Asymptotically Safe Quantum Gravity on foliated spacetimes

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F. Saueressig, J. Wang, arXiv:2306.10408

G. Korver, F. Saueressig, J. Wang, arXiv:2402.01260

F. Saueressig, J. Wang, arXiv:2501.03752

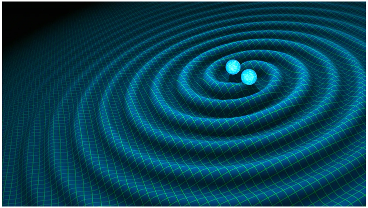
FAU^2 workshop on quantum gravity across scales

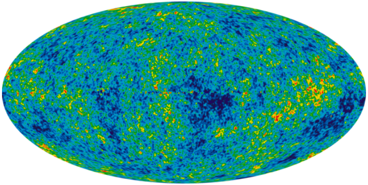
May 20$^{\rm th}$, 2025

Outline

1. motivation
2. basics of asymptotic safety
3. the renormalized graviton 2-point function
   - phase diagram of the graviton mass
   - IR fixed points
   - results for Lorentzian signature
4. summary and outlook

Motivation I: phenomenology from first principles





- graviton propagators in a flat background [today]
- tensor fluctuations in the CMB [future]

can we use these observations to constrain quantum gravity?

Motivation II: Asymptotic Safety at Lorentzian signature

Asymptotic Safety transit: from Euclidean to Lorentzian signature

- ADM-decomposition providing a foliation of spacetime

  [E. Manrique, S. Rechenberger, F. Saueressig, arXiv:1102.5012]

- spectral functions on flat background

  [J. Fehre, D. F. Litim, J. M. Pawlowski, M. Reichert, arXiv:2111.13232]

- algebraic quantum field theory + renormalization group

  [E. D'Angelo, N. Drago, N. Pinamonti, K. Rejzner, arXiv:2202.07580]

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natural connections:

- Causal Dynamical Triangulations (CDT)

  [J. Ambjorn, R. Loll, arXiv:2401.09399]

- canonical quantum gravity

  [T. Thiemann, arXiv:2404.18220]
  [R. Ferrero, T. Thiemann, arXiv:2404.18224]

Basics of the

gravitational asymptotic safety program

Asymptotic Safety in the Handbook of Quantum Gravity

[editors: C. Bambi, L. Modesto and I.L. Shapiro, Springer Singapore]

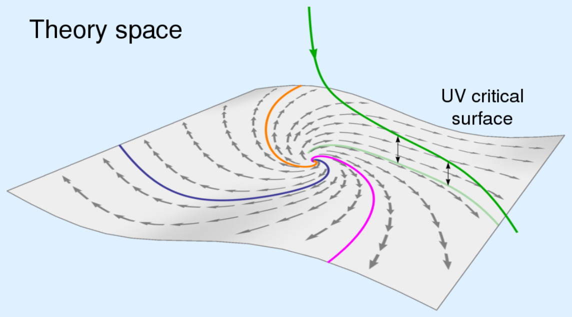
[R. Percacci - organizing the Asymptotic Safety Section]

- *The Functional Renormalization Group in Quantum Gravity*,
  F. Saueressig, arXiv:2302.14152
- *Quantum Gravity from dynamical metric fluctuations*,
  J.M. Pawlowski, M. Reichert, arXiv:2309.10785
- *Asymptotic safety of gravity with matter*,
  A. Eichhorn, M. Schiffer, arXiv:2212.07456
- *Form Factors in Asymptotically Safe Quantum Gravity*,
  B. Knorr, C. Ripken, F. Saueressig, arXiv:2210.16072
- *The Functional $f(R)$ Approximation*,
  T.R. Morris, D. Stulga, arXiv:2210.11356
- *Perturbative approaches to non-perturbative quantum gravity*,
  R. Martini, G.P. Vacca, O. Zanusso, arXiv:2210.13910
- *Quantum gravity and scale symmetry in cosmology*,
  C. Wetterich, arXiv:2211.03596.
- *Black Holes in Asymptotically Safe Gravity*,
  A. Platania, arXiv:2302.04272.

