EFFECTIVE QUANTUM CORRECTIONS TO BIANCHI MODELS AND ITS CONNECTION TO BKL SCENARIOS

Ana Alonso-Serrano
Humboldt-Universität zu Berlin
And
Max-Planck-Institut für Gravitationsphysik
(Albert-Einstein-Institut)

In collaboration with:
Marek Liska *(Dublin Institute for Advance Studies)*

MOTIVATION

- Behaviour of spacetime near singularities
- Classical approach to singularities: BKL conjecture
- Quantum-gravity modifications?

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- General effective quantum-gravity dynamics
- Preliminary analysis of Bianchi I and Bianchi II models

CLASSICAL DYNAMICS TOWARDS THE SINGULARITY

BKL CONJECTURE

- Evolution towards the singularity: Asymptotic expansion of Einstein equations
  - Locally disconnected and chaotic behaviour of spacetime
  - Behaviour dictated by the vaccum solution (except stiff matter)

[Belinskii, Khalatnikov, Lifshitz]

- Open question but numerous numerical confirmations

[Berger, Garfinkle, Andersson...]

- Quantum effects: Works in the literature in different scenarios

[Nicolai, Henneaux, Kleinschmidth, Kuchar, Ashtekar, Bojowald, Wilson-Ewing, Berger, Montani, Garfinkle, Malkiewicz.....]

- General anisotropic and homogeneous: Expansion near $t \to 0$

$$ds^2 = -dt^2 + \sum_{i,j,k=1}^{3} t^{2p_k} l_i^k l_j^k dx^i dx^j$$

Bianchi-type cosmologies

- Solving Einstein equations:

$$\sum_{k=1}^{3} p_k = 1 \qquad \sum_{k=1}^{3} p_k^2 = 1$$

Extra constraint $$\sum_{i,j,k=1}^{3} \left(l_{i,j}^k - l_{j,i}^k\right) l_n^i l_m^j = 0$$

Kasner universes

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Kasner universes

but..
.

$$\sum_{i,j,k=1}^{3} \left(l_{i,j}^k - l_{j,i}^k\right) l_n^i l_m^j \neq 0.$$

Transforms Kasner into another Kasner

- Oscillatory approach to singularity $\longrightarrow$ Consequtive Kasner epochs

Kasner exponents $-1/3 \leq p_1 \leq 0$ $\quad 0 \leq p_2 \leq 2/3$ $\quad 2/3 \leq p_3 \leq 1$

One-dimensional map

$$p_1 = \frac{-u}{1+u+u^2} \qquad p_2 = \frac{1+u}{1+u+u^2} \qquad p_3 = \frac{u(1+u)}{1+u+u^2}$$

- Transition rules $u \to u-1$ $\longrightarrow$ chaotic map

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- Transition rules $u \to u-1$ $\longrightarrow$ chaotic map

