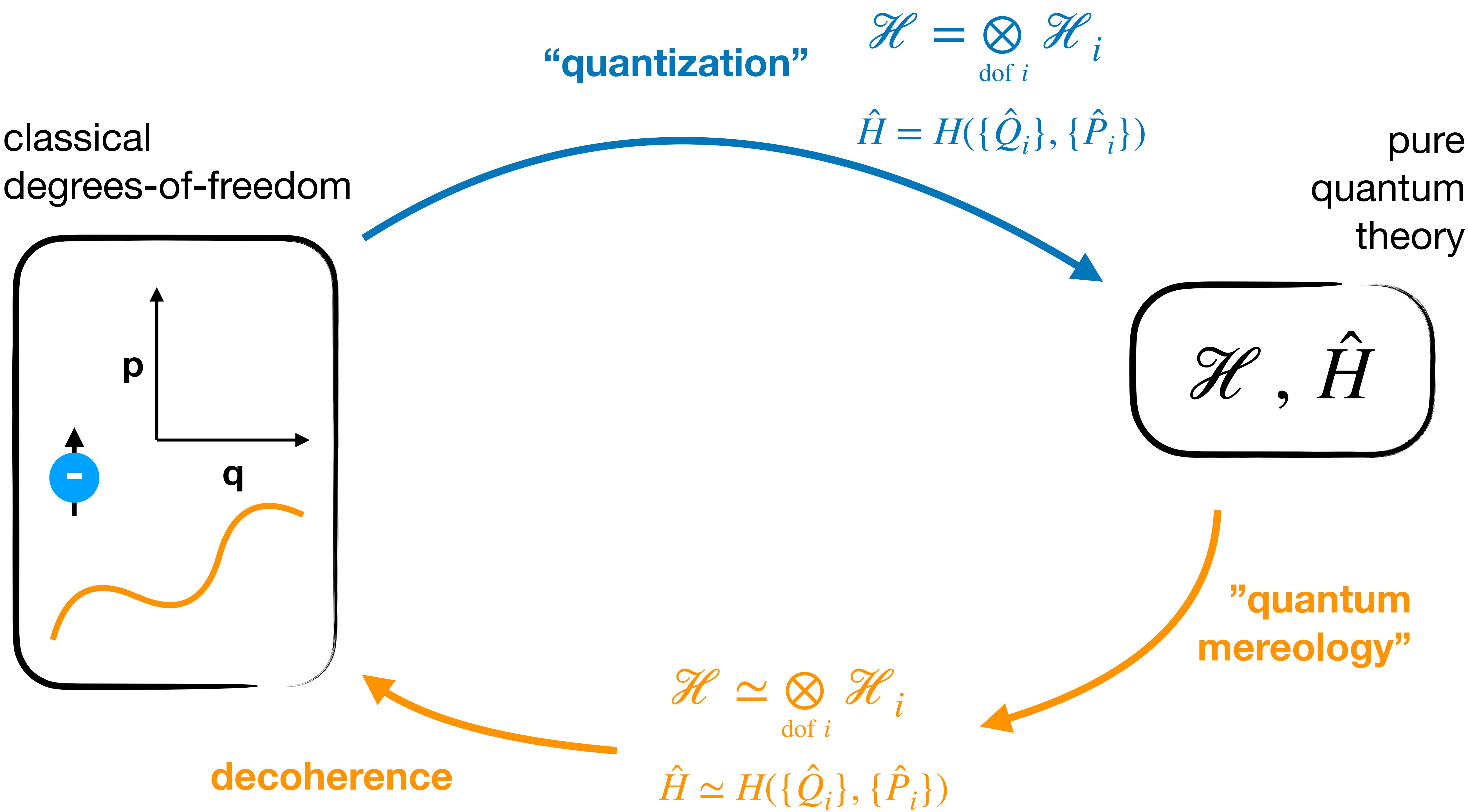
Fraunhofer-Schwarzschild Fellow @ LMU Munich
FAU^2 Quantum Gravity across scales, 20 May 2025

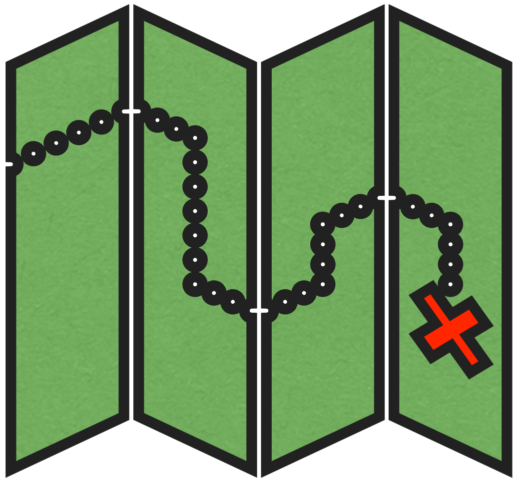
Work with:
Varun Kushwaha, Kristina Giesel, ChunJun Cao, Sean Carroll, Gong Cheng, Ashmeet Singh, Nicolas Loizeau, Marin Girard



arXiv:2402.11016
Friedrich++ in prep
Kushwaha++ in prep

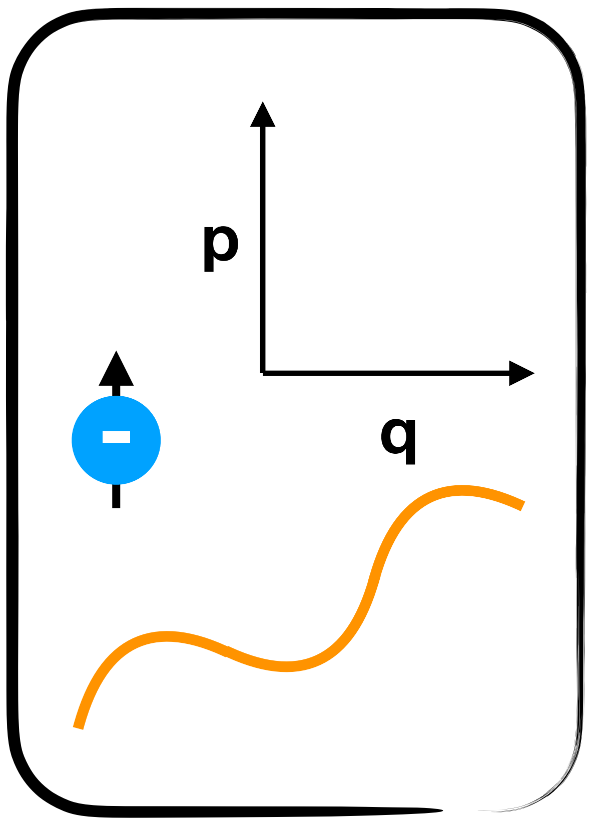


classical
degrees-of-freedom
p
q
"quantization"
$\mathscr{H} = \bigotimes_{\text{dof } i} \mathscr{H}_i$
$\hat{H} = H(\{\hat{Q}_i\}, \{\hat{P}_i\})$
pure
quantum
theory
$\mathscr{H}\ ,\ \hat{H}$
"quantum
mereology"
$\mathscr{H} \simeq \bigotimes_{\text{dof } i} \mathscr{H}_i$
$\hat{H} \simeq H(\{\hat{Q}_i\}, \{\hat{P}_i\})$
decoherence



Plan for this talk

A) Do this for quantum fields in a way motivated by
- holographic entropy bound
- operator dressing



$$\mathscr{H} \,,\, \hat{H}$$

$$\mathscr{H} \simeq \bigotimes_{\text{dof } i} \mathscr{H}_i$$

$$\hat{H} \simeq H(\{\hat{Q}_i\}, \{\hat{P}_i\})$$

B) Explore whether features of **A)** can be motivated / re-discovered in mereology

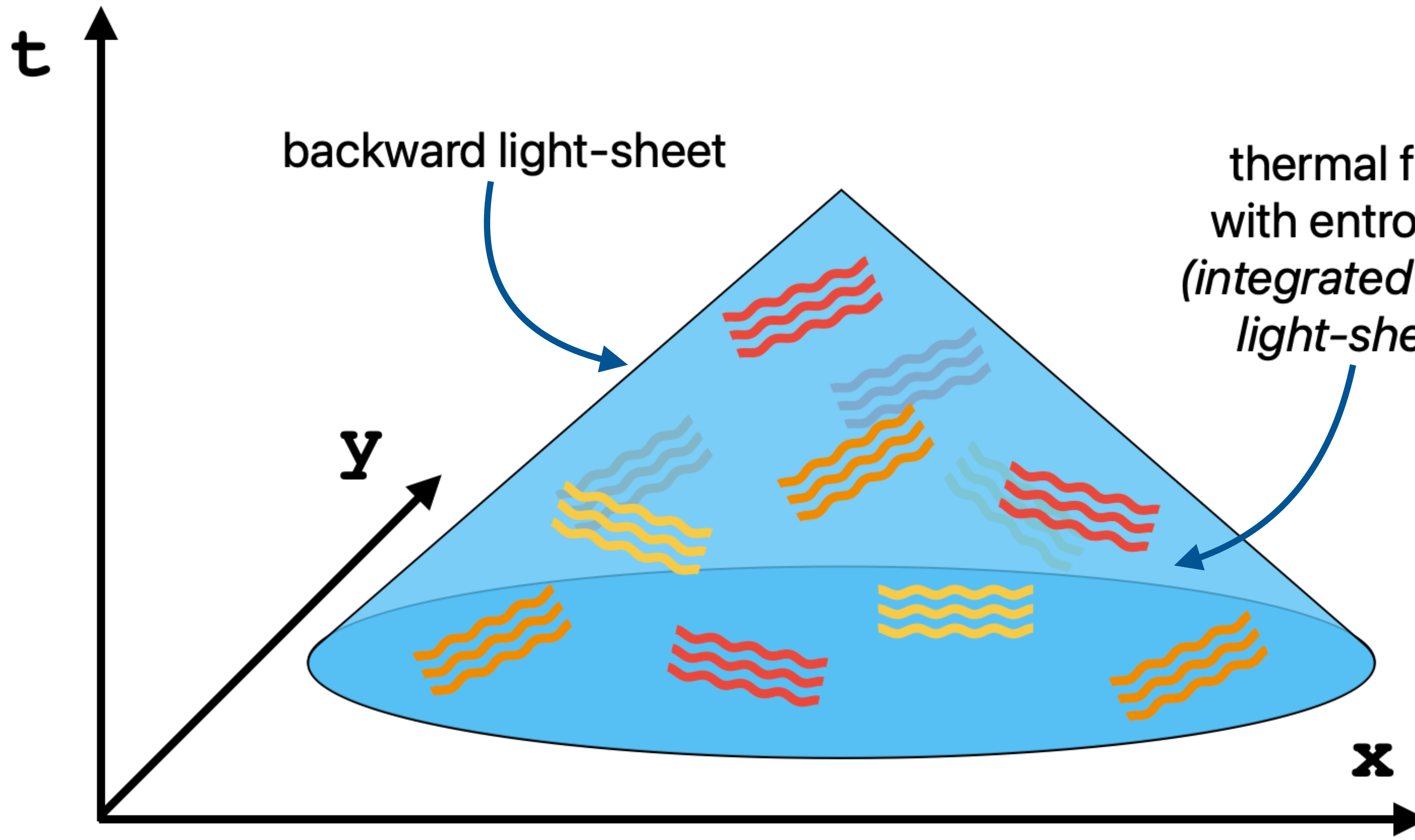
quantization with
overlapping dofs
/
QFT that satisfies
holographic entropy
bounds

Motivation I:
The holographic principle (not AdS/CFT)

How much entropy can you amass on your backward light-cone (before gravity cuts that light-cone with a horizon) ?

Answer: covariant holographic bound (“Bousso bound” ; arxiv:0203101):

$$S < \frac{A}{4}$$



- Consequence when building a quantum theory for matter on that sheet:

Hilbert space should have finite dimension

$$\log \dim \mathscr{H} \equiv S_{\max} < \frac{A}{4}$$

This is in stark contradiction with standard quantum field theory where $S_{\max} \propto$ volume!

Motivation II: operator dressing

Consider scalar field φ with local U(1) gauge symmetry;

$\Rightarrow$ $\varphi(\mathbf{x})$ is not gauge invariant / not an observable!

But the dressed operators

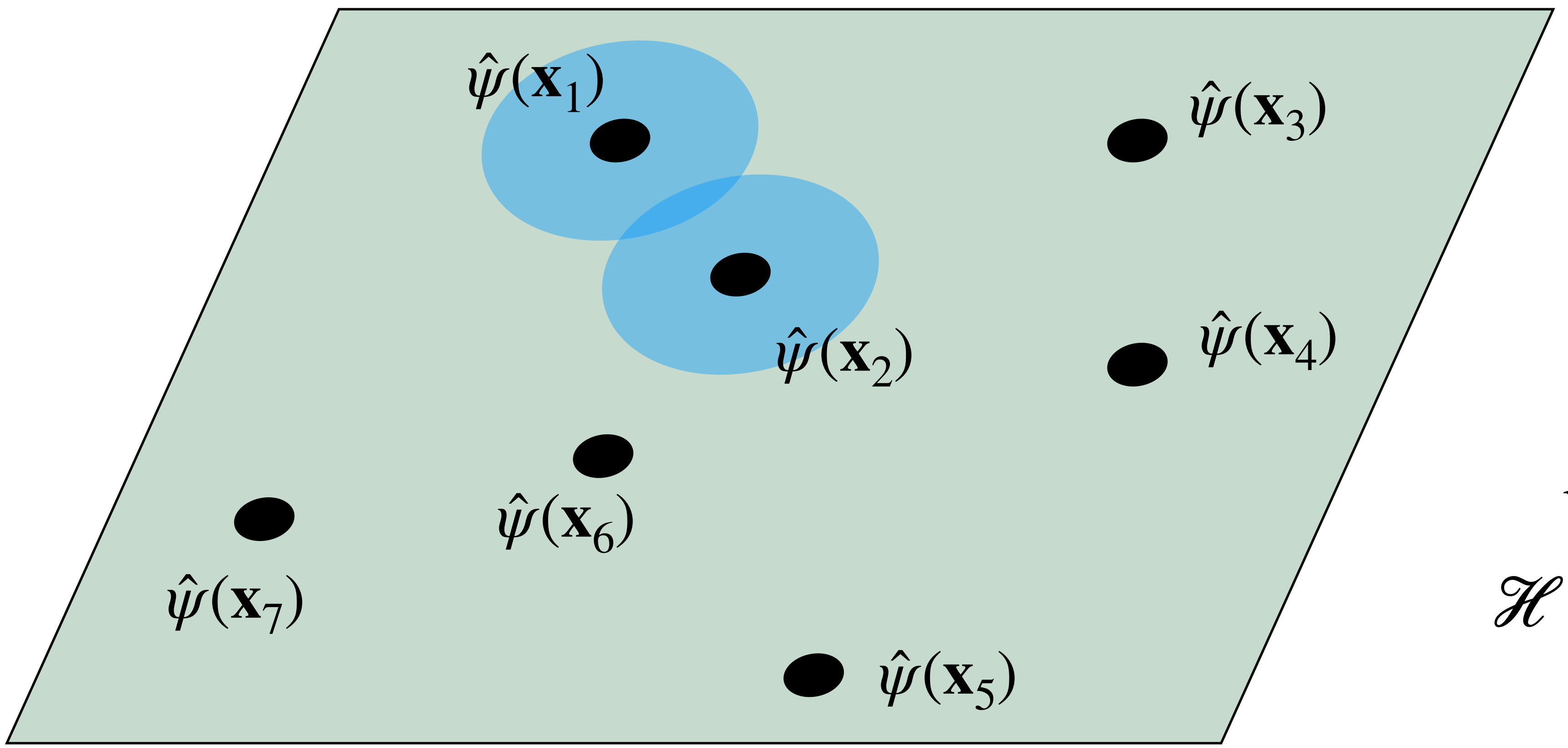
$$\Phi(x) \equiv \exp\left(iq\int \mathrm{d}^4x' f^\mu(x,x')A_\mu(x')\right)\varphi(x)$$

are gauge invariant as long as $\partial_\mu f^\mu(x,x') = \delta^{(4)}(x-x')$.

(see e.g. https://arxiv.org/abs/1507.07921 , including analogous procedure for diff-invariance)

Motivation II:
operator dressing

dressed field in flat space



$$\begin{aligned} \mathscr{H} &\simeq \mathscr{H}_{\mathbf{x}_1} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_1)} \\ &\simeq \mathscr{H}_{\mathbf{x}_2} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_2)} \\ &\simeq \mathscr{H}_{\mathbf{x}_3} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_3)} \\ &\simeq \ldots \end{aligned}$$

but :

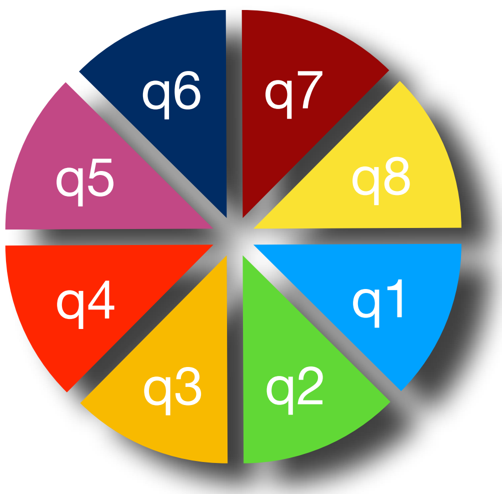
$$\mathscr{H} \not\simeq \mathscr{H}_{\mathbf{x}_1} \otimes \mathscr{H}_{\mathbf{x}_2} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_1,\mathbf{x}_2)}$$

Gravitational dressing causes non-canonical equal-time commutators!
("overlaps")

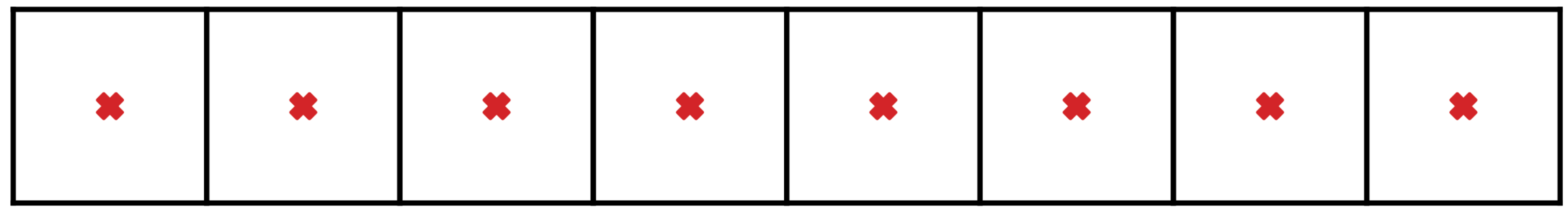
At same time: systems with non-canonical commutators may be embedded in reduced Hilbert space!

What are “overlapping dofs”?

Inspiration from https://arxiv.org/abs/1701.01062 who considered imperfect quantum computer with “overlapping qubits”.