The arena

- input
  - a) field content (e.g., spacetime metric $g_{\mu\nu}$)
  - b) symmetries (e.g., coordinate transformations)
- actions $=$ combinations of interaction monomials $\mathcal{O}$
  - build from the field content
  - compatible with symmetries (e.g., $\mathcal{O}[g] = \int d^4x \sqrt{g} R$)
- theory space $=$ space containing all actions
  - coordinates: couplings $\{u_i\}$ (e.g., G, Λ)
- Wilsonian renormalization group flow:
  - couplings run when integrating out quantum fluctuations (e.g., $k\partial_k u_i = \beta_i(\{u_i\})$)

The asymptotic safety conjecture

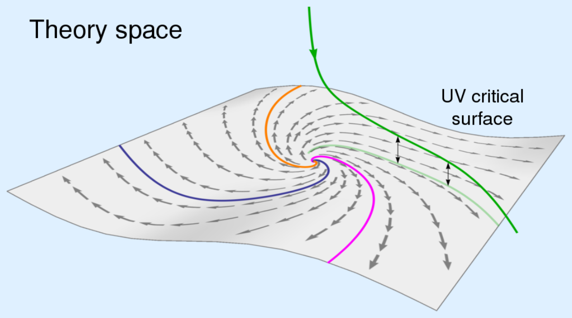


high-energy completion of gravity in $d = 4$

controlled by an interacting renormalization group fixed point

The asymptotic safety conjecture

fixed point provides predictive power:



- fixed points are saddle points:
  - relevant RG trajectories = end at the fixed point in the UV
  - irrelevant RG trajectories = go somewhere else

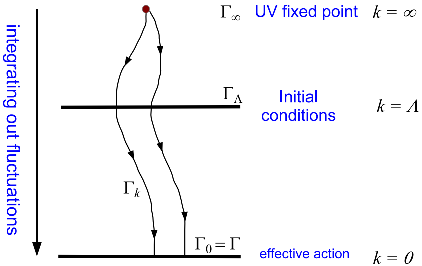
Computing renormalization group flows

Wetterich equation

[C. Wetterich, Phys. Lett. **B301** (1993) 90]
[M. Reuter, Phys. Rev. D **57** (1998) 971, hep-th/9605030]

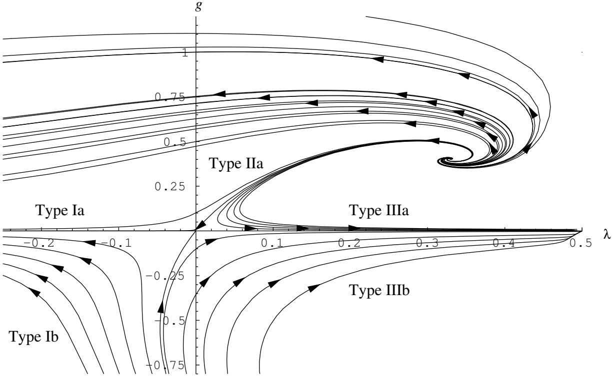
$$k\partial_k \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

- flow equation for the effective average action Γ_k
- integrates out fluctuations shell-by-shell in momentum space



Wilsonian RG flow of the Einstein-Hilbert action

[M. Reuter, Phys.Rev.D 65 (2002) 065016]



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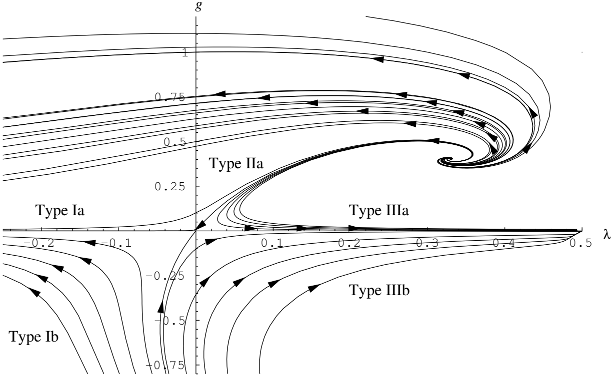