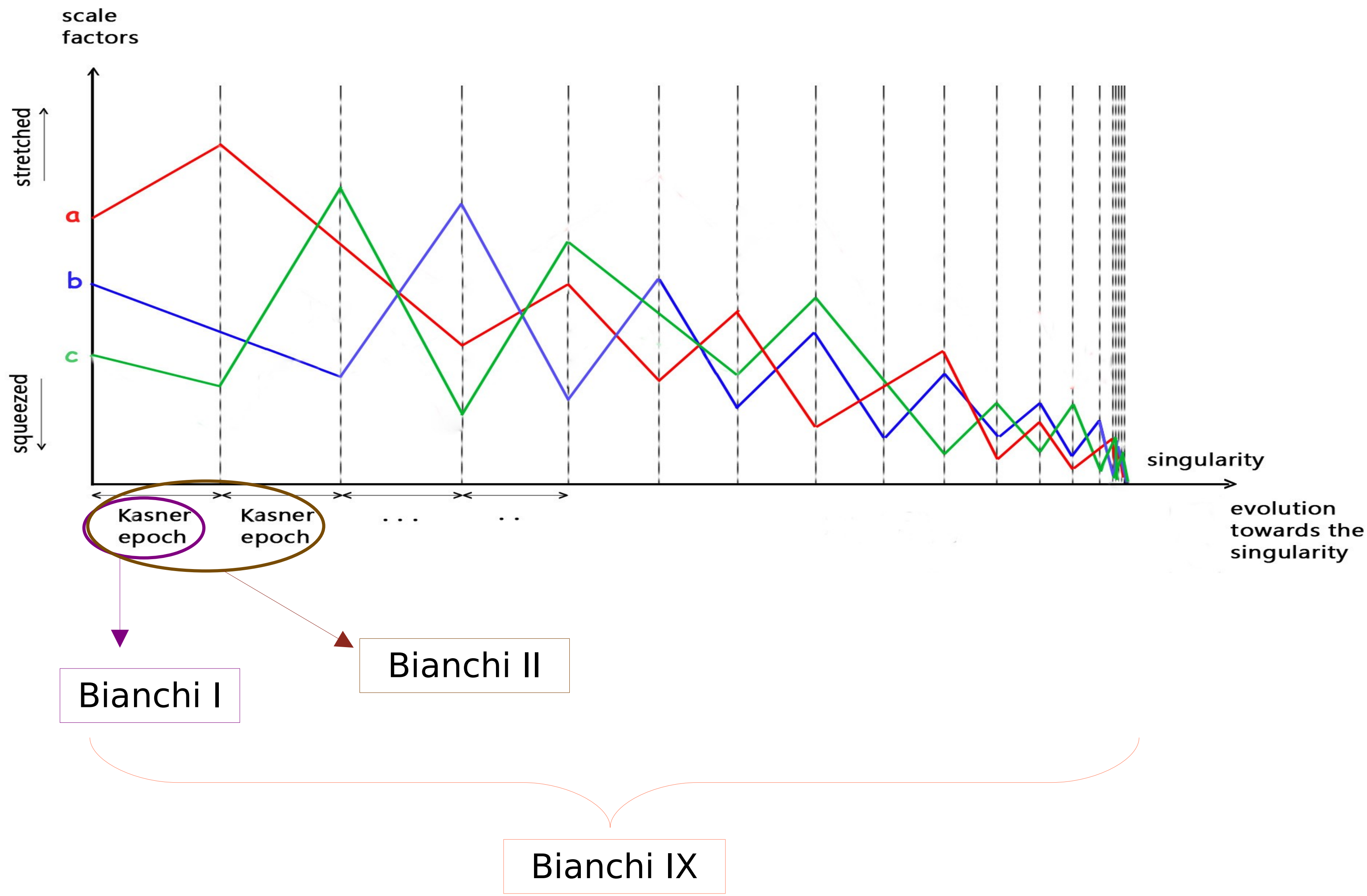
- No influence of matter $\longrightarrow$ but stiff matter stops oscillations at some point

- Locallity: negligible spatial derivatives $\longrightarrow$ particle horizons

- Mixmaster universe $\longrightarrow$ in terms of Misner variables

[C. W. Misner, PRL 22 (1969)]

- We can qualitatively see the evolution towards the singularity as



BIANCHI I AND BIANCHI II MODELS

- Bianchi-type universes

$$ds^2 = -N(t)^2 dt^2 + \sum_{i,j,k=1}^{3} a_k(t)^2 l_i^k l_j^k dx^i dx^j$$

- Bianchi I: $$\sum_{i,j,k=1}^{3} \left(l_{i,j}^k - l_{j,i}^k\right) l_n^i l_m^j = 0$$

- Bianchi II: $$\sum_{i,j,k=1}^{3} \left(l_{i,j}^k - l_{j,i}^k\right) l_n^i l_m^j = \delta_{k3}\epsilon_{knm}$$

- Change to Misner variables $a_k(t) = e^{\alpha(t)+\beta_k(t)}$
  - Spatial volume: $e^{\alpha} = (a_1 a_2 a_3)^{1/3}$
  - Anisotropies: $\beta_k \longrightarrow \beta_1 + \beta_2 + \beta_3 = 0$

Two shape parameters:

$$\begin{cases} \beta_+ := -\dfrac{1}{2}\beta_3 = -\dfrac{1}{2}\ln\left[\dfrac{a_3}{e^{\alpha}}\right] \\ \beta_- := \dfrac{1}{2\sqrt{3}}(\beta_1 - \beta_2) = \dfrac{1}{2\sqrt{3}}\ln\left(\dfrac{a_1}{a_2}\right) \end{cases}$$

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Bianchi I

$$\mathrm{d}s^2 = -e^{6\alpha}\mathrm{d}t^2 + e^{2\alpha}\left[e^{2\beta_+}\left(e^{2\sqrt{3}\beta_-}\mathrm{d}x^2 + e^{-2\sqrt{3}\beta_-}\mathrm{d}y^2\right) + e^{-4\beta_+}\mathrm{d}z^2\right]$$