Perfect (i.e. non-overlapping) set of 8 qubits:

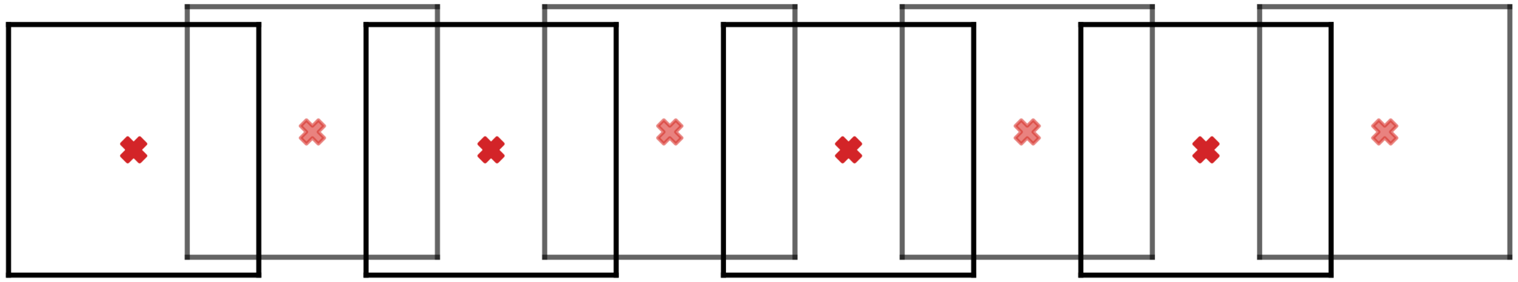


$$\mathscr{H} = \mathscr{H}_{q_1} \otimes \mathscr{H}_{q_2} \otimes \mathscr{H}_{q_3} \otimes \ldots \quad ; \quad \dim \mathscr{H} = 2^8$$

In particular, measurements on different qubits commute: $\left[\sigma_{Z,q_i} \;,\; \sigma_{Z,q_j}\right] = 0$

What are "overlapping dofs"?

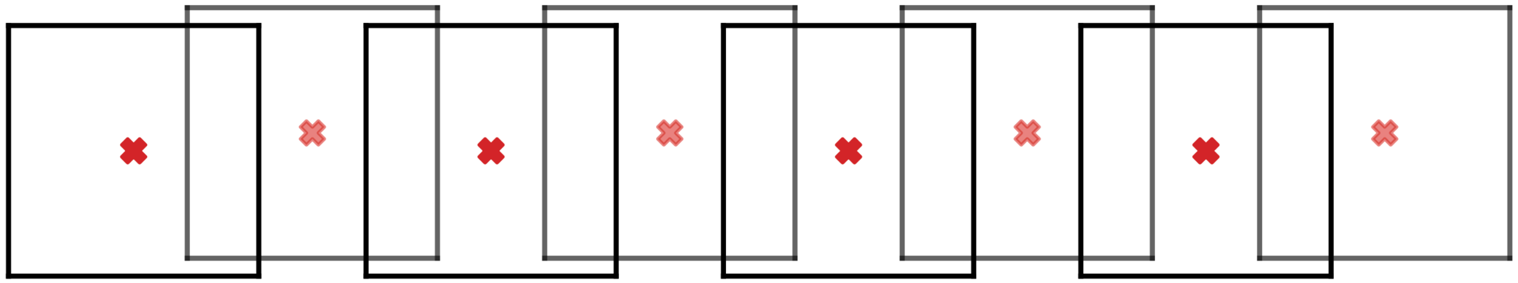
Instead, consider imperfect (i.e. overlapping) set of 8 qubits:



In particular, measurements on different qubits don't commute:

$$\left[\sigma_{Z,q_i}\ ,\ \sigma_{Z,q_j}\right] \neq 0$$

Instead, consider imperfect (i.e. overlapping) set of 8 qubits:

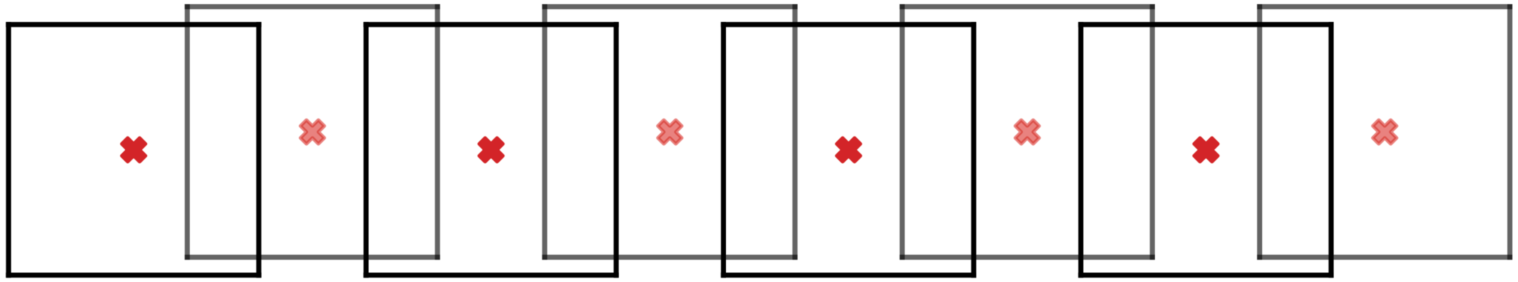


In particular, measurements on different qubits don’t commute:

$$\left[\sigma_{Z,q_i}\ ,\ \sigma_{Z,q_j}\right] \neq 0$$

Side effect: these qubits now fit into smaller Hilbert space!
(i.e. $\dim \mathscr{H} = 2^n < 2^8$)

Instead, consider imperfect (i.e. overlapping) set of 8 qubits:



In particular, measurements on different qubits don’t commute:

$$\left[\sigma_{Z,q_i}\ ,\ \sigma_{Z,q_j}\right] \neq 0$$

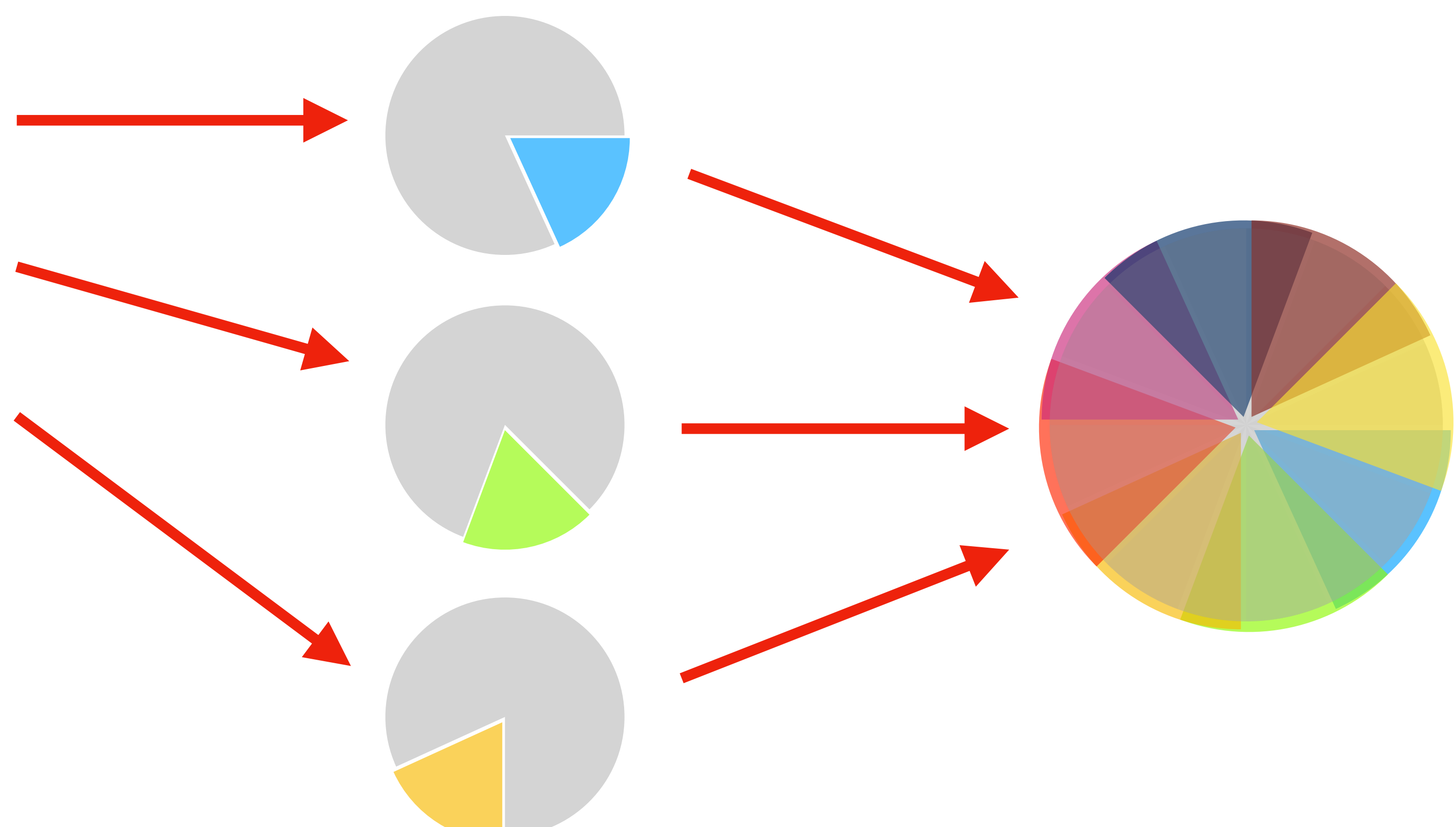
$$\mathscr{H} \neq \mathscr{H}_{q_1} \otimes \mathscr{H}_{q_2} \otimes \mathscr{H}_{q_3} \otimes \dots \quad ; \quad \dim \mathscr{H} = 2^n < 2^8$$

but : $\mathscr{H} \simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{r_1}$

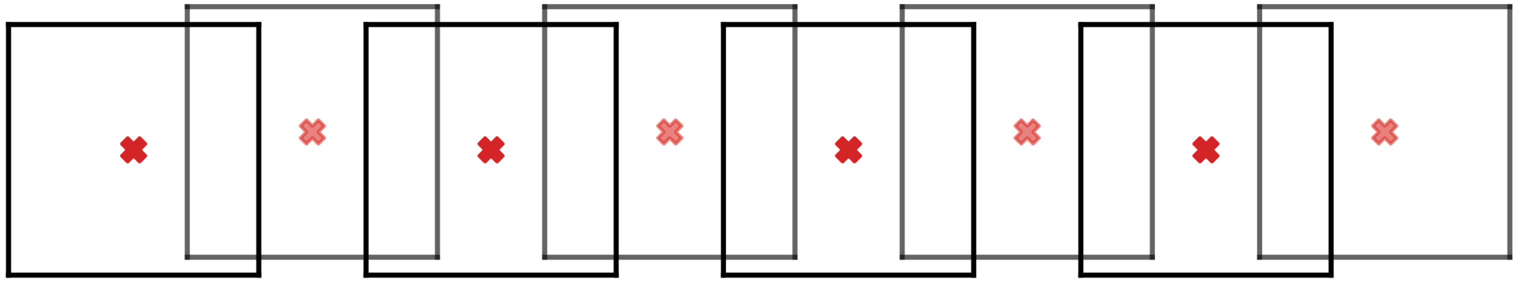
$\simeq \mathscr{H}_{q_2} \otimes \mathscr{H}_{r_2}$

$\simeq \mathscr{H}_{q_3} \otimes \mathscr{H}_{r_3}$

$\dots$



Instead, consider imperfect (i.e. overlapping) set of 8 qubits:



In particular, measurements on different qubits don’t commute:

$$\left[\sigma_{Z,q_i}\ ,\ \sigma_{Z,q_j}\right] \neq 0$$

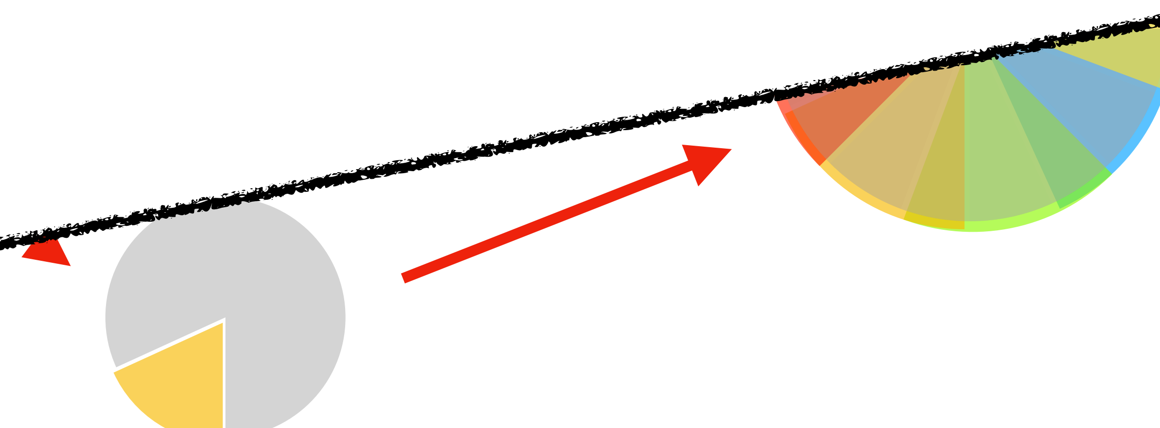
$$\mathcal{H} \neq \mathcal{H}_{q_1} \otimes \mathcal{H}_{q_2} \otimes \mathcal{H} \ldots$$

b

$\Rightarrow$ What number of qubits N_q can we squeeze into Hilbert space with $\dim \mathcal{H} = 2^n \ (n < N_q)$ while still keeping $||[\sigma_{Z,q_i}\ ,\ \sigma_{Z,q_j}]|| < \epsilon$?

Answer (using Johnson-Lindenstrauss theorem) : **exponentially many!**

$$N_q \sim \exp(\epsilon^2 n)$$



Classical and Quantum Gravity

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Holographic phenomenology via overlapping degrees of freedom

Oliver Friedrich[1,2,*], ChunJun Cao[3,4,5], Sean M Carroll[6,7], Gong Cheng[5,8] and Ashmeet Singh[9]

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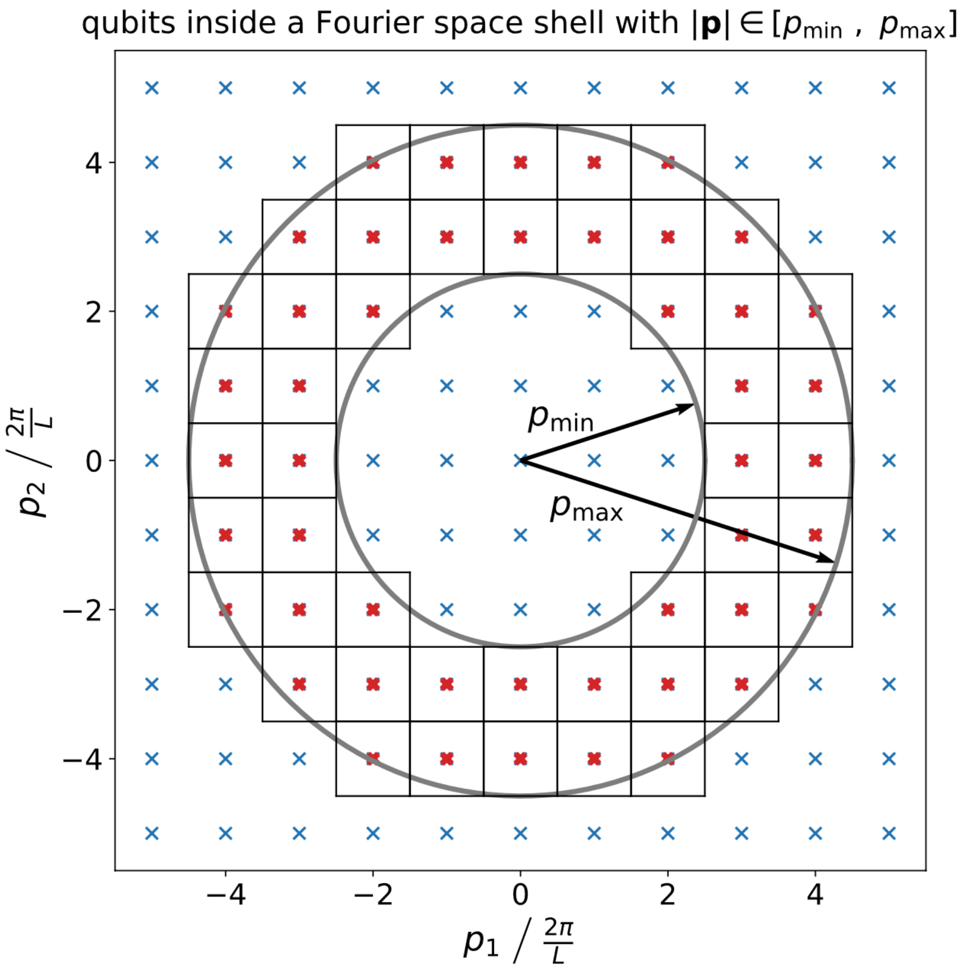
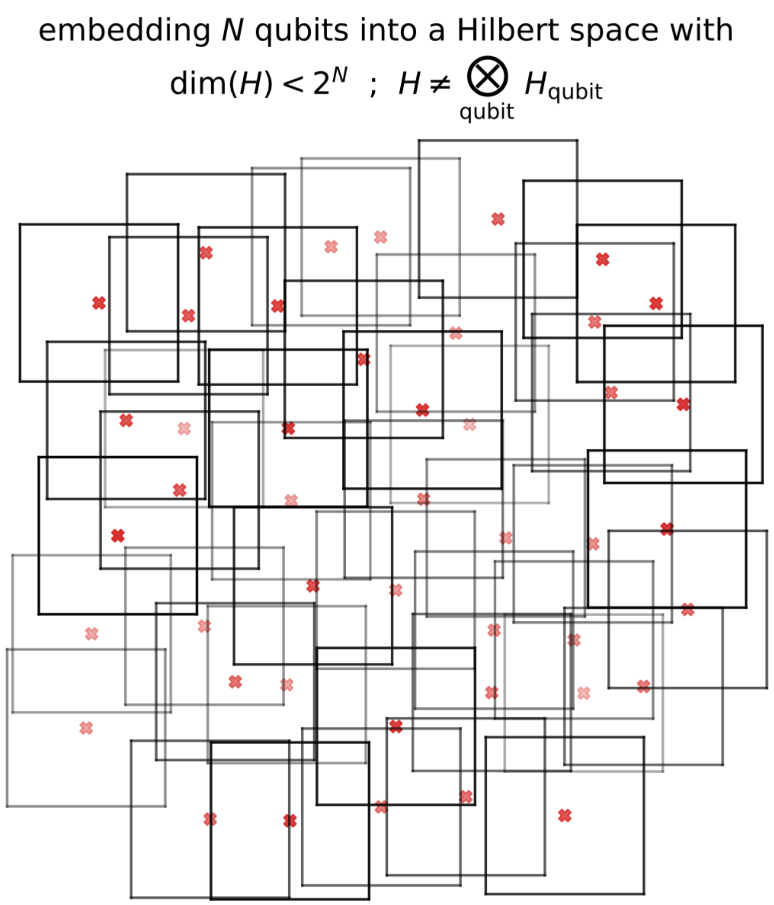
DOI 10.1088/1361-6382/ad6e4d

Article PDF

Numerical tools publicly available at:
https://github.com/OliverFHD/GPUniverse





Building a holographic quantum field

(here, a Weyl field = Fermion)

$$\hat{\psi}(\mathbf{x}, t) = \sum_{\mathbf{p}} \frac{1}{(|\mathbf{p}|L^3)^{\frac{1}{2}}} \left\{ \hat{c}_{\mathbf{p}}(t)\ u(\mathbf{p})\ e^{i\mathbf{px}} + \hat{d}_{\mathbf{p}}(t)^{\dagger}\ u(\mathbf{p})\ e^{-i\mathbf{px}} \right\} .$$

Consider Fourier space shell s of radius k_s and width Δ_s .