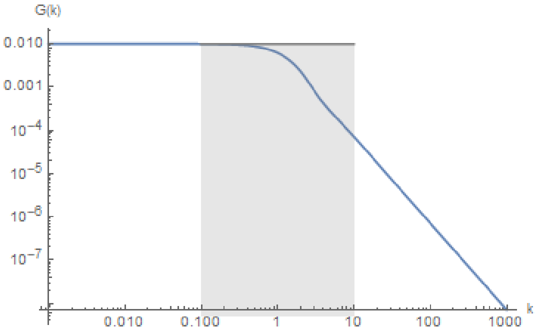
fixed point persists for a wide range of gravity-matter systems

[A. Eichhorn, Front.Astron.Space Sci. 5 (2019) 47]

Connection to classical general relativity

renormalization group flow connects classical and quantum phase:

- Newton's coupling changes with the coarse-graining scale k:



sub-Planckian | trans-Planckian

classical phase | fixed point scaling

Asymptotic Safety - today

1. gravity has the interacting fixed point (Weinberg's conjecture)

   Reuter fixed point

2. fixed point delivers predictive power
   - potential relations to couplings in particle physics

3. effective field theory: reached after crossover

4. profound consequences for cosmology and black holes

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Asymptotic Safety is an attractive Quantum Gravity program

Physics from the effective average action

physics should be analyzed based on $\Gamma = \lim_{k \to 0} \Gamma_k$

quantum field theory:

- all quantum fluctuations should be integrated out

conceptual consequences for the energy dependence of couplings:

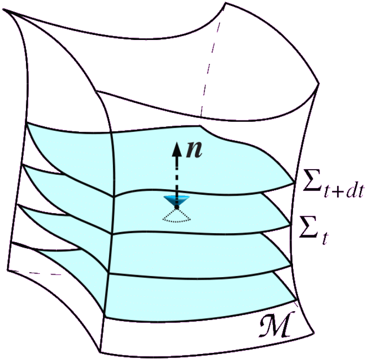
- k-dependence is not observable
- energy-dependence of couplings $\Longrightarrow$ form factors in Γ
  - Newton's coupling: no energy dependence
  - cosmological constant: no energy dependence
- resolves puzzle about gravity-mediated scattering processes

[J. Donoghue, Front. in Phys. 8 (2020) 56]

Asymptotic Safety

on foliated spacetimes

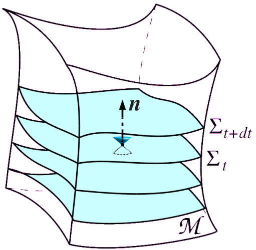
Equip spacetime with a foliation



Arnowitt-Deser-Misner (ADM) decomposition of the metric:

$$
\begin{aligned}
ds^2 =& g_{\mu\nu} dx^\mu dx^\nu \\
=& -N^2 dt^2 + \sigma_{ij} \left(N^i dt + dy^i\right) \left(N^j dt + dy^j\right)
\end{aligned}
$$

ADM formalism



virtues:

- allows to transit between Lorentzian and Euclidean signature
    - provides a preferred "time"-direction
- closely related to observables (Bardeen variables)

Where is the graviton?