Bianchi II

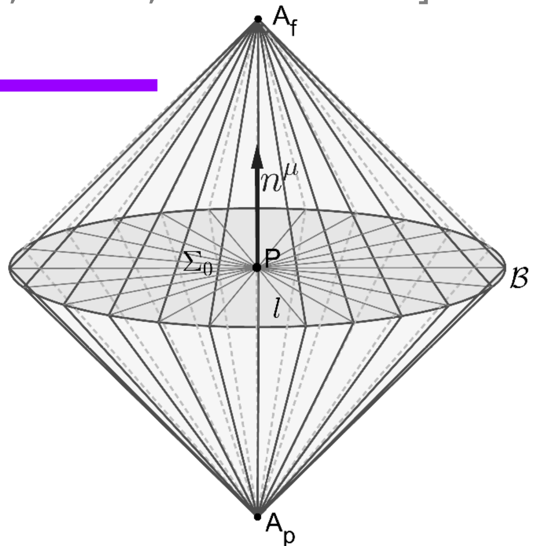
$$\mathrm{d}s^2 = -e^{6\alpha}\mathrm{d}t^2 + e^{2\alpha}\left[e^{2\beta_+}\left(e^{2\sqrt{3}\beta_-}\mathrm{d}x^2 + e^{-2\sqrt{3}\beta_-}\mathrm{d}y^2\right) + e^{-4\beta_+}\left(\mathrm{d}z + x\mathrm{d}y\right)^2\right]$$

EFFECTIVE QUANTUM-GRAVITY DYNAMICS

QUANTUM EFFECTIVE DYNAMICS

- Based on a result for classical dynamics [T. Jacobson, T. Padmanabhan, M. Parikh, A. Svesko, A. J. Speranza, M. Visser, G. Chirco, S. Liberati, A.A-S, M. Liska.....]

Conformal Killing horizon



- Temperature (Unruh effect) $\Rightarrow$ matter entropy
- Entaglement entropy/Bekenstein entropy: $S = \eta \mathcal{A}$

- Local equilibrium condition: $\boxed{\delta S = 0}$ $\Rightarrow$ Einstein equations

$$\delta S_{\mathrm{B}} + \delta S_{\mathrm{m}} = 0 \quad \Rightarrow \quad R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = \frac{8\pi G}{c^4}\left(\delta\langle T_{\mu\nu}\rangle - \frac{1}{4}\delta\langle T\rangle g_{\mu\nu}\right)$$

$$\delta S_{\mathrm{B}} \propto G_{\mu\nu} n^\mu n^\nu$$
$$S_{\mathrm{m}} \propto T_{\mu\nu} n^\mu n^\nu$$

and imposing $T_\mu{}^\nu{}_{;\nu} = 0$

➢ New approach for an effective quantum dynamics

[A.A-S, M. Liska, JHEP 12 (2020)]

Logarithmic corrections to entropy

General expression: $\boxed{S_{\mathrm{q}} = \mathcal{A}/4 + \mathcal{C} \ln\left(\mathcal{A}/l_{\mathrm{P}}^2\right)}$

- Quantum gravity theories (LQG, String theory, Ads/CFT...), phenomenological approaches, entanglement entropy, statistical fluctuations….

[Majumdar, Kaul, Meissner, Sen, Gupta, Carlip, Fareghbal, Karimi, Solodukhin, Adler, Chen….]

➢ We follow an analogous derivation $\delta S_q + \delta S_m = 0$

- We found general effective equations of motion (encoding low-energy quantum gravity effects)

$$\boxed{S_{\mu\nu} - Dl_{\mathrm{P}}^2 \left(S_{\mu\lambda} S_{\nu}{}^{\lambda} - \frac{1}{4} S_{\lambda\rho} S^{\lambda\rho} g_{\mu\nu} \right) = 8\pi G \mathcal{T}_{\mu\nu}}$$

$\sim C$

where $S_{\mu\nu} = R_{\mu\nu} - R g_{\mu\nu}/4$ and $\mathcal{T}_{\mu\nu} = T_{\mu\nu} - T g_{\mu\nu}/4$

- Do not directly generalize Einstein equations of motions (they are traceless)