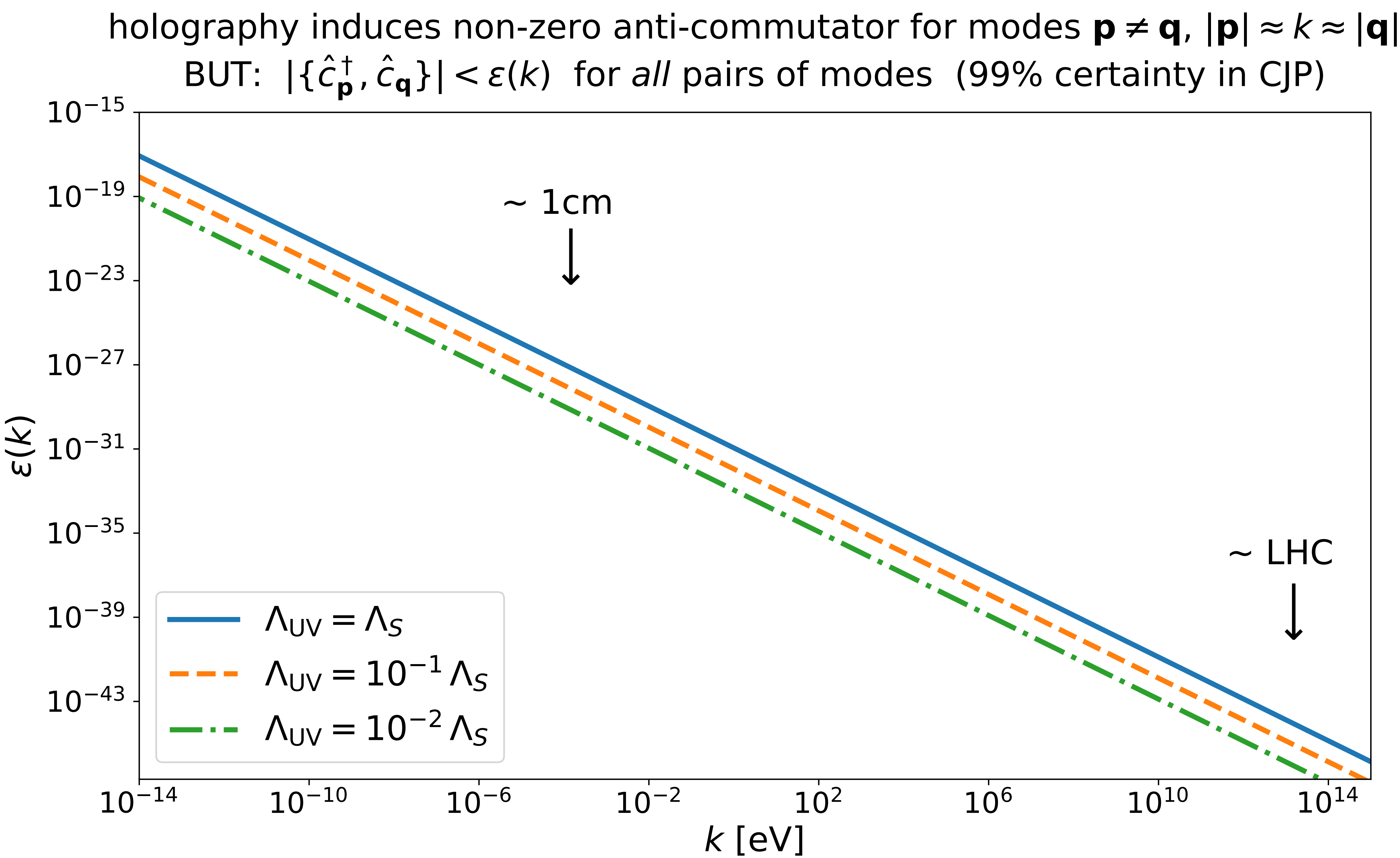
$\Rightarrow$ number of modes $\boxed{N_s \propto k_s^2 \Delta_s}$

volume scaling

but we want holographic scaling, i.e. effective number of modes $\boxed{n_s \propto k_s \Delta_s}$

area scaling

To achieve holography, we only have to allow for very low overlaps:



$$\Lambda_S \equiv \Lambda_{\rm Planck}\sqrt{N_{\rm dof,\psi}/N_{\rm dof,total}}$$ "species scale"

Main result: scattering of plane wave in vacuum

- can view this dispersion over time as “cosmic fog”
- the severity of this effect depends on UV-cut; “the more modes, the thicker the fog”

=> Use this + neutrino Observations to
test (our version of) the holographic principle:

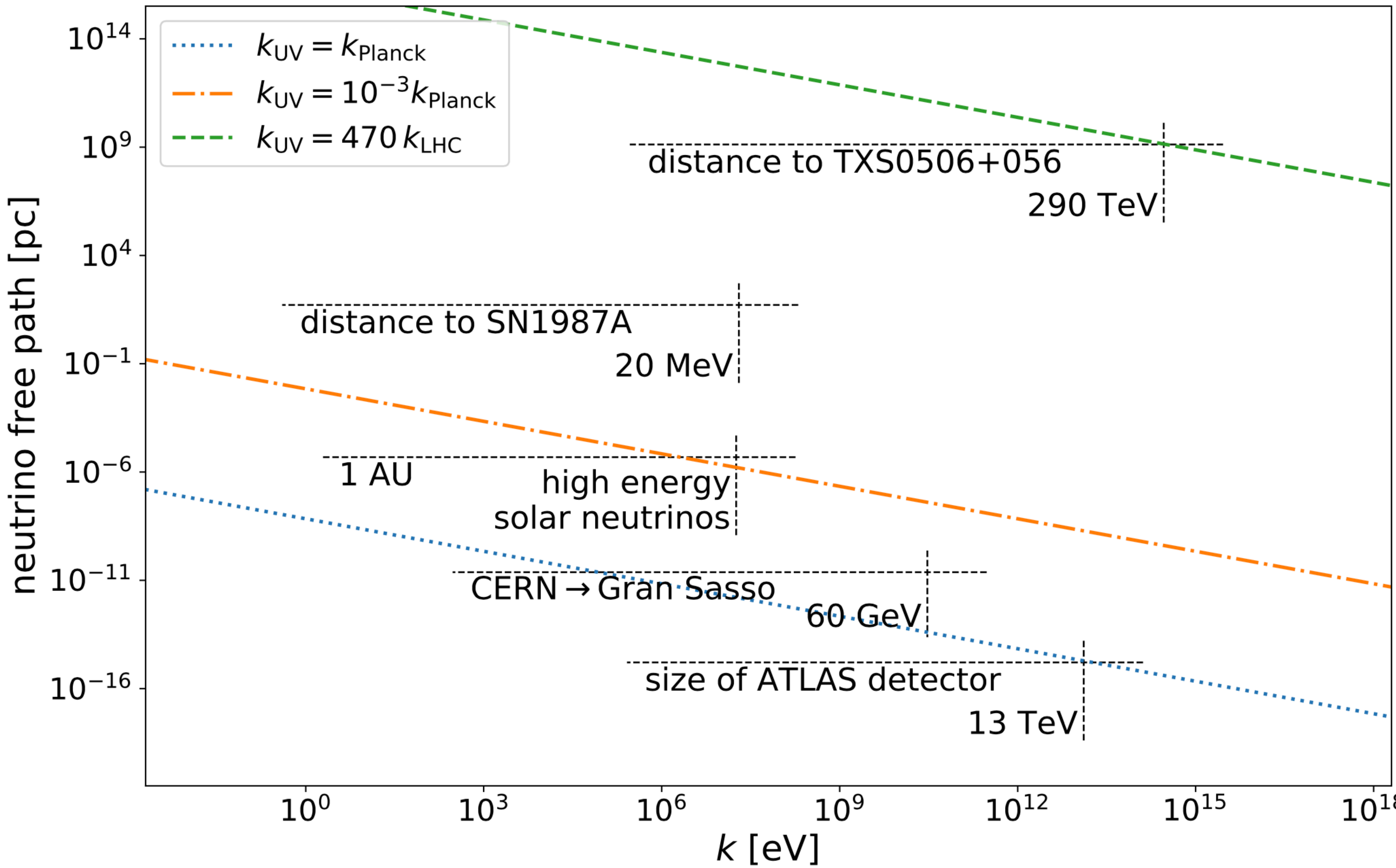
Step A: use neutrinos with know source

=> must have flewn on straight line
=> can use this to constrain UV-cut = “thickness of the fog”

Step B: use any neutrinos

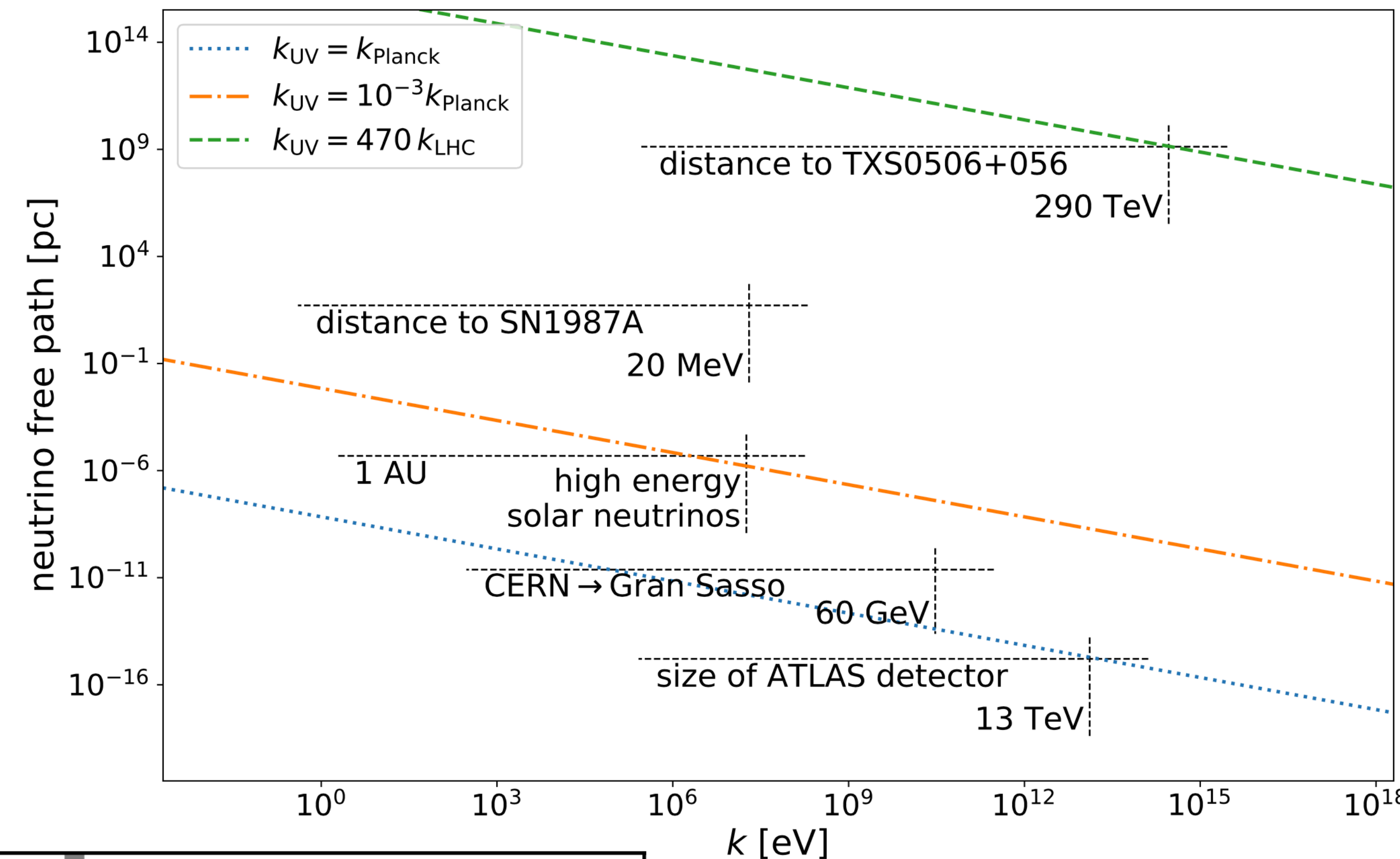
=> are there neutrinos beyond the UV-cut derived in step A)?

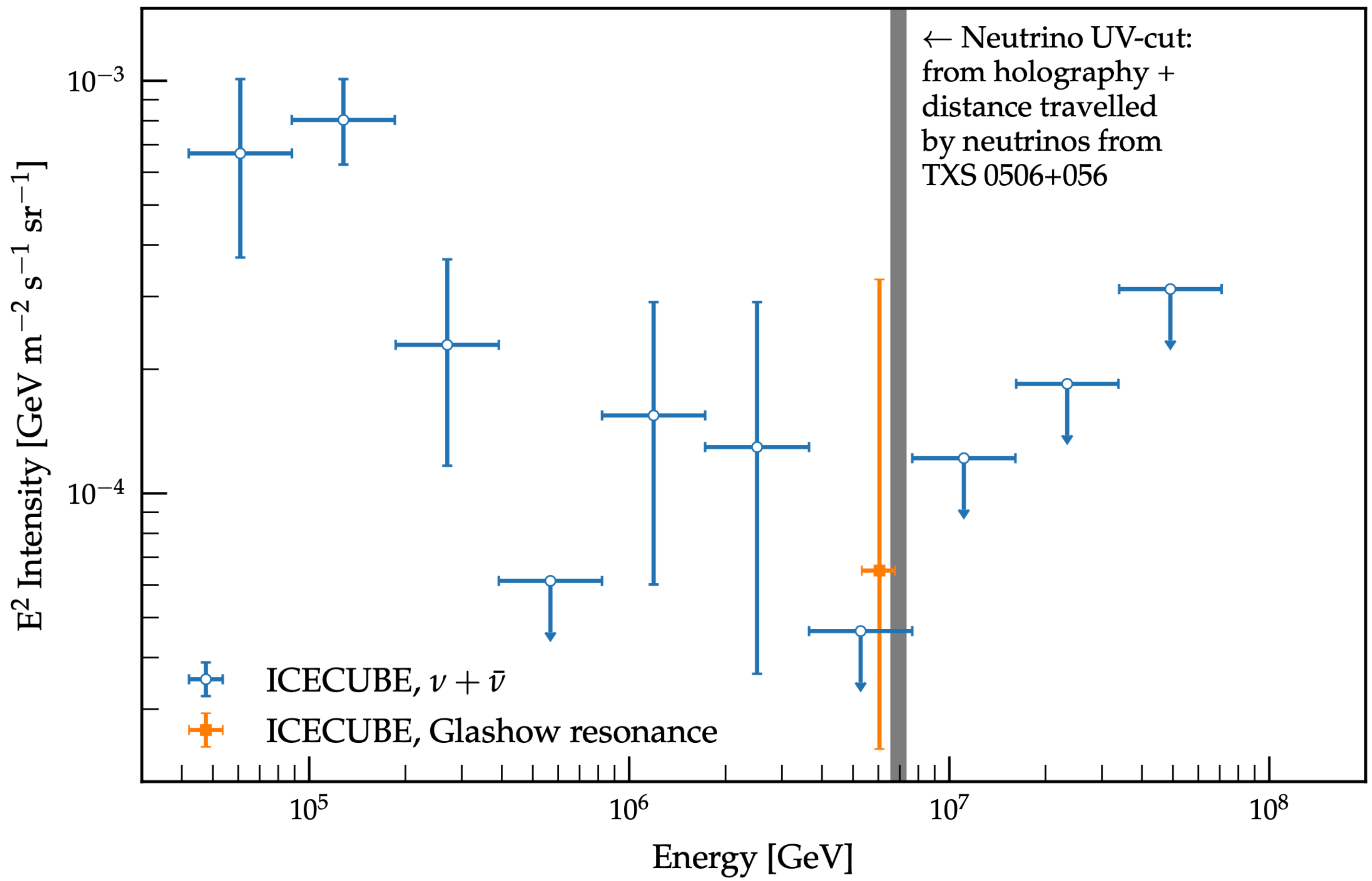
Testing the holographic principle: step A)



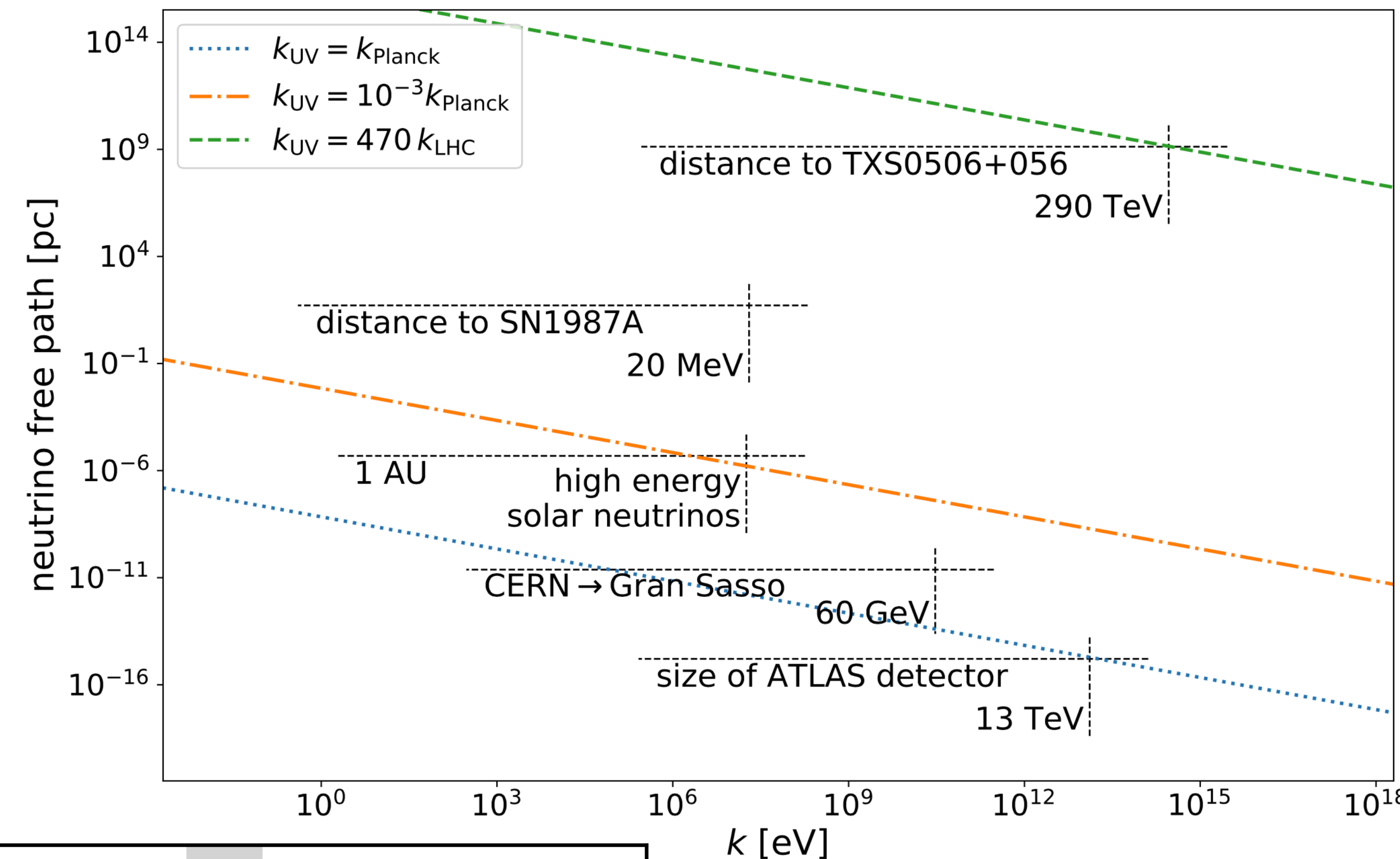
Testing the holographic principle: step B)

Is the cut we derive consistent with highest measured neutrino energies?