- ADM-fields: $g_{\mu\nu} \mapsto \{N, N_i, \sigma_{ij}\}$

- introduce fluctuations in a flat background

$$\sigma_{ij} = \bar{\sigma}_{ij} + \hat{\sigma}_{ij}, \quad N_i = \bar{N}_i + \hat{N}_i, \quad N = \bar{N} + \hat{N}$$

with

$$\bar{\sigma}_{ij} = \delta_{ij}, \quad \bar{N} = 1, \quad \bar{N}_i = 0$$

- transverse-traceless decomposition of $\hat{\sigma}_{ij}$: $(\Delta = -\partial^2)$

$$\hat{\sigma}_{ij} = h_{ij} + \partial_i \frac{1}{\sqrt{\Delta}} v_j + \partial_j \frac{1}{\sqrt{\Delta}} v_i + \partial_i \partial_j \frac{1}{\Delta} E + \frac{1}{3} \delta_{ij} E + \frac{1}{3} \delta_{ij} \psi$$

  - h_{ij} carries 2 degrees of freedom
  - h_{ij} is gauge-invariant

Target: graviton-two point function

object of interest: 2-point correlation function of h_{ij}

$$\Gamma = \frac{1}{32\pi G} \int_x h_{ij} \Big[-\partial^2 + \underbrace{\mu^2}_{\text{graviton mass}} \Big] h^{ij}$$

- gives the graviton propagator on a flat background
- variation with respect to $h_{ij} \Longrightarrow$ massive wave equation

$$\left(-\partial^2 + \mu^2\right) h_{ij} = 0$$

phenomenology: strong constraints on the graviton mass

$$\mu \lesssim 8.6 \times 10^{-24}\,\mathrm{eV}$$

Graviton propagation in quantum gravity

left-hand side of the flow equation:

- 2-point correlation function of h_{ij}

$$\Gamma_k = \frac{1}{64\pi G_k} \int_p h_{ij} \left[p^2 + \mu_k^2 \right] h^{ij}$$

tracks the RG flow of two couplings:

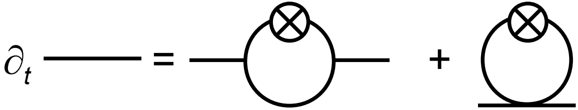
- G_k: wave function renormalization
- μ_k^2: graviton mass

physics: end-point of a RG trajectory ($k = 0$)

Flow of a two-point function

take two functional derivatives with respect to the graviton h_{ij}:

$$\partial_t \Gamma_k^{(2)} = \mathrm{STr}\Big[\mathcal{G}\,\Gamma_k^{(3)}\,\mathcal{G}\Gamma_k^{(3)}\,\mathcal{G}\,\partial_t \mathcal{R}_k\Big] - \frac{1}{2}\mathrm{STr}\Big[\mathcal{G}\,\Gamma_k^{(4)}\,\mathcal{G}\,\partial_t \mathcal{R}_k\Big] \, .$$



- $\mathcal{G}$: regulated propagator
- flow of $\Gamma_k^{(2)}$ determined by 3-point and 4-point vertex

! warning active !

technical interlude ahead

Let's talk about: gauge fixing

a) common choice: gauge-fixing lapse and shift

$$\hat{N} = 0 \,, \qquad \hat{N}_i = 0$$

$\Longrightarrow$ propagators for other fields are ill-defined

b) harmonic gauge condition (linearized; flat background)

$$\Gamma_k^{\rm gf} = \frac{1}{16\pi G_k} \int d\tau d\vec{y} \, \left[\delta^{ij} F_i F_j + F^2\right]$$

with

$$\begin{aligned} F =& \partial_\tau \hat{N} + \partial^i \hat{N}_i - \frac{1}{2}\partial_\tau \psi \\ F_i =& \partial_\tau \hat{N}_i + \partial_i \hat{N} - \frac{1}{2}\partial_i \psi + \partial^j \hat{\sigma}_{ij} \end{aligned}$$

$\Longrightarrow$ all propagators are relativistic!

Let's talk about: Lorentz symmetry breaking

fluctuation fields are defined via the linear split

$$N = \bar{N} + \hat{N}\,, \qquad N_i = \bar{N}_i + \hat{N}_i\,, \qquad \sigma_{ij} = \bar{\sigma}_{ij} + \hat{\sigma}_{ij}$$

a) gauge fixing $\Longrightarrow$ Lorentz-invariant 2-point functions

3- and 4-point vertices break Lorentz invariance