We need to additionally impose $\nabla_{\nu} T_{\mu}{}^{\nu} = 0$

$$\Downarrow$$

$$8\pi G \nabla_{\nu} T_{\mu}{}^{\nu} = \nabla_{\mu} \left(2\pi G T + \frac{1}{4} R + \frac{Dl_{\mathrm{P}}^2}{4} R_{\lambda\nu} R^{\lambda\nu} - \frac{Dl_{\mathrm{P}}^2}{32} R^2 \right) - Dl_{\mathrm{P}}^2 R^{\lambda\nu} \nabla_{\nu} R_{\mu\lambda}$$

- Cosmological solution

[A.A-S, M. Liska, A. Vicente-Becerril PLB 839 (2023) ,
A.A-S, G. A. Mena-Marugán, A. Vicente-Becerril IJMPD 14 (2023)]

Modified Friedmann equation $$H^2 = \frac{8\pi G}{3}\rho\left(1 - 2\pi\gamma D\frac{\rho}{\rho_{\mathrm{P}}}\right)$$

→ Singularity resolution

- Black hole solution

[A.A-S, M. de Cesare, M. Del Piano, (2025)]

EFFECTIVE BIANCHI I AND II MODELS

EFFECTIVE BIANCHI MODELS

- We couple it to a perfect fluid $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$

  with $p = (\gamma - 1)\,\rho$

  - Local conservation of the energy-momentum tensor classically $\rho = \rho_0 e^{-3\gamma\alpha}$

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EFFECTIVE BIANCHI I AND BIANCHI II

$$3\dot{\alpha}^2 - 3\dot{\beta}_+^2 - 3\dot{\beta}_-^2 - \frac{k}{4}e^{4\alpha - 8\beta_+} = \rho_0 e^{3(2-\gamma)\alpha} - D\frac{\gamma}{4}\rho_0^2 e^{-6(\gamma-1)\alpha}$$

$$-2\ddot{\alpha} + \ddot{\beta}_+ + \sqrt{3}\ddot{\beta}_- + 3\dot{\alpha}^2 - 3\dot{\beta}_+^2 - 3\dot{\beta}_-^2 - \frac{k}{4}e^{4\alpha - 8\beta_+} = (\gamma - 1)\,\rho_0 e^{3(2-\gamma)\alpha} - D\frac{\gamma}{4}\left(2\gamma - 1\right)\rho_0^2 e^{-6(\gamma-1)\alpha}$$

$$-2\ddot{\alpha} - 2\ddot{\beta}_+ + 3\dot{\alpha}^2 - 3\dot{\beta}_+^2 - 3\dot{\beta}_-^2 + \frac{3k}{4}e^{4\alpha - 8\beta_+} = (\gamma - 1)\,\rho_0 e^{3(2-\gamma)\alpha} - D\frac{\gamma}{4}\left(2\gamma - 1\right)\rho_0^2 e^{-6(\gamma-1)\alpha}$$

And then $\nabla_\nu T_\mu{}^\nu = 0$

PRELIMINARY RESULTS

Choosing $N = e^{3\alpha}$ → Volume scale time τ

EFFECTIVE BIANCHI I

- Simple solution to anisotropy variables for any perfect fluid

$$\beta_+ = k_+ + p_+\tau$$
$$\beta_- = k_- + p_-\tau$$

→ Classical solution

➡ No quantum effect on the evolution of anisotropy

- Solution for volume variable for dust $\gamma = 1$

$$P = \sqrt{p_+^2 + p_-^2} \qquad R = 8\pi G \rho_0 \qquad Q = 16\pi^2 \gamma D G \rho_0^2 / \rho_{\mathrm{P}}$$

- Regime: $P^2 < Q/3$

$$e^\alpha = \left(\frac{Q - 3P^2}{R}\right)^{1/3} \cos^{-2/3}\left[\pm\frac{3}{2}\sqrt{\frac{Q}{3} - P^2}\,(\tau - \tau_0)\right]$$