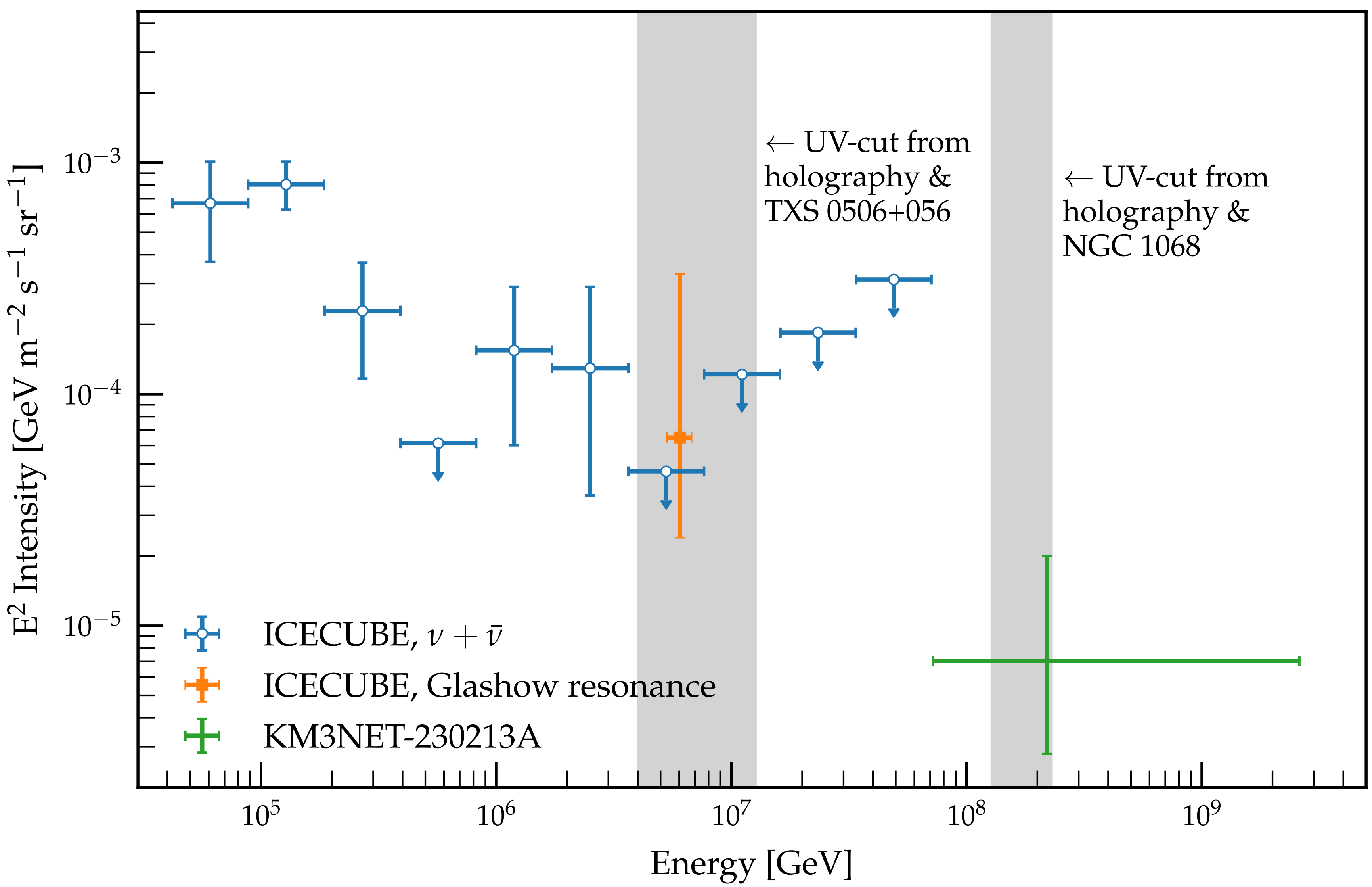




Testing the holographic principle: **step B)**



Is the cut we derive consistent with highest measured neutrino energies?



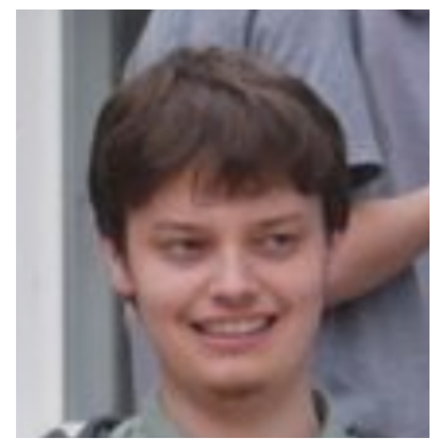
Team of students looking into improving our model:

Varun Kushwaha
(PhD)

- holographic scalars & photons
- helping me supervise the team

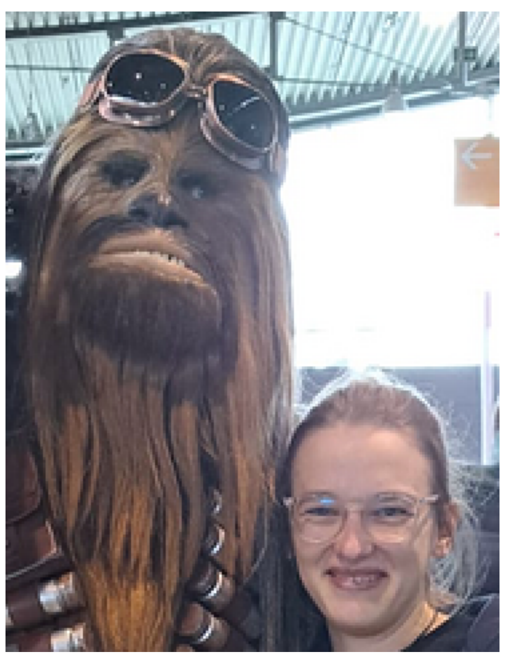
Paul Schneidewind-Telge
(Master)

S-matrix of holographic fields



Laurenz Kohlbach
(Master)

propagation of wave packets



Sarah Joswig
(Master)

Holographic Weyl field
on expanding background

With Varun & Kristina Giesel:
overlapping degrees-of-freedom
in classical field theory

Thank You!

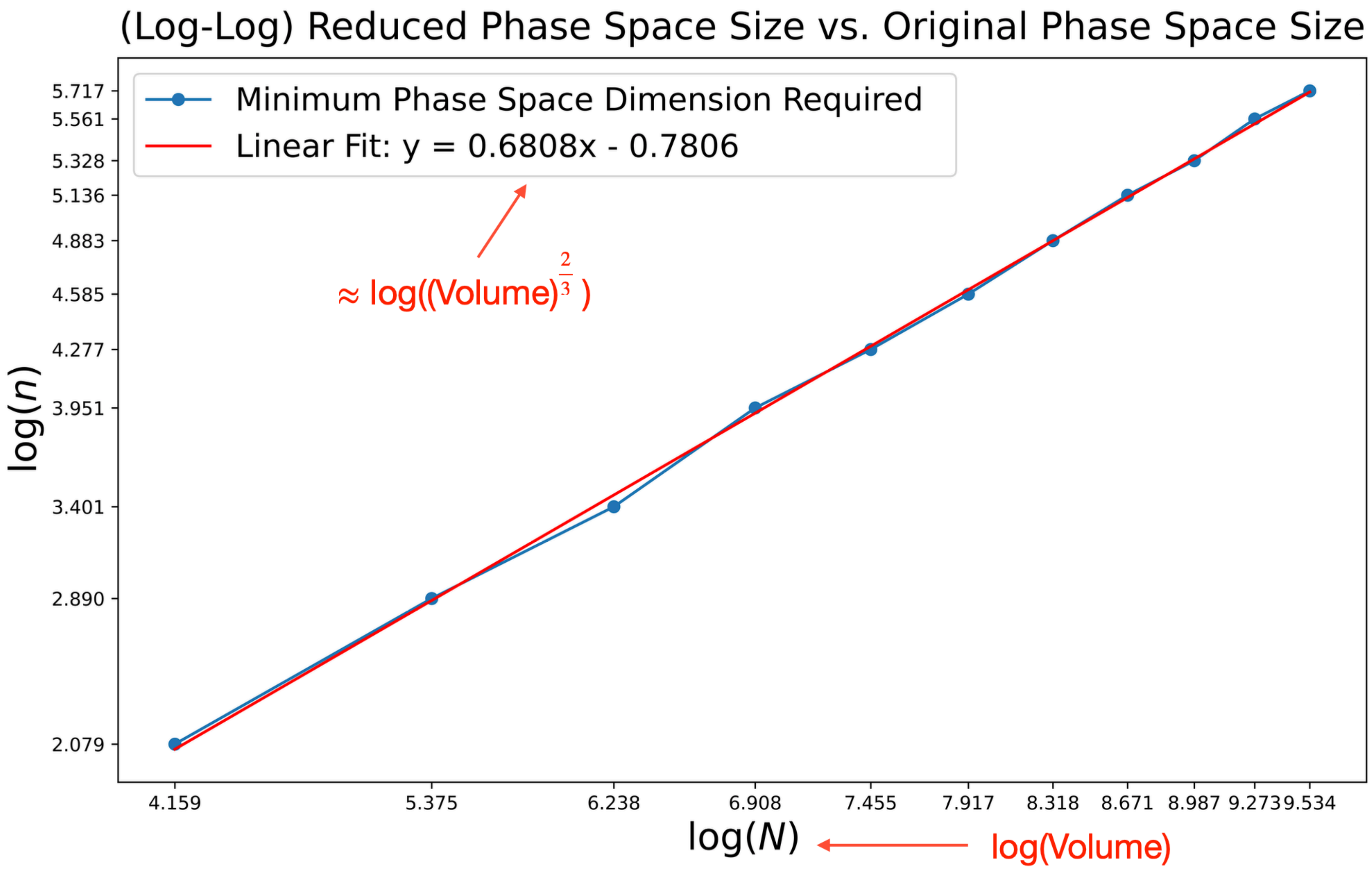
Classical holography
(Varun, Kristina & me)

Compress classical phase space while preserving symplectic structure:

$$\begin{pmatrix} \vec{Q} \\ \vec{P} \end{pmatrix}_{2N\times 1} \approx A_{2N\times 2n} \cdot \begin{pmatrix} \vec{q} \\ \vec{p} \end{pmatrix}_{2n\times 1}$$

see Varun's poster for more details

How small can we make n while still preserving dynamics?



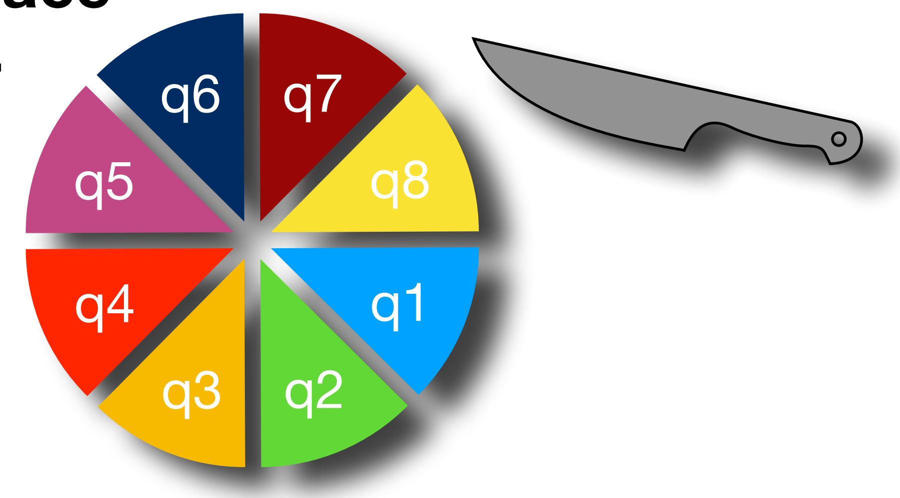
Area scaling?! :-D

overlapping dofs / proto-dressing from mereology

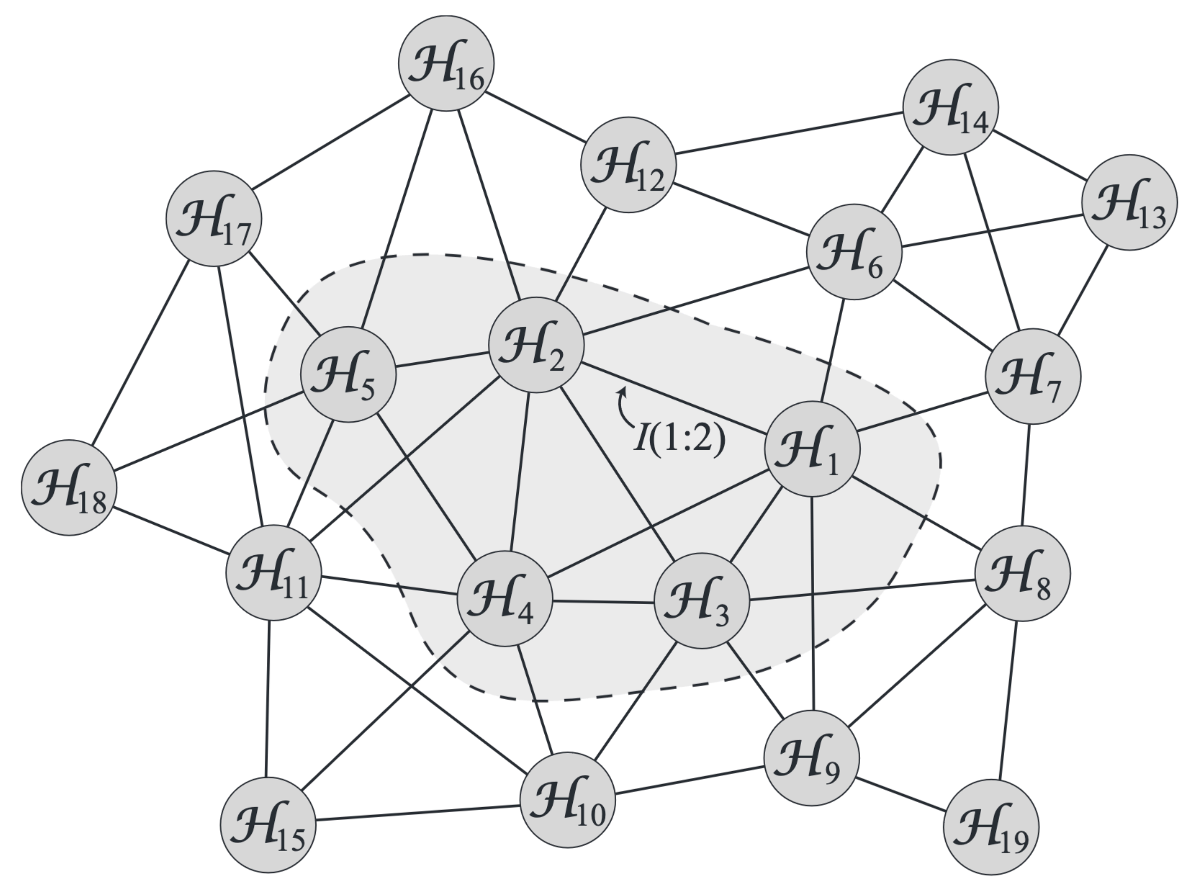
Quantum mereology approach for Emergence of gravity + EFT

ChunJun Cao, Sean Carroll, Ashmeet Singh, Marin Girard, Nicolas Loizeau, Arsalan Adil

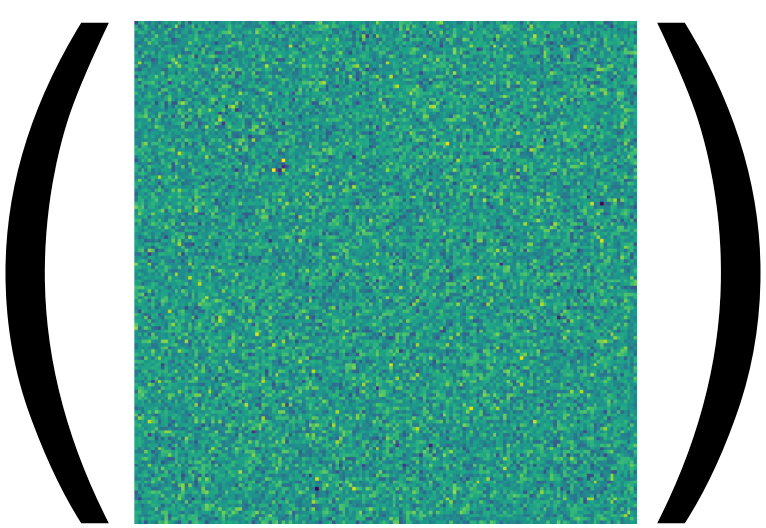
2. Split Hilbert space into "degrees-of-freedom"

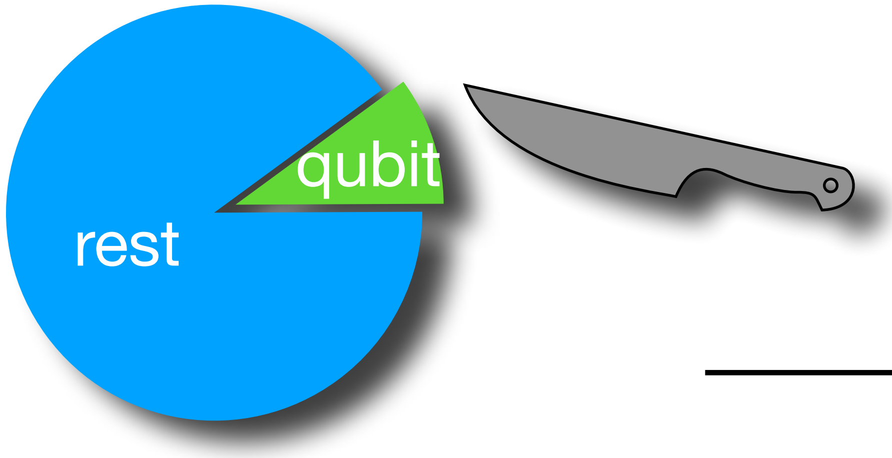


3. Study under which these dofs resemble "Bulk entanglement gravity" (Cao & Carrol arxiv:1712.02803)



1. Start from random matrix model (Hamiltonian)





How to split $\mathscr{H} \simeq \mathscr{H}_q \otimes \mathscr{H}_r$ such that $\mathscr{H}_q$ is a "good" degree of freedom?

Can always decompose Hamiltonian as

$$\hat{H} = \hat{H}_q \otimes \mathbb{1}_r + \mathbb{1}_q \otimes \hat{H}_r + \frac{\mathrm{Tr}(\hat{H})}{d} \cdot \mathbb{1} + \hat{H}_{\mathrm{int}}$$

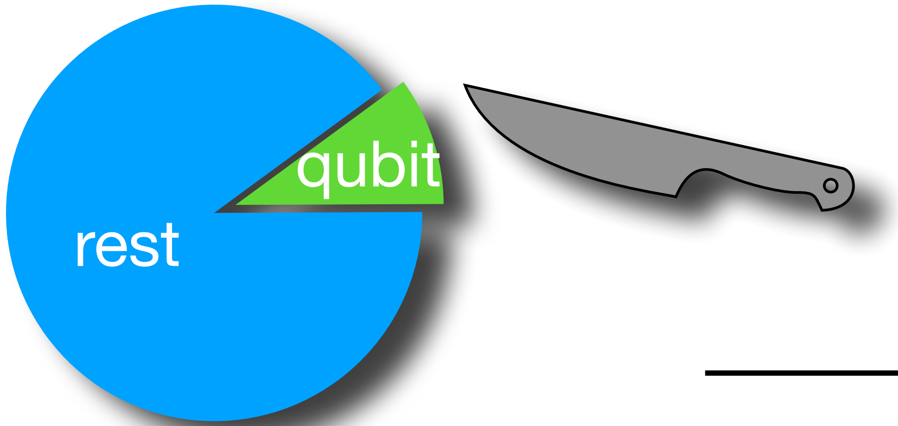
where

$$\mathrm{Tr}_q \hat{H}_q = 0 \ \ , \ \ \mathrm{Tr}_r \hat{H}_r = 0 \ \ , \ \ \mathrm{Tr}_r \hat{H}_{\mathrm{int}} = 0 = \mathrm{Tr}_q \hat{H}_{\mathrm{int}} \ .$$

Now find split that minimizes interaction strength!

i.e. in space of factorizations, minimize the "loss function"

$$\mathscr{L}(\hat{H}) \equiv \mathrm{Tr}(\hat{H}_{\mathrm{int}}^2)$$



How to split $\mathscr{H} \simeq \mathscr{H}_q \otimes \mathscr{H}_r$ such that $\mathscr{H}_q$ is a "good" degree of freedom?

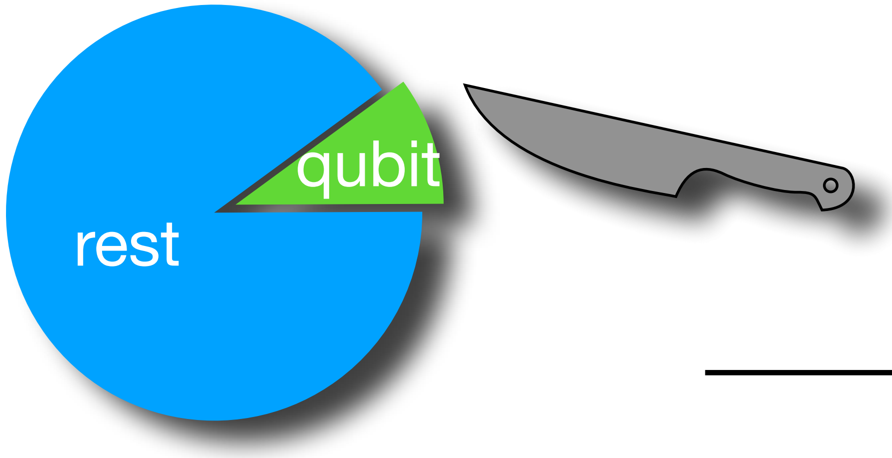
In practice:

Keep factorisation $\mathscr{H} = \mathscr{H}_q \otimes \mathscr{H}_r$ fixed and instead change $\hat{\rho}_T \to U \hat{\rho}_T U^\dagger$.

$\Rightarrow$ Find all local minima $U \in \mathrm{SU}(N)$ of $\mathrm{Tr}(\hat{H}_{\mathrm{int}}[U\,\hat{H}\,U^\dagger]^2)$!

Once a minimum U is found, the Pauli algebra of the emergent qubit is given by:

$$
\begin{aligned}
\Sigma_X &= U^\dagger\,(\sigma_X \otimes \mathbb{1}_r)\,U \\
\Sigma_Y &= U^\dagger\,(\sigma_Y \otimes \mathbb{1}_r)\,U \\
\Sigma_Z &= U^\dagger\,(\sigma_Z \otimes \mathbb{1}_r)\,U
\end{aligned}
$$



How to split $\mathscr{H} \simeq \mathscr{H}_q \otimes \mathscr{H}_r$ such that $\mathscr{H}_q$ is a "good" degree of freedom?

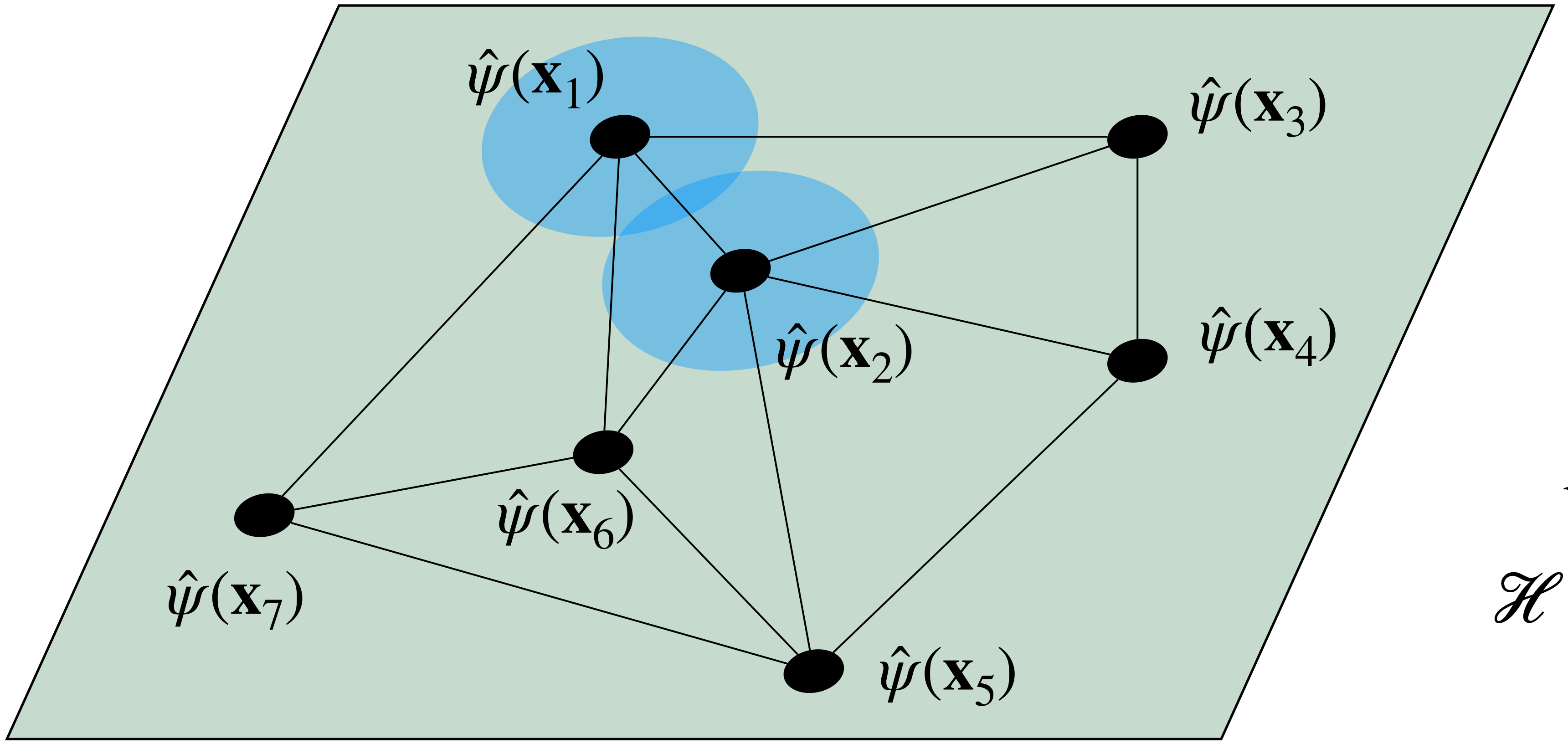
Idea: find all factorisations

$$
\begin{aligned}
\mathscr{H} &\simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{\mathrm{rest}_1} \\
&\simeq \mathscr{H}_{q_2} \otimes \mathscr{H}_{\mathrm{rest}_2} \\
&\simeq \mathscr{H}_{q_3} \otimes \mathscr{H}_{\mathrm{rest}_3} \\
&\simeq \ldots
\end{aligned}
$$

that minimize $\mathrm{Tr}(\hat{H}_{\mathrm{int}}^2)$!

→ do these factorisations overlap?
→ do they behave like a collection of dressed field operators?

dressed field in flat space

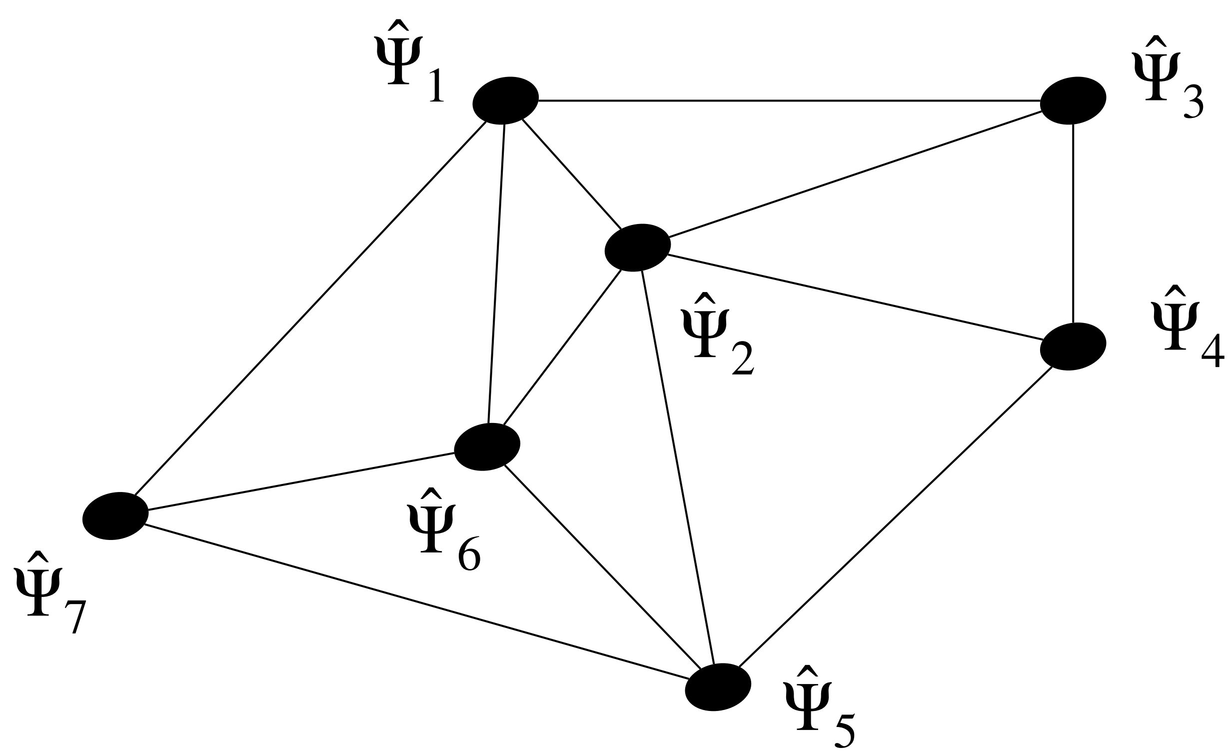


$$\begin{aligned}\mathscr{H} &\simeq \mathscr{H}_{\mathbf{x}_1} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_1)}\\ &\simeq \mathscr{H}_{\mathbf{x}_2} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_2)}\\ &\simeq \mathscr{H}_{\mathbf{x}_3} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_3)}\\ &\simeq \ldots\end{aligned}$$

but :

$$\mathscr{H} \not\simeq \mathscr{H}_{\mathbf{x}_1} \otimes \mathscr{H}_{\mathbf{x}_2} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_1,\mathbf{x}_2)}$$

pre–geometry of overlapping dofs

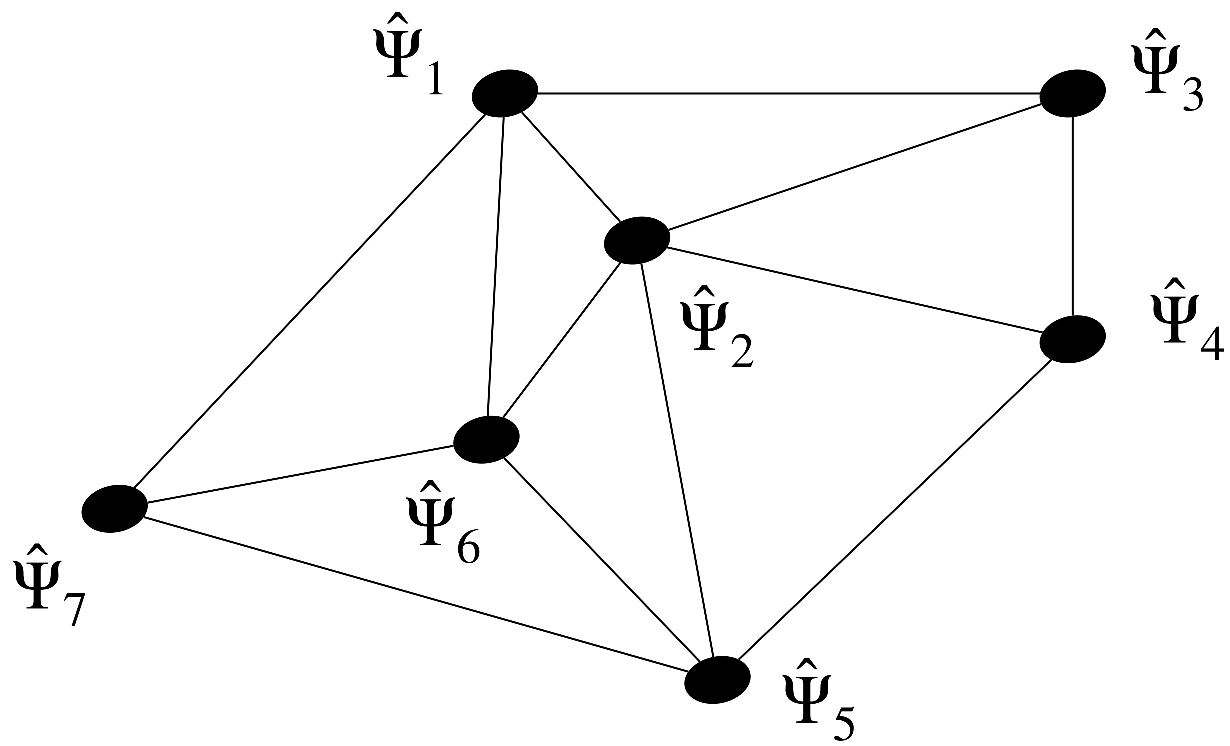


$$\begin{aligned}\mathscr{H} &\simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{\mathrm{rest}_1}\\ &\simeq \mathscr{H}_{q_2} \otimes \mathscr{H}_{\mathrm{rest}_2}\\ &\simeq \mathscr{H}_{q_3} \otimes \mathscr{H}_{\mathrm{rest}_3}\\ &\simeq \ldots\end{aligned}$$

but :

$$\mathscr{H} \not\simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{q_2} \otimes \mathscr{H}_{\mathrm{rest}_{12}}$$

pre−geometry of overlapping dofs



$$
\begin{aligned}
\mathscr{H} &\simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{\text{rest}_1} \\
&\simeq \mathscr{H}_{q_2} \otimes \mathscr{H}_{\text{rest}_2} \\
&\simeq \mathscr{H}_{q_3} \otimes \mathscr{H}_{\text{rest}_3} \\
&\simeq \ldots
\end{aligned}
$$

but :

$$\mathscr{H} \not\simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{q_2} \otimes \mathscr{H}_{\text{rest}_{12}}$$

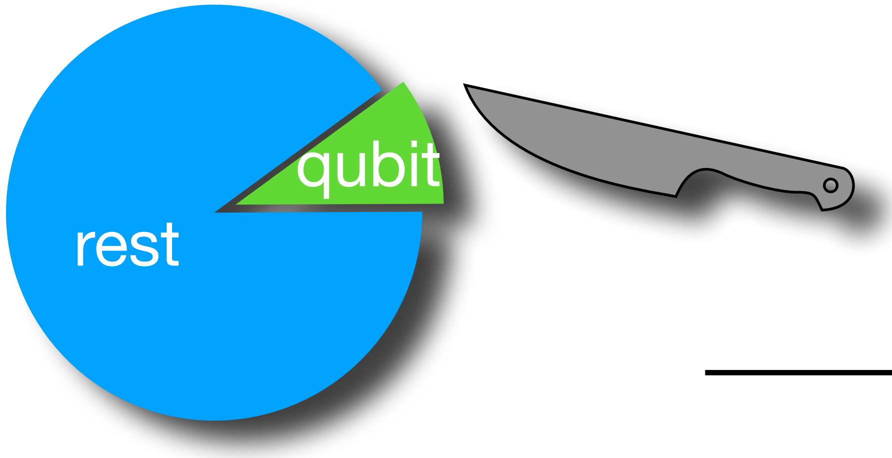
Why think of this as real space & not momentum space?

For interacting theory, real-space modes minimize interaction
https://arxiv.org/abs/hep-th/0506124 (Piazza, 2005)

contains real space H_int

$$
\begin{aligned}
\hat{H} &\sim \int \mathrm{d}^3x \left[\dot{\phi}^2 + \boxed{(\boldsymbol{\nabla}\phi)^2} + \lambda\phi^4\right] \\
&\sim \int \mathrm{d}^3k \left[|\dot{\phi}|^2 + k^2|\phi|^2 + \lambda \boxed{\int \mathrm{d}^3k_{234}\, \phi(\boldsymbol{k}_1 - \boldsymbol{k}_2 - \boldsymbol{k}_3)\phi(\boldsymbol{k}_3)\phi(\boldsymbol{k}_2 - \boldsymbol{k}_4)\phi(\boldsymbol{k}_4)}\right]
\end{aligned}
$$

contains Fourier space H_int



How to split $\mathscr{H} \simeq \mathscr{H}_q \otimes \mathscr{H}_r$ such that $\mathscr{H}_q$ is a “good” degree of freedom?

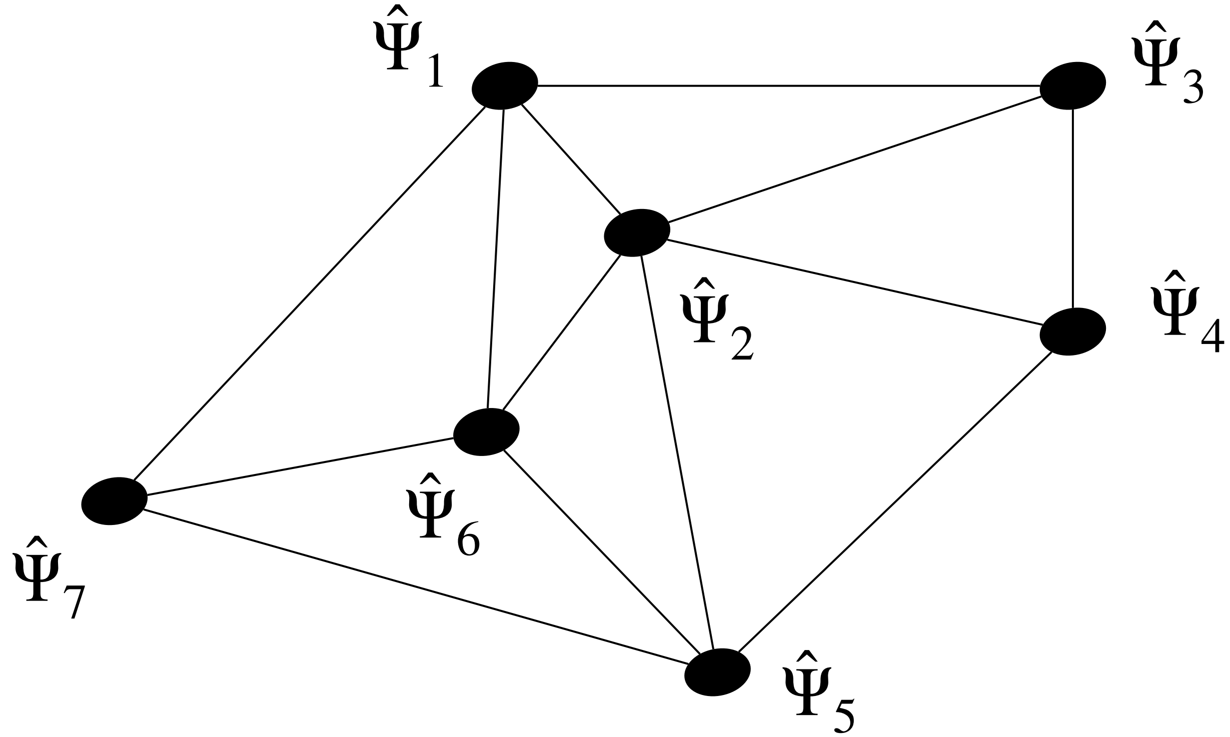
(of course!) results for general Hamiltonians are limited..
One can show: at minima of $\mathrm{Tr}(\hat{H}_{\mathrm{int}}^2)$ the Hamiltonian becomes

$$\hat{H} = \hat{H}_q \otimes \mathbb{I}_r + \mathbb{I}_q \otimes \hat{H}_r + \frac{\mathrm{Tr}(\hat{H})}{d} \cdot \mathbb{I} + \hat{H}_q \otimes \hat{X}_{\mathrm{int}}$$

$$\simeq \frac{E_q}{2} \sigma_z \otimes \mathbb{I}_r + \mathbb{I}_q \otimes \hat{H}_r + \frac{\mathrm{Tr}(\hat{H})}{d} \cdot \mathbb{I} + \sigma_z \otimes \hat{X}_{\mathrm{int}} \ ,$$

and even that $[\hat{H}_r, \hat{X}_{\mathrm{int}}] = 0$.