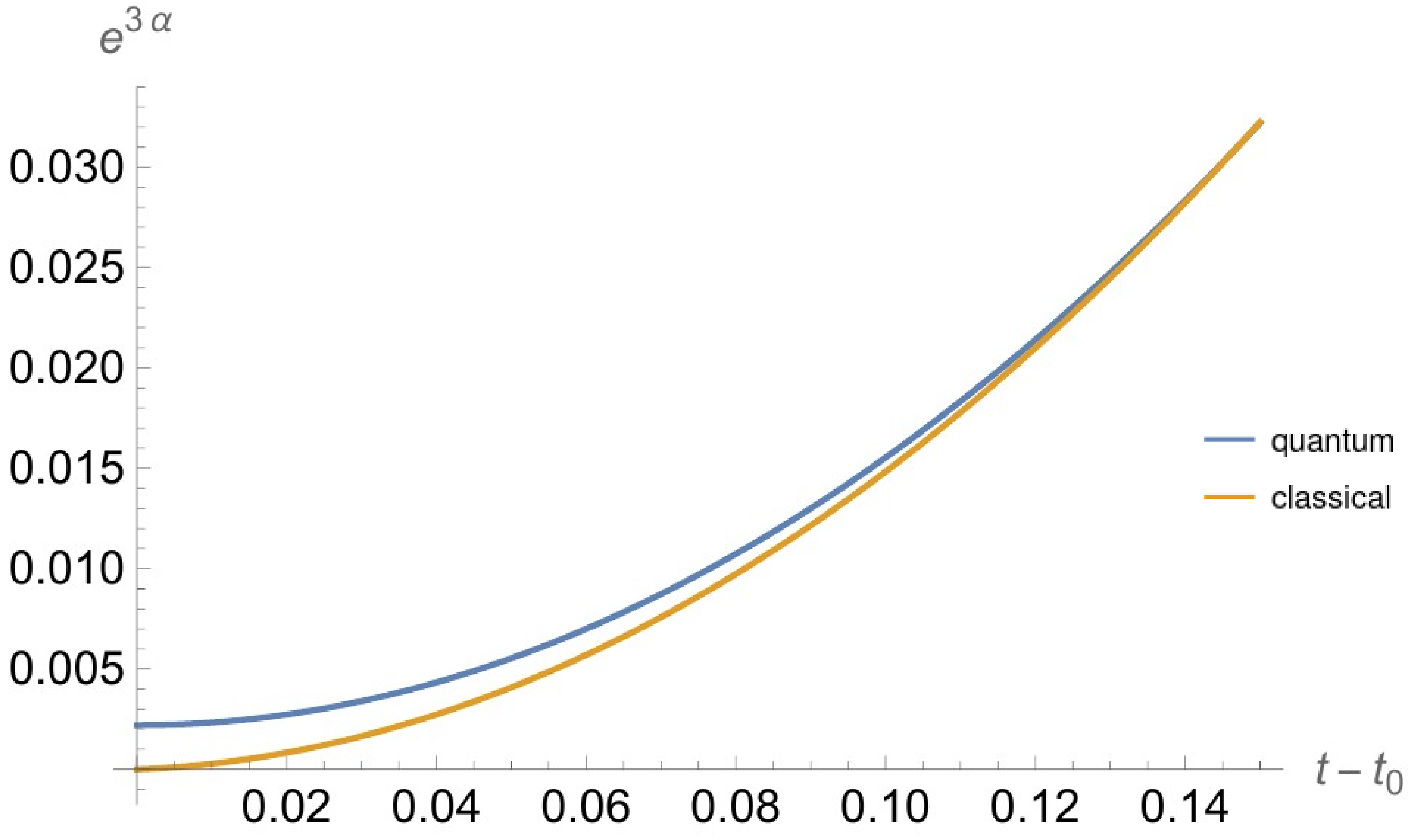
It has a positive minumum at $\tau = \tau_0$

- Regime: $P^2 > Q/3$

$$e^\alpha = \left(\frac{3P^2 - Q}{R}\right)^{1/3} \sinh^{-2/3}\left[\pm\frac{3}{2}\sqrt{P^2 - \frac{Q}{3}}\,(\tau - \tau_0)\right]$$

It goes to zero at $\tau = \tau_0$

Classical Bianchi I + dust + stiff matter



Effective Bianchi I coupled to dust

- Solution for volume variable for stiff matter $\gamma = 2$

  - Regime: $D > 0$

$$e^{\alpha} = \left\{ \frac{\rho_{\mathrm{P}} \left(p_+^2 + p_-^2 + \frac{8\pi G}{3}\rho_0\right)}{32\pi^2 G^2 D \rho_0^2} \cosh^{-2}\left[\pm 3\sqrt{p_+^2 + p_-^2 + \frac{8\pi G}{3}\rho_0}\,(\tau - \tau_0)\right] \right\}^{-1/6}$$