pre−geometry of overlapping dofs

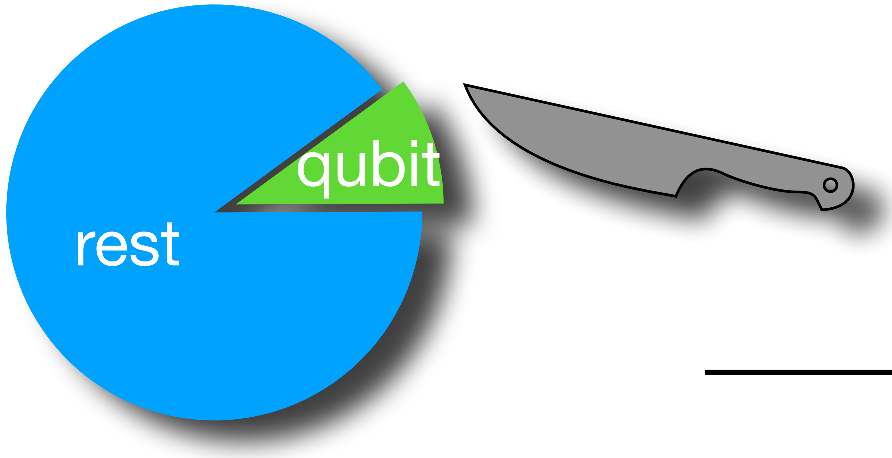


$$\begin{aligned}\mathscr{H} &\simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{\mathrm{rest}_1} \\ &\simeq \mathscr{H}_{q_2} \otimes \mathscr{H}_{\mathrm{rest}_2} \\ &\simeq \mathscr{H}_{q_3} \otimes \mathscr{H}_{\mathrm{rest}_3} \\ &\simeq \dots\end{aligned}$$

but :

$$\mathscr{H} \not\simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{q_2} \otimes \mathscr{H}_{\mathrm{rest}_{12}}$$

at each site : $\hat{H} = \frac{E_q}{2}\sigma_z \otimes \mathbb{I}_r + \mathbb{I}_q \otimes \hat{H}_r + \frac{\mathrm{Tr}(\hat{H})}{d} \cdot \mathbb{I} + \sigma_z \otimes \hat{X}_{\mathrm{int}}$



How to split $\mathscr{H} \simeq \mathscr{H}_q \otimes \mathscr{H}_r$ such that $\mathscr{H}_q$ is a "good" degree of freedom?

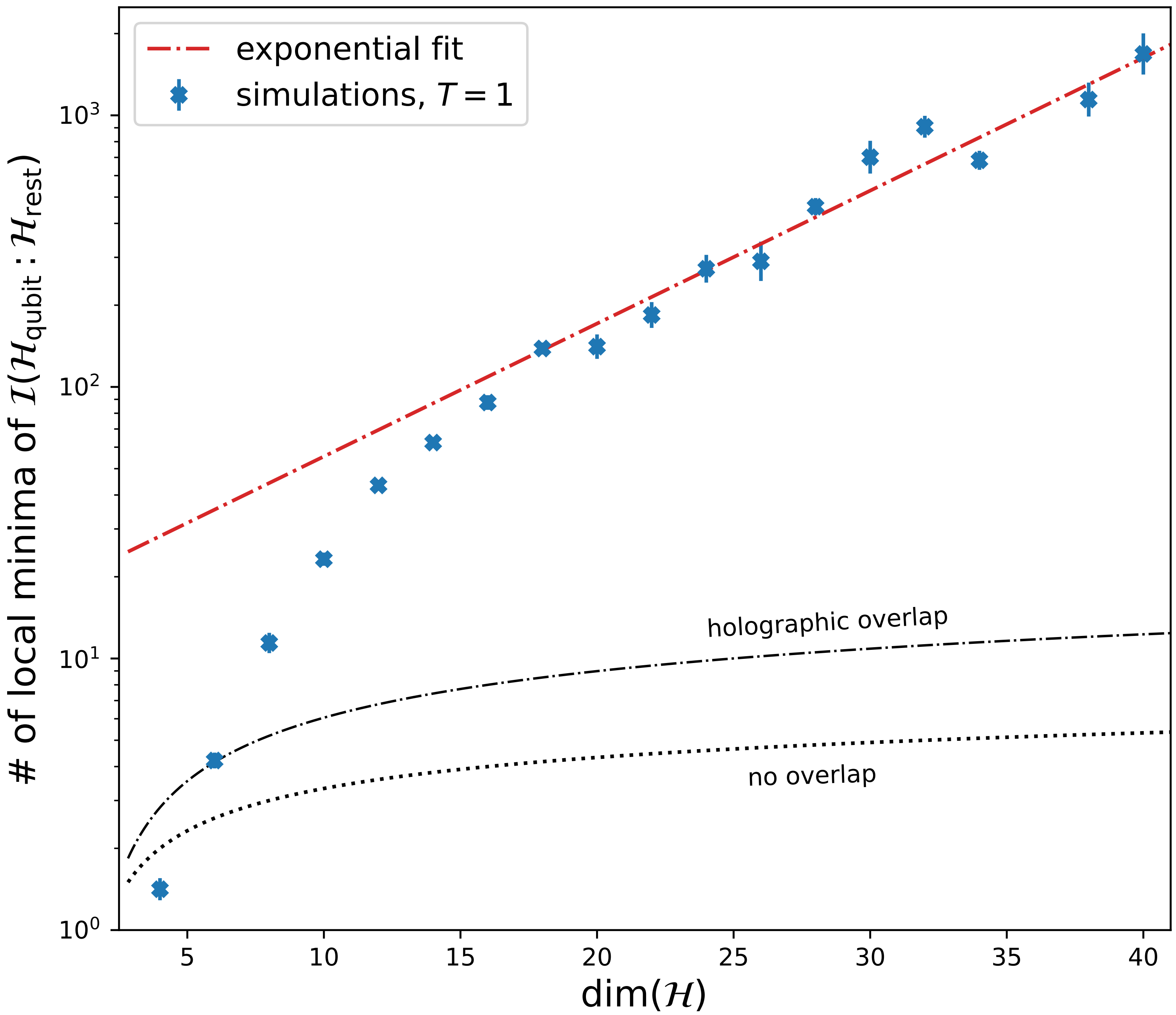
For more results: pick concrete Hamiltonian; e.g.:

Step A: Draw random Hamiltonian (for now from Gaussian ensemble)

Step B: Find all local minima of $\mathrm{Tr}(\hat{H}^2_{\mathrm{int}})$
("local" in space of possible factorisations $\mathscr{H} \simeq \mathscr{H}_q \otimes \mathscr{H}_r$)

Questions:

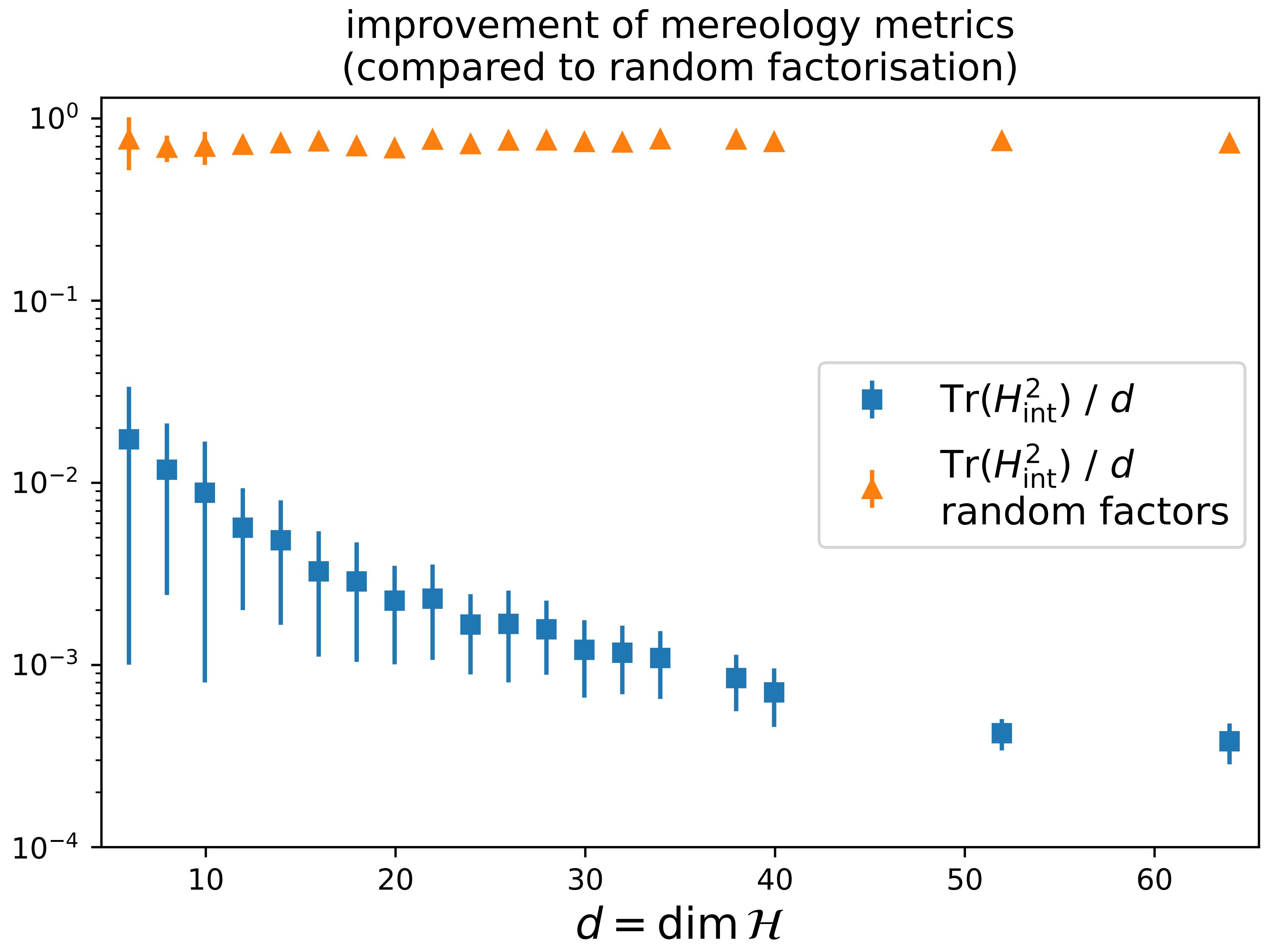
Are the minima "overlapping"?
Are they only local minima, or legitimate dof-candidate globally?
"Choice of dressing" ?



Result No. 1:

There are a lot of local minima!

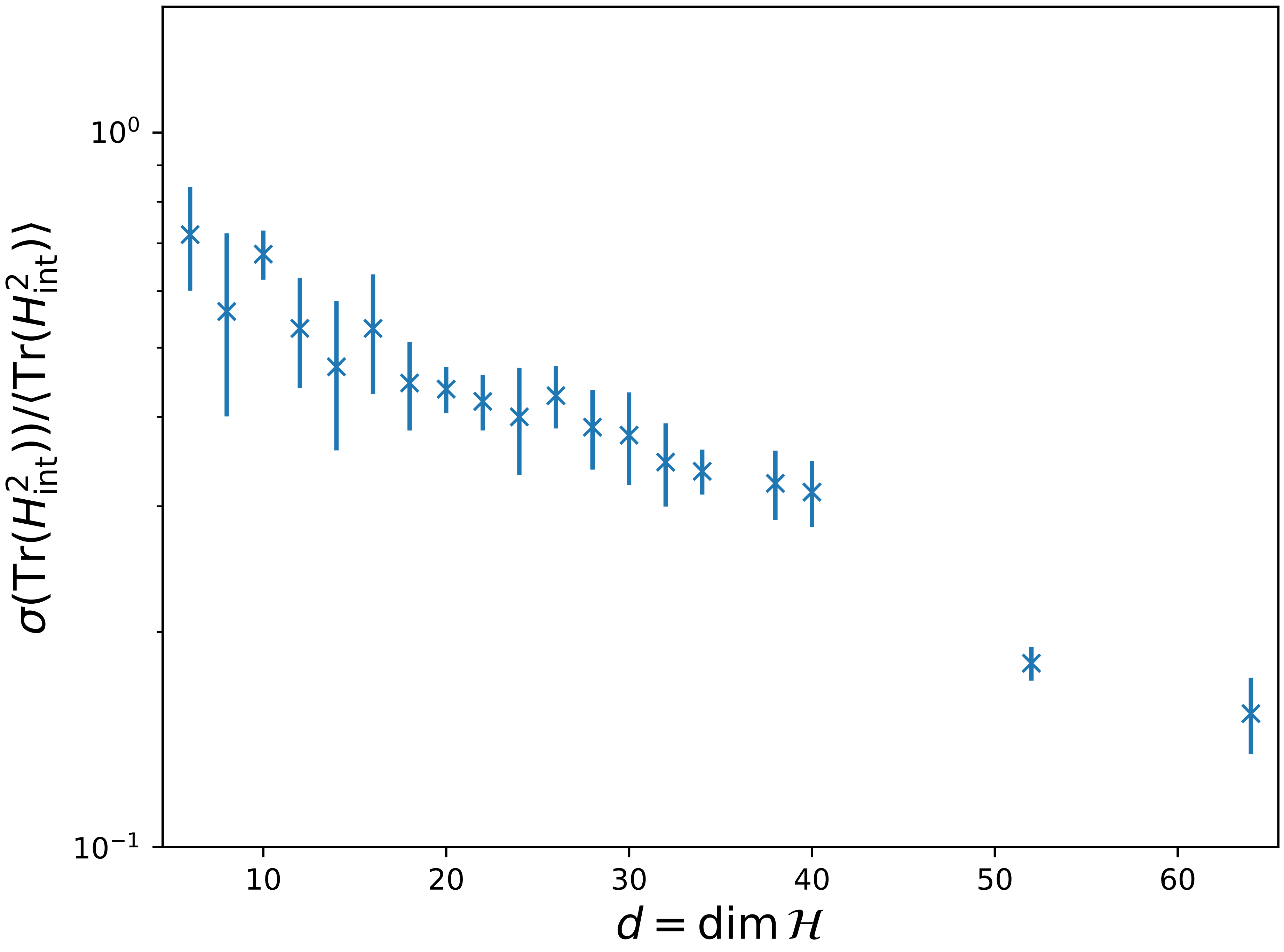
- the number of emergent qubits increases ~ exponentially with dimension; in particular $> \log_2 \dim \mathscr{H}$, i.e. these are overlapping qubits!



Result No. 2:
Each minimum achieves very low interaction

- i.e. qualitatively: there is indeed very little interaction between $\mathscr{H}_q$ and $\mathscr{H}_r$
(compared to random factorisation)

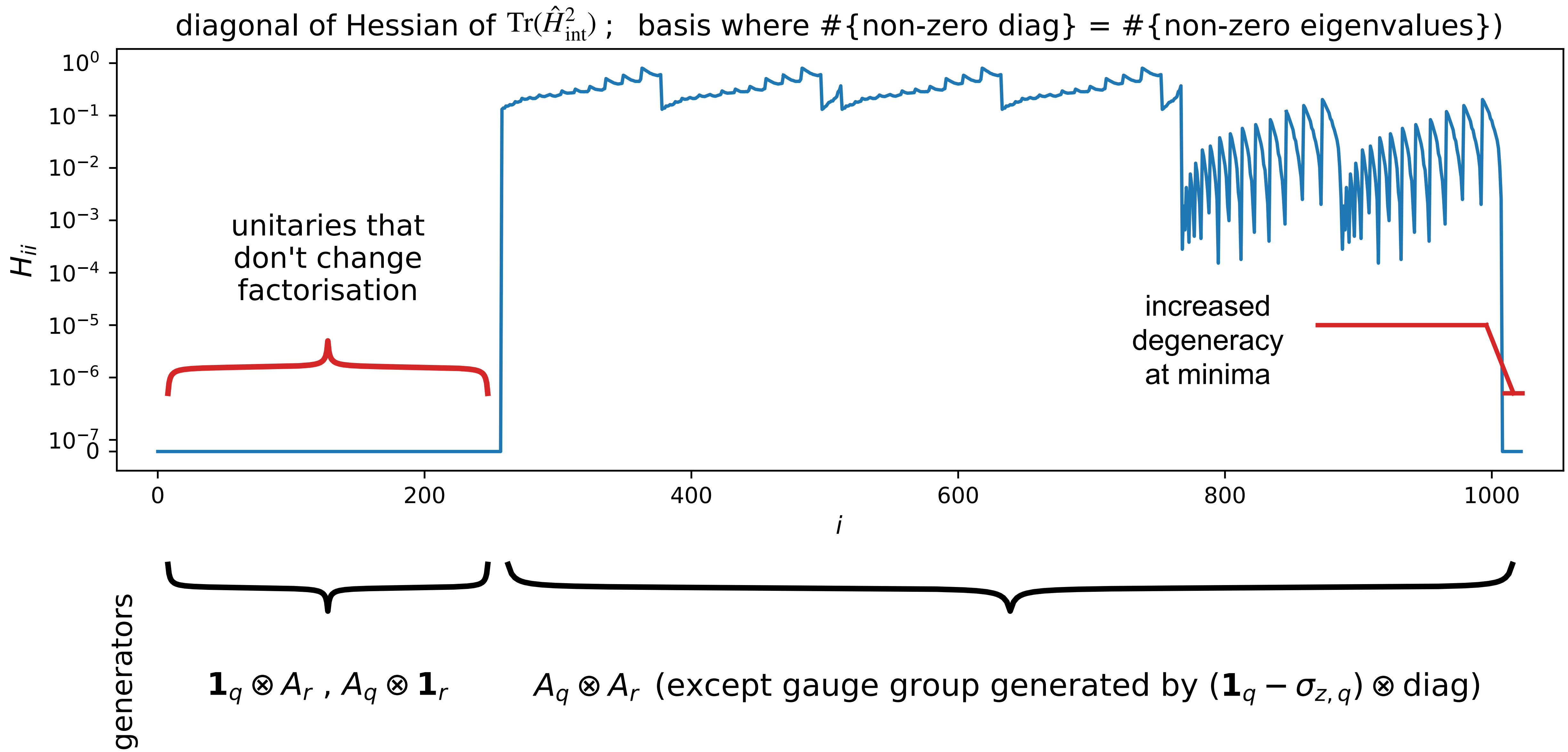
$\Rightarrow$ plausible to view $\mathscr{H}_q$ as “degrees-of-freedom”



Result No. 3:

In limit of large d, all minima have similar coupling

- i.e. coupling structure “translation invariant”?



Result No. 4: An unexpected degeneracy emerges

- remember: minimisation is done in $\mathrm{SU}(N)$
  $\Rightarrow$ minima will be degenerate, because some U don't change factorisation

- But degeneracy is bigger than expected. An additional symmetry group changes factorisation, but does not change self-Hamiltonians and interaction Hamiltonians.

The Hamiltonian

$$\hat{H} = \frac{E_q}{2}\sigma_z \otimes \mathbb{I}_r + \mathbb{I}_q \otimes \hat{H}_r + \frac{\mathrm{Tr}(\hat{H})}{d} \cdot \mathbb{I} + \sigma_z \otimes \hat{X}_{\mathrm{int}} \ ,$$

only depends on $\sigma_z \otimes \mathbb{I}_r \equiv \Sigma_z$, but not on the rest of the Pauli algebra in the qubit.

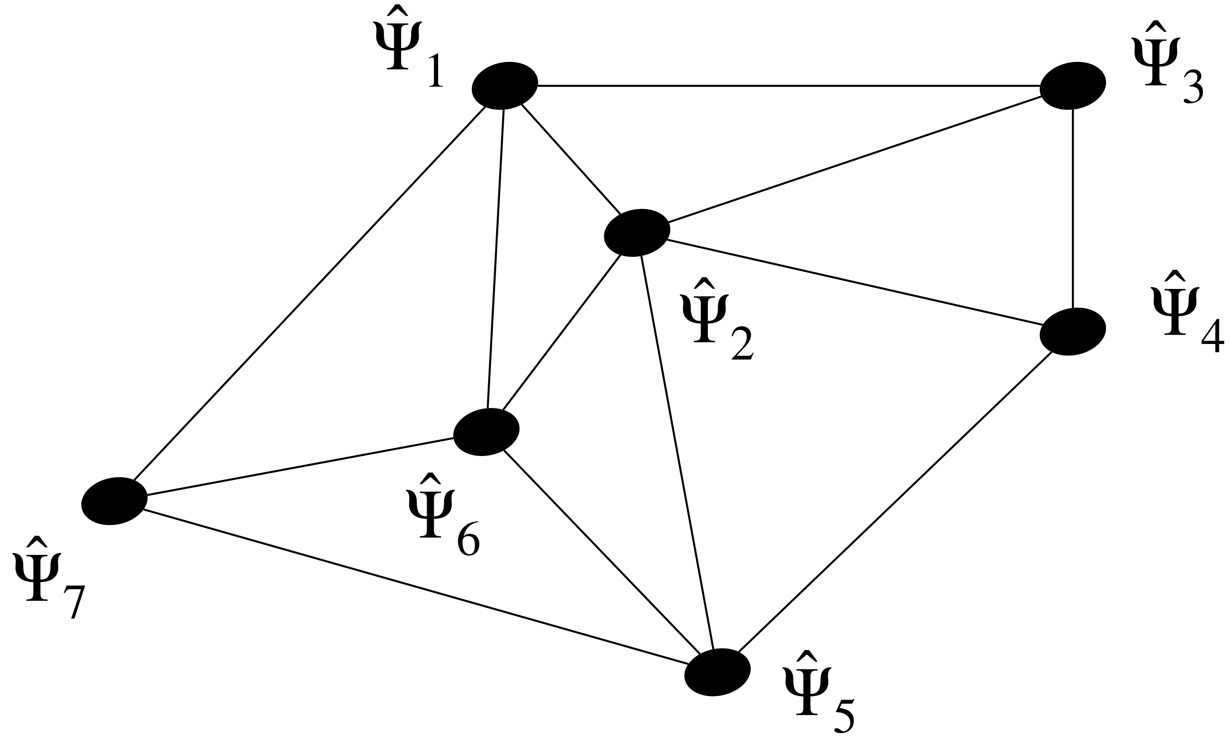
$\Rightarrow$ we have freedom with which Σ_x, Σ_y to complete the Pauli algebra

Define "field" $\hat{\Psi} \equiv \frac{1}{2}(\Sigma_x + i\Sigma_y)$

$\Rightarrow$ this is not unique \ choice of "dressing;
But Hamiltonian is independent of this choice:

$$\hat{H} = \frac{E_q}{2}\hat{\Psi}^\dagger\hat{\Psi} + \hat{\Psi}^\dagger\hat{\Psi} \cdot (\mathbb{I}_q \otimes \hat{X}_{\mathrm{int}}) + \ldots$$

pre−geometry of overlapping dofs



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but :

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$$\text{at each site :} \quad \hat{H} = \frac{E_q}{2} \hat{\Psi}^\dagger \hat{\Psi} + \hat{\Psi}^\dagger \hat{\Psi} \cdot (\mathbb{I}_q \otimes \hat{X}_{\mathrm{int}}) + \ldots$$

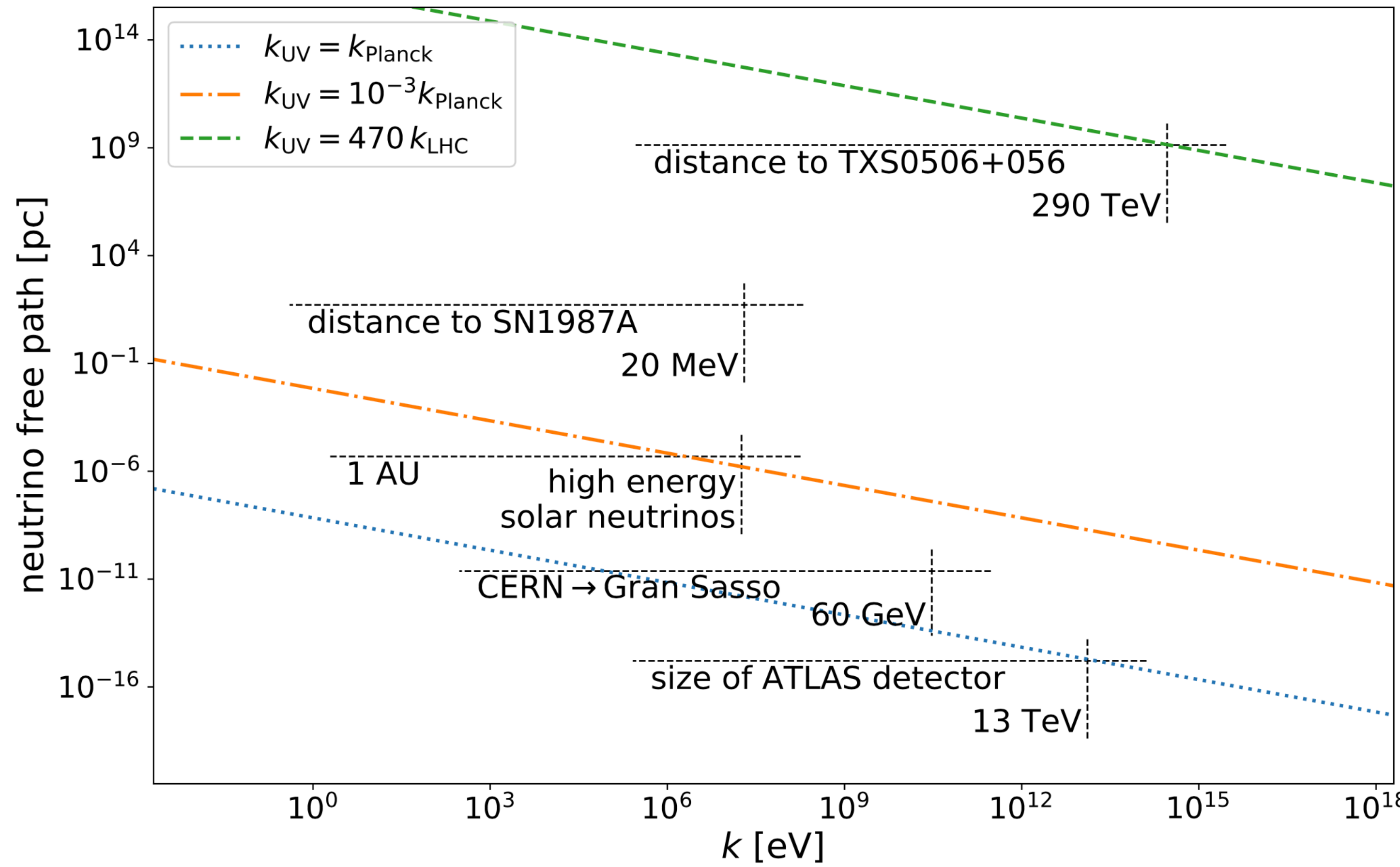
Summary & Outlook

Summary

There…

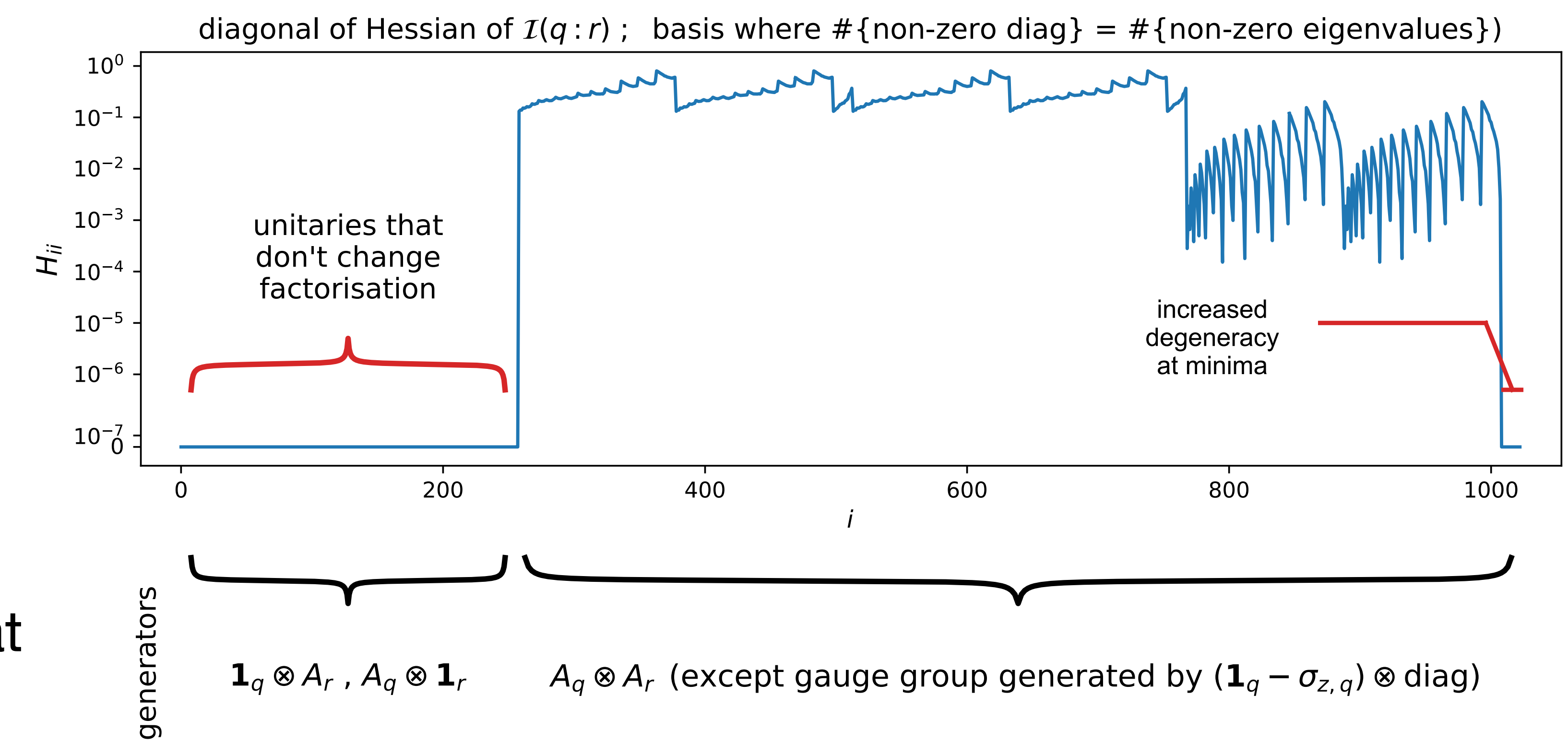
- We built a holographic Fermion field that also satisfies a cosmic Bousso bound
- We analytically calculated lifetime of plane wave excitations (and more..)
- We compared plane wave lifetime to Neutrino observations

=> Comparing a core principle of QG to data!!



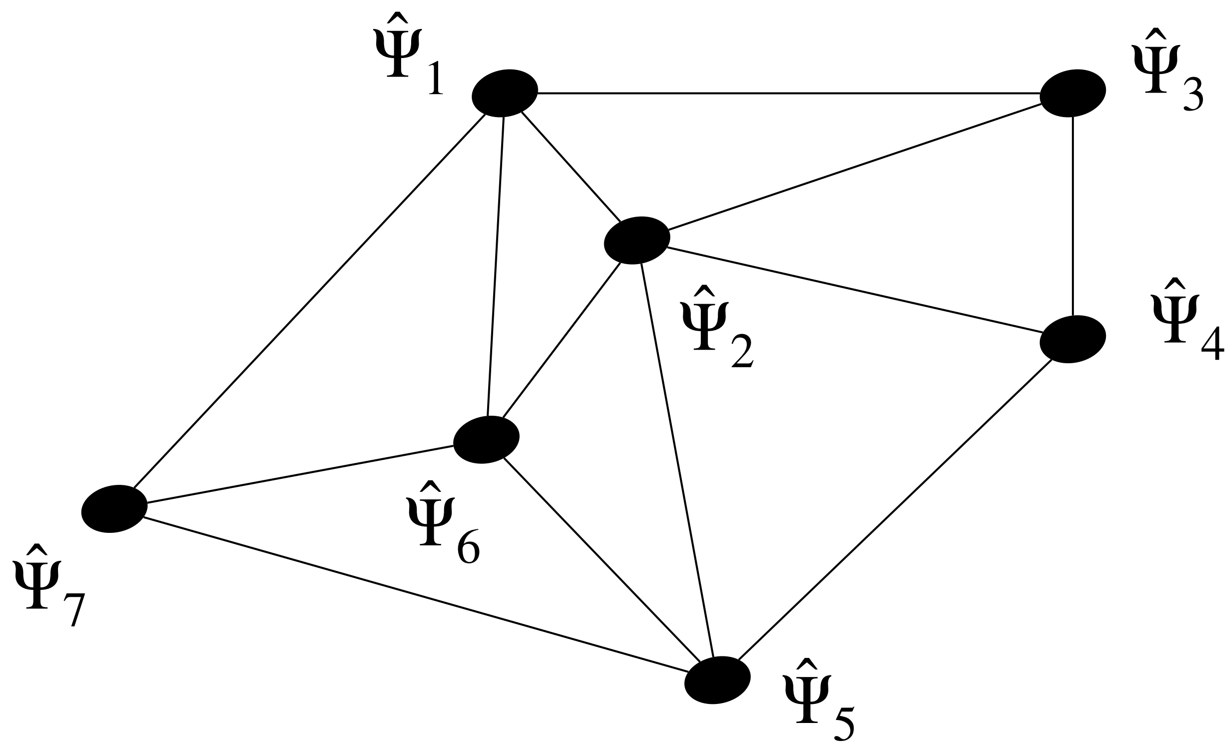
… and back again:

- Ising-like structure when looking for minimum-interaction dofs
- numerous local minima in random matrix model, all with similar & low interaction
- operator algebra not uniquely fixed at minima (“proto-dressing choice”)



backup slides

pre−geometry of overlapping dofs



$$
\begin{aligned}
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\end{aligned}
$$

but :

$$\mathscr{H} \not\simeq \mathscr{H}_{q_1} \otimes \mathscr{H}_{q_2} \otimes \mathscr{H}_{\mathrm{rest}_{12}}$$

Why think of this as real space & not momentum space?

For interacting theory, real-space modes minimize interaction
https://arxiv.org/abs/hep-th/0506124 (Piazza, 2005)

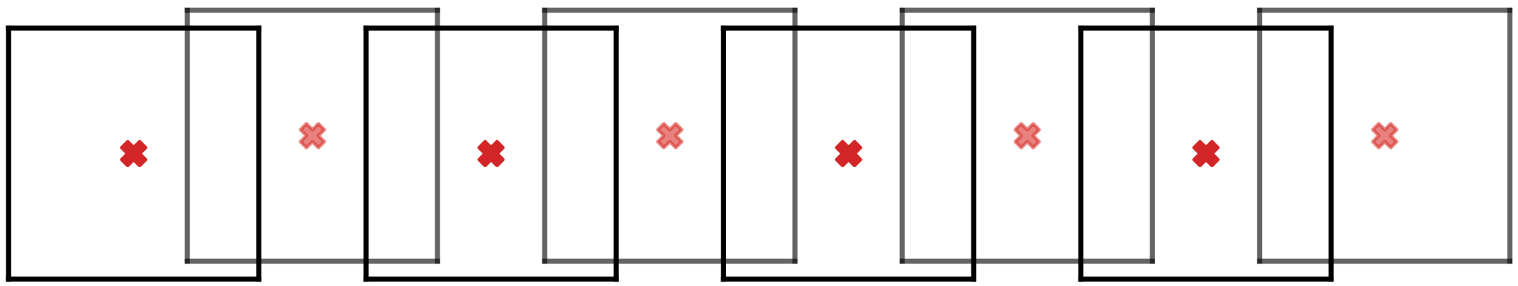
contains real space H_int

$$
\begin{aligned}
\hat{H} &\sim \int \mathrm{d}^3 x \left[\dot{\phi}^2 + \boxed{(\boldsymbol{\nabla}\phi)^2} + \lambda\phi^4\right] \\
&\sim \int \mathrm{d}^3 k \left[|\dot{\phi}|^2 + k^2|\phi|^2 + \lambda \boxed{\int \mathrm{d}^3 k_{234}\ \phi(\boldsymbol{k}_1 - \boldsymbol{k}_2 - \boldsymbol{k}_3)\phi(\boldsymbol{k}_3)\phi(\boldsymbol{k}_2 - \boldsymbol{k}_4)\phi(\boldsymbol{k}_4)}\right]
\end{aligned}
$$

contains Fourier space H_int

What are “overlapping dofs”?

Instead, consider imperfect (i.e. overlapping) set of 8 qubits:



How does this work in practice?

Quoting from Friedrich et al. 2024:

1. Choose generators $\boldsymbol{C}_1, \ldots, \boldsymbol{C}_{2n}$ for the Clifford algebra in the Hilbert space of dimension $2^n < 2^N$.
2. Choose a pair $\boxed{\boldsymbol{v}, \boldsymbol{w}}$ of orthonormal vectors in $\mathbb{R}^{2n}$.
3. Define

$$\boxed{\boldsymbol{\sigma}_x \equiv \sum_{j=1}^{2n} \langle e_j|v\rangle \boldsymbol{C}_j \ , \ \boldsymbol{\sigma}_y \equiv \sum_{j=1}^{2n} \langle e_j|w\rangle \boldsymbol{C}_j \ , \ \boldsymbol{\sigma}_z = -i\boldsymbol{\sigma}_x\boldsymbol{\sigma}_y \ ,}$$

   where $\langle e_j|v\rangle$ and $\langle e_j|w\rangle$ are the jth components of $\boldsymbol{v}, \boldsymbol{w}$ in some orthonormal basis $\{\boldsymbol{e}_j\}$ of $\mathbb{R}^{2n}$.
4. Finally, define raising and lowering operators as

$$\boldsymbol{c} = \frac{1}{2}(\boldsymbol{\sigma}_x + i\boldsymbol{\sigma}_y) \ , \ \boldsymbol{c}^\dagger = \frac{1}{2}(\boldsymbol{\sigma}_x - i\boldsymbol{\sigma}_y) \ .$$

each pair of vectors defines
Pauli-algebra of one qubit
=> now choose 8 such random
vector pairs

Further results from studying this model:

A. overlaps lead to long-range correction to real-space propagator

$$\rightarrow \quad \frac{1}{2}\{i\psi^{\alpha}(\mathbf{x})^{\dagger}, \psi_{\alpha}(\mathbf{y})\} = 2i\delta_D(\mathbf{x}-\mathbf{y}) + 2iC(\mathbf{x},\mathbf{y})$$

B. computed full energy spectrum of the holographic field

→ vacuum energy density suppressed by mode overlaps

C. mode overlaps generate a "cosmic fog", i.e. plane waves in the field scatter in vacuum
=> comparison to neutrino observations allows test of holographic principle

holographic principle almost fails the test!

scattering of plane wave in vacuum

- Mode overlaps cause scattering of plane wave excitations in the vacuum!