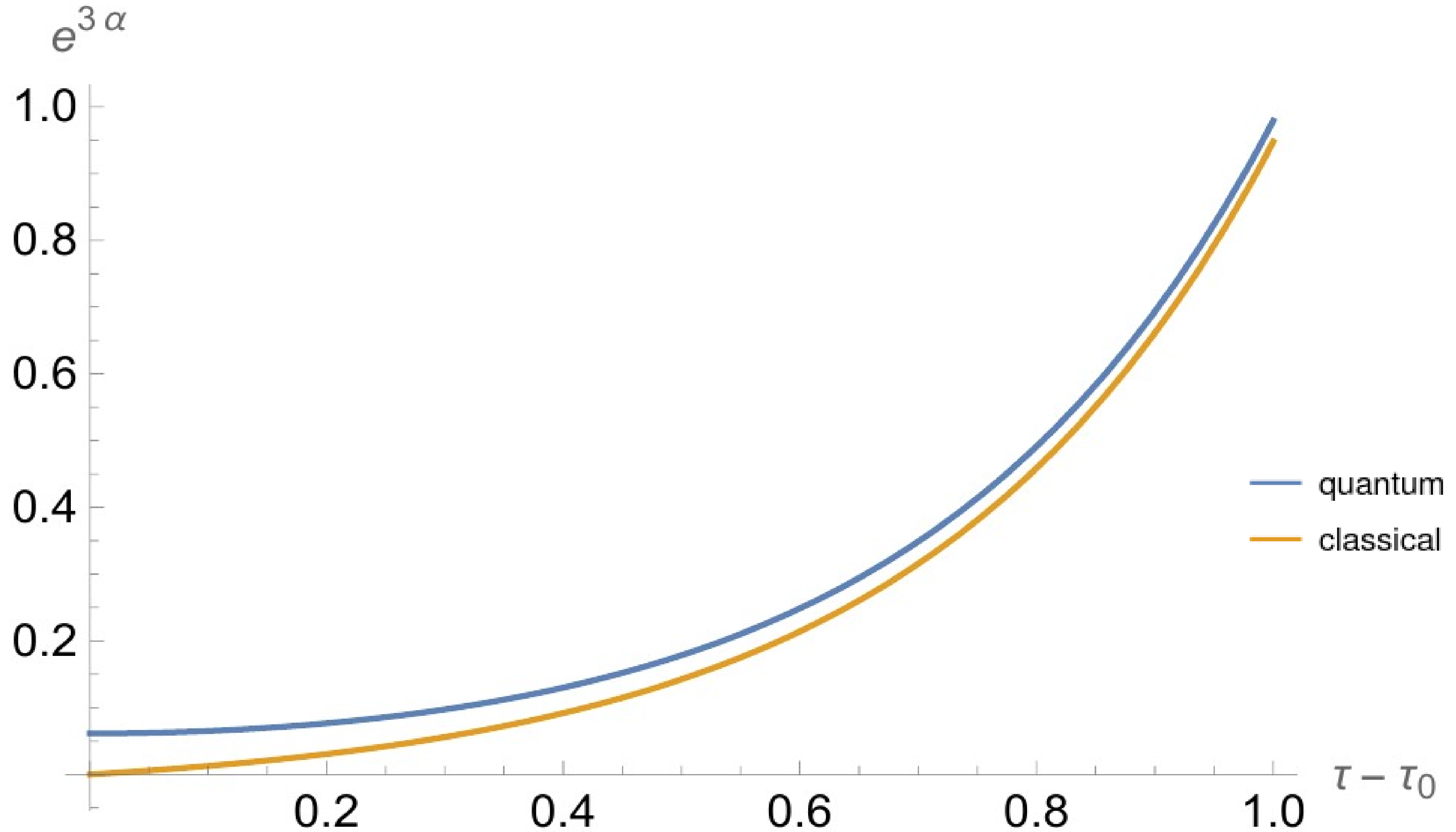
It has a positive minumum at $\tau = \tau_0$

  - Regime: $D < 0$

$$e^{\alpha} = \left\{ -\frac{\rho_{\mathrm{P}} \left(p_+^2 + p_-^2 + \frac{8\pi G}{3}\rho_0\right)}{32\pi^2 G^2 D \rho_0^2} \sinh^{-2}\left[\pm 3\sqrt{p_+^2 + p_-^2 + \rho_0/3}\,(\tau - \tau_0)\right] \right\}^{-1/6}$$

It goes to zero at $\tau = \tau_0$

➡ Numerical solution for any other perfect fluid



Effective Bianchi I coupled to stiff matter

- Bianchi I + dust in cosmic time parametrization

$$\beta_+ = p_+ \frac{1}{\sqrt{Q/3 - P^2}} \arctan\left[\frac{R\,(t-t_0)}{4\sqrt{Q/3-P^2}}\right] + k_+$$

$$\beta_- = p_- \frac{1}{\sqrt{Q/3 - P^2}} \arctan\left[\frac{R\,(t-t_0)}{4\sqrt{Q/3-P^2}}\right] + k_-$$

- Regime: $P^2 < Q/3$

$$e^{3\alpha} = \frac{3R}{4}(t-t_0)^2 + \frac{Q-3P^2}{R}$$ minimum value at $t = t_0$

- Regime: $P^2 > Q/3$

$$e^{3\alpha} = \frac{3R}{4}(t-t_0)^2 \pm \frac{\sqrt{3}\sqrt{3P^2-Q}}{2}(t-t_0)$$ singularity at $t = t_0$

Effective metric:

$$ds^2 = -dt^2 + \left[\frac{3R}{4}(t-t_0)^2 \pm \frac{\sqrt{3}\sqrt{3P^2-Q}}{2}(t-t_0)\right]^{2/3}\left[e^{2\beta_+}\left(e^{2\sqrt{3}\beta_-}dx^2 + e^{-2\sqrt{3}\beta_-}dy^2\right) + e^{-4\beta_+}dz^2\right]$$

CLASSICAL BIANCHI II + STIFF MATTER

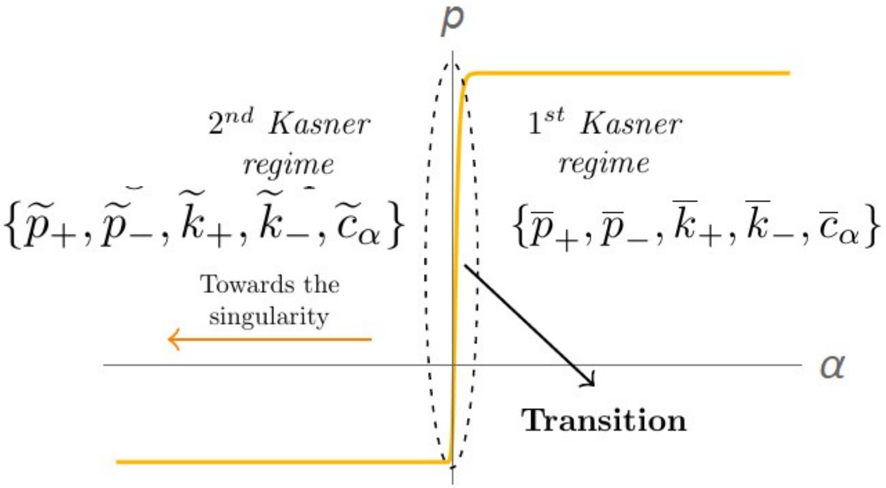
➢ It was found classically that in presence of stiff matter, the solution

$$\beta_+ = k_+ + p_+\tau$$
$$\beta_- = k_- + p_-\tau$$

$$\alpha = \pm\left(p_+^2 + p_-^2 + \frac{\kappa}{3}\rho_{0\mathrm{stiff}}\right)^{1/2}\tau + c_\alpha$$

- The change of epoch follow the transition rules given by

[D. Brizuela, S. F. Uria, PRD (2024)]



[AA-S, D. Brizuela, S. F. Uria, PRD (2021)]

$$\widetilde{p}_+ = \frac{1}{3}\left(4\overline{P} - 5\overline{p}_+\right),$$
$$\widetilde{k}_+ = \overline{k}_+,$$
$$\widetilde{c}_\alpha = \overline{c}_\alpha,$$
$$\widetilde{p}_- = \overline{p}_-,$$
$$\widetilde{k}_- = \overline{k}_-,$$