- We estimate the lifetime to be

$$T_{\text{scatter}} \approx 2\pi^2 \sqrt{\alpha} \left(\frac{\Lambda_{\text{sp}}}{\Lambda_{\text{UV}}} \right)^2 \sqrt{\frac{L}{|\mathbf{p}|}} \, .$$

scattering of plane wave in vacuum

- Mode overlaps cause scattering of plane wave excitations in the vacuum!

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$$\Lambda_{\rm sp} \equiv \Lambda_{\rm Planck}\sqrt{N_{{\rm dof},\psi}/N_{\rm dof,total}}$$

$$T_{\rm scatter} \approx 2\pi^2\sqrt{\boxed{\alpha}}\left(\frac{\boxed{\Lambda_{\rm sp}}}{\boxed{\Lambda_{\rm UV}}}\right)^2\sqrt{\frac{\boxed{L}}{\boxed{|\mathbf{p}|}}}\,.$$

How much overlaps mix energies

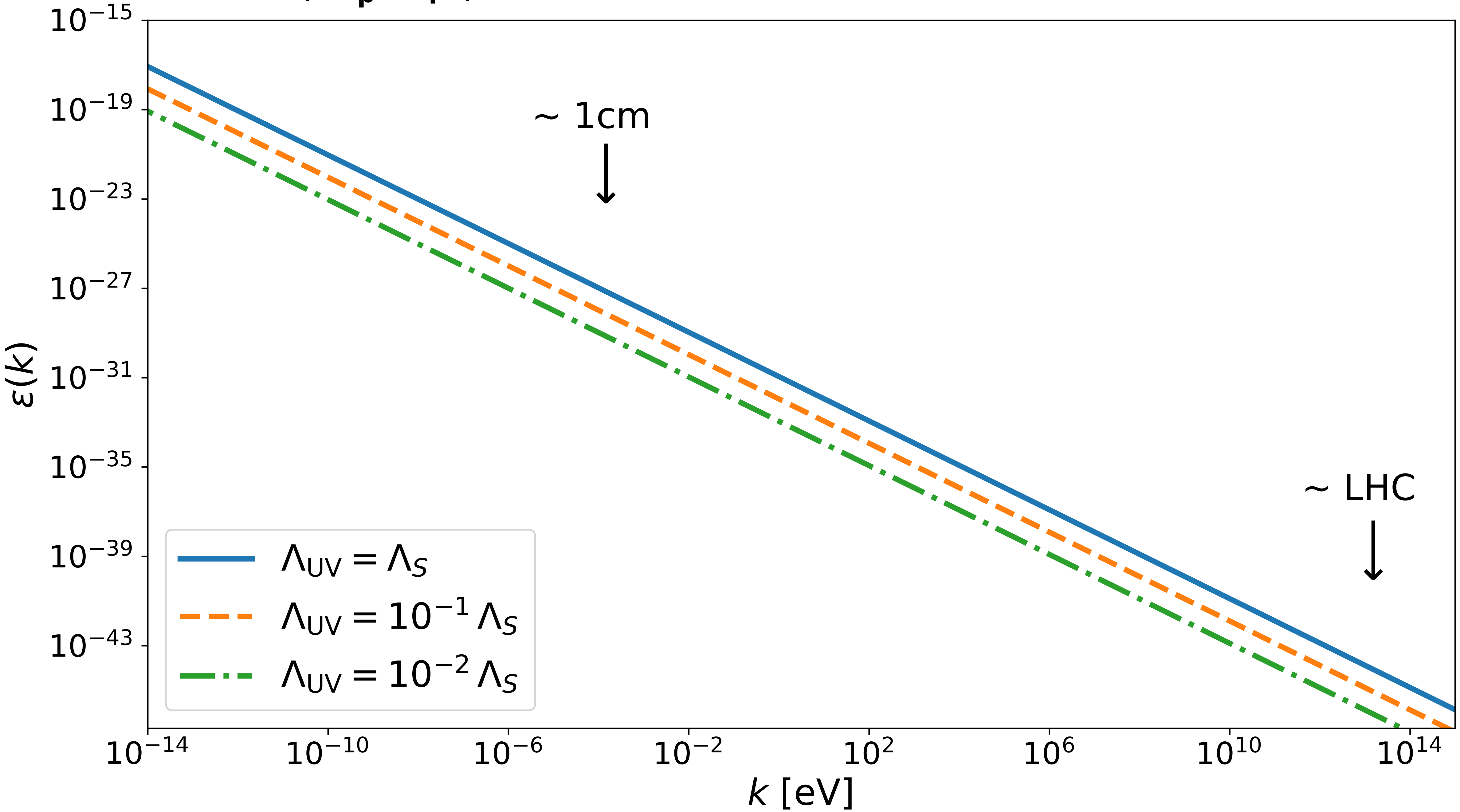
UV cut
i.e. no particles beyond this energy!

IR cut
=
size of cosmic horizon

momentum of particle

To achieve holography, we only have to allow for very low overlaps:

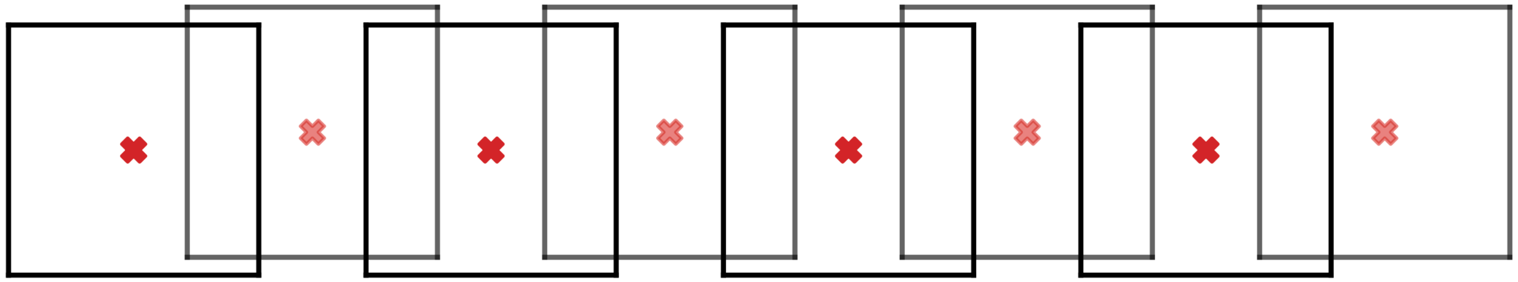
holography induces non-zero anti-commutator for modes $\mathbf{p} \neq \mathbf{q}$, $|\mathbf{p}| \approx k \approx |\mathbf{q}|$
BUT: $|\{\hat{c}^\dagger_{\mathbf{p}}, \hat{c}_{\mathbf{q}}\}| < \varepsilon(k)$ for *all* pairs of modes (99% certainty in CJP)



$$\Lambda_S \equiv \Lambda_{\rm Planck}\sqrt{N_{{\rm dof},\psi}/N_{\rm dof,total}}$$ "species scale"

What are "overlapping dofs"?

Imperfect (i.e. overlapping) set of 8 qubits:



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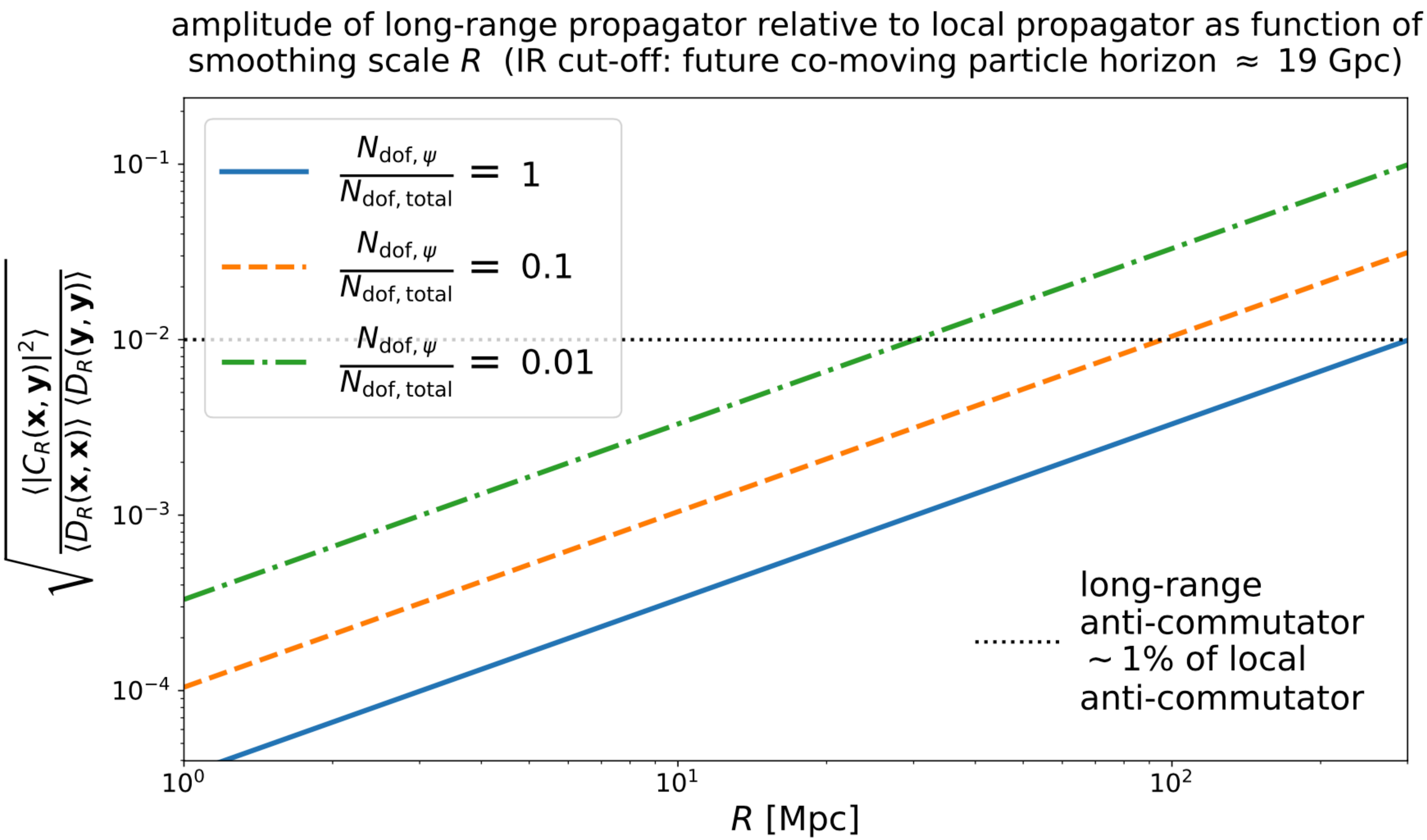
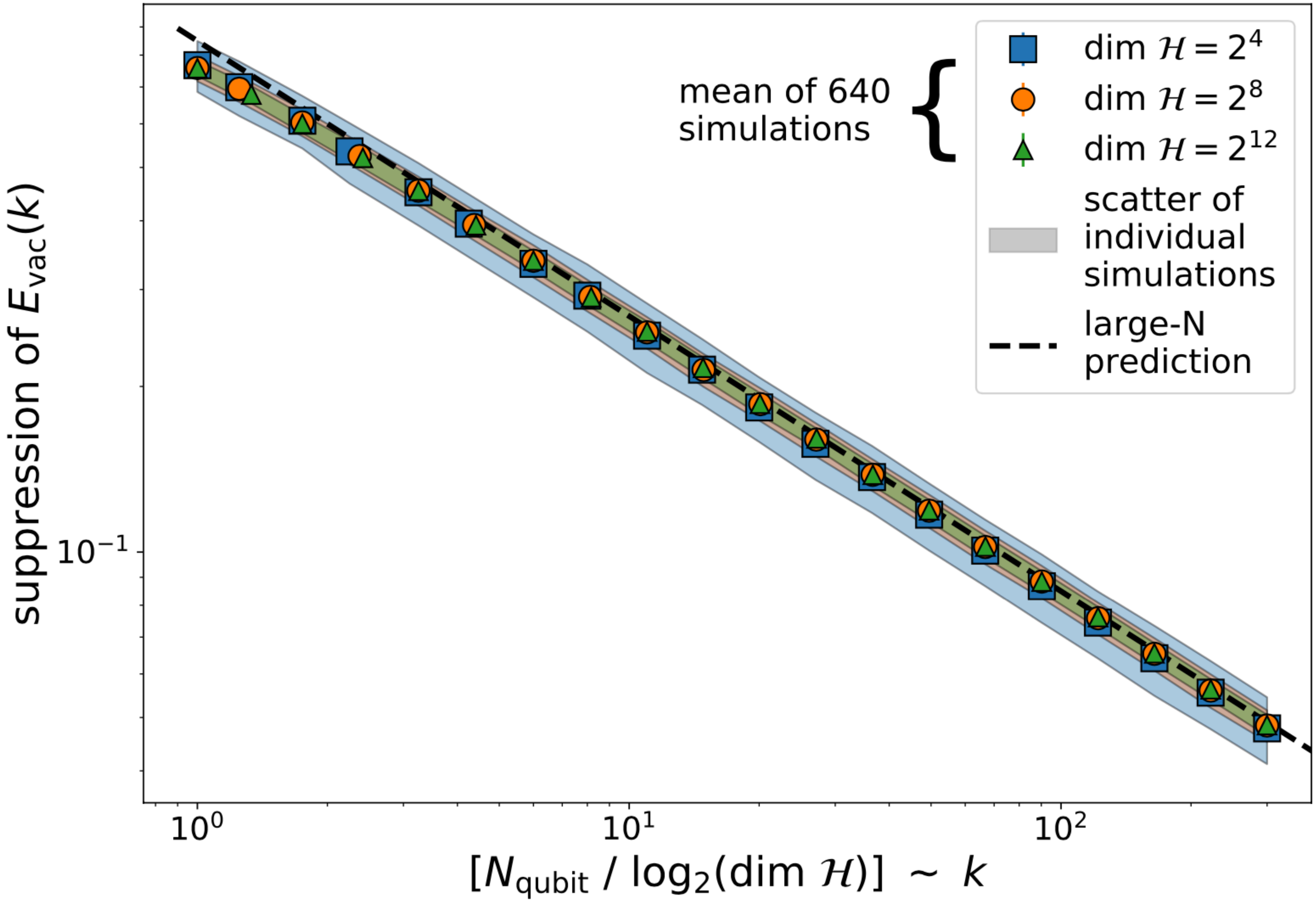
each pair of vectors defines Pauli-algebra of one qubit
=> now choose 8 such random vector pairs

A. Long-range correction to real-space propagator

- field propagator obtains **long-range, stochastic correction**;
  e.g. after tracing out spin dofs: $\{\psi^{\alpha}(\mathbf{x})\ ,\ \psi^{\dagger}_{\alpha}(\mathbf{y})\} \;=\; 2i\delta_D(\mathbf{x}-\mathbf{y}) \;+\; \boxed{iC(\mathbf{x},\mathbf{y})}$
- after smoothing field by radius R, compare local and long-distance propagator:



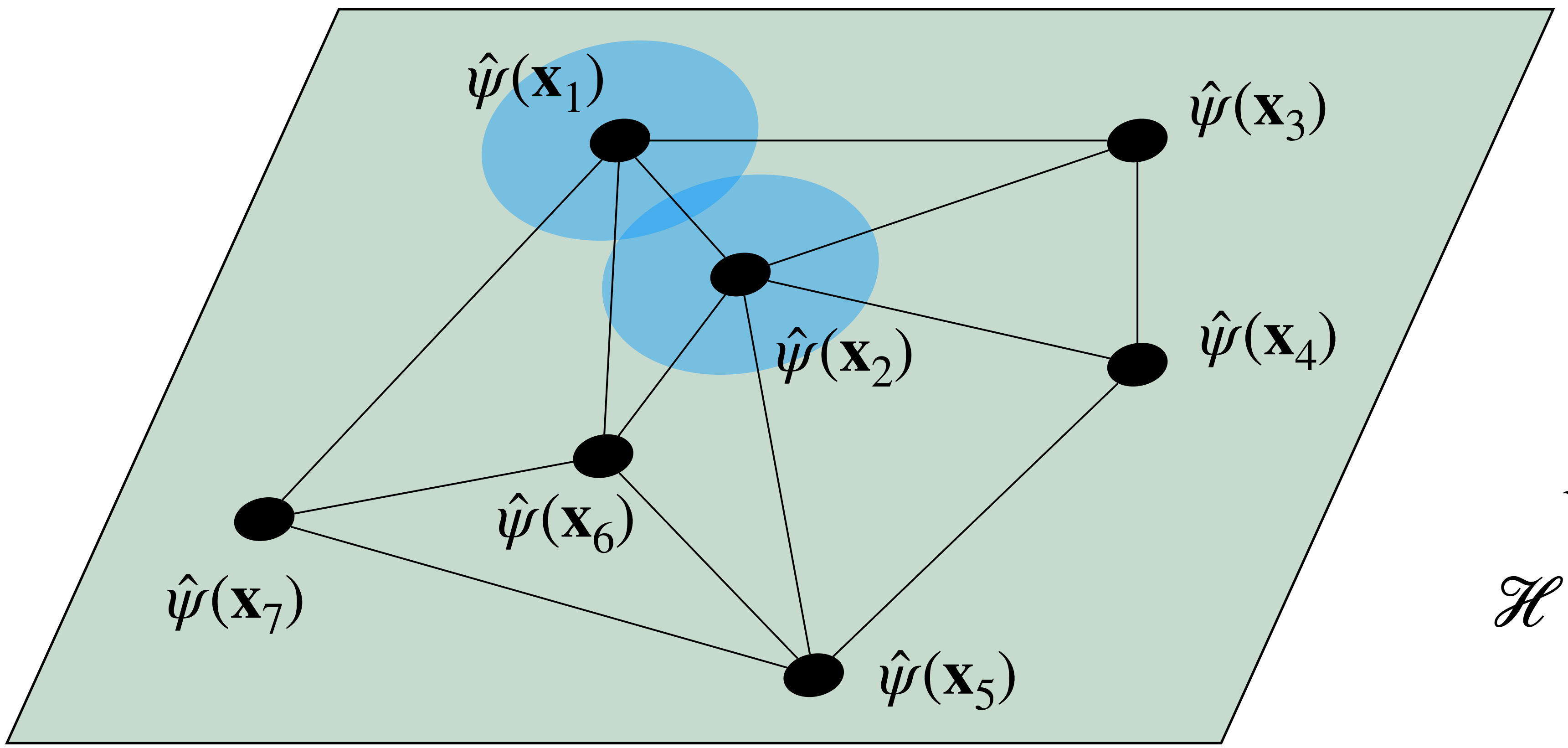
B. Vacuum energy density suppression



- both low-dimensional simulations of overlapping qubits & analytical result indicate suppression of vacuum Energy (compared to standard Weyl field)
  —> analytical prediction is from random matrix theory => will be even more accurate at realistic dimensions
- vacuum energy in shell s is $E_{\mathrm{min},s} \approx -\frac{N_s k_s}{2} \cdot \boxed{\left(\frac{8}{3\pi}\sqrt{\frac{n_s}{N_s}}\right)}$ suppression due to holography $\sim \frac{1}{\sqrt{k_s}}$

Motivation II: operator dressing

dressed field in flat space



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but :

$$\mathscr{H} \not\simeq \mathscr{H}_{\mathbf{x}_1} \otimes \mathscr{H}_{\mathbf{x}_2} \otimes \mathscr{H}_{\mathrm{rest}(\mathbf{x}_1,\mathbf{x}_2)}$$

Gravitational dressing causes non-canonical equal-time commutators!
(“overlaps”, see e.g. https://arxiv.org/abs/1507.07921)

At same time: systems with non-canonical commutators may be embedded in reduced Hilbert space!