EFFECTIVE BIANCHI II

- We have preliminary analyzed a dust model

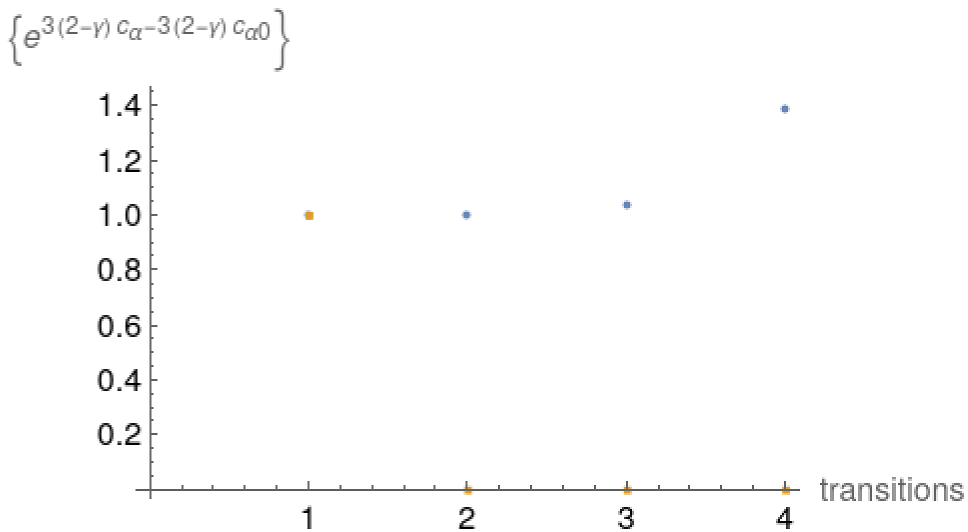
  - Evolution of the anisotropy is affected by quantum corrections

  - Effective Bianchi II + dust = classical Bianchi II + stiff matter + dust

  - Classical transition rules are modified

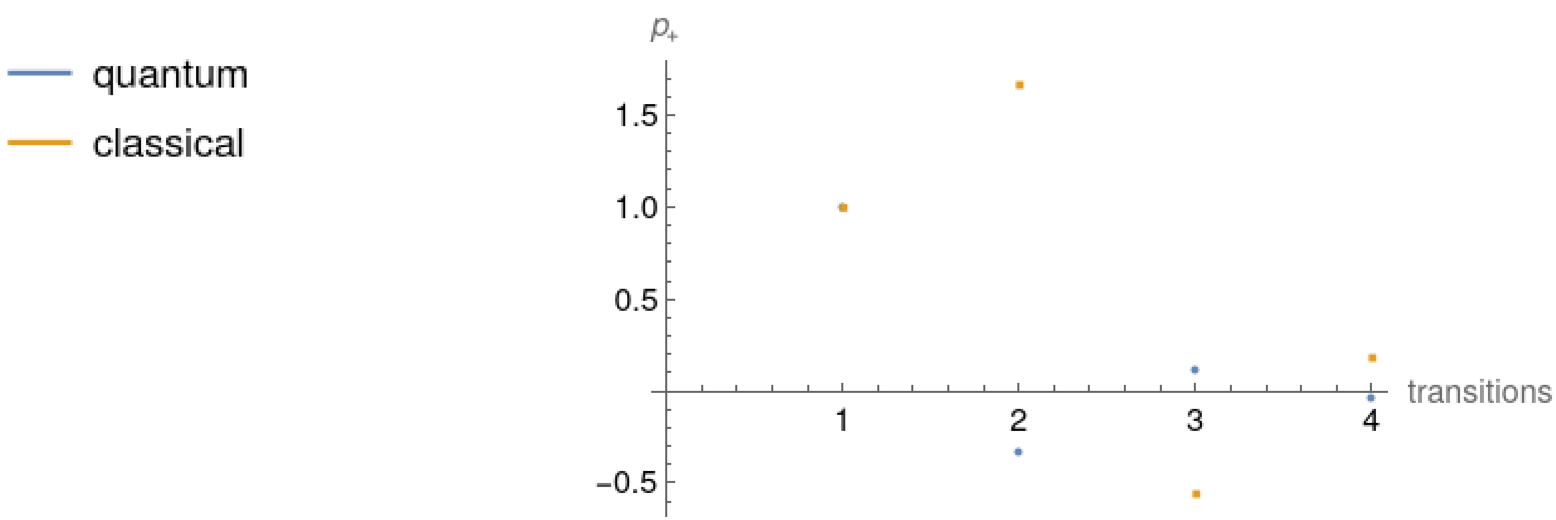
… Still work in progress

- Preliminary graphs for fluids $\gamma > 1$



Strength of matter corrections

With each transition, the correction gets stronger



FUTURE PERSPECTIVE

- Complete general solution for the effective Bianchi I coupled to a perfect fluid

- Full solution for Bianchi II and expression for the transition rules

- Extension to Bianchi IX complete derivation

- Could we derive a BKL asymptotic behaviour from general equations?

Thank you very much for your attention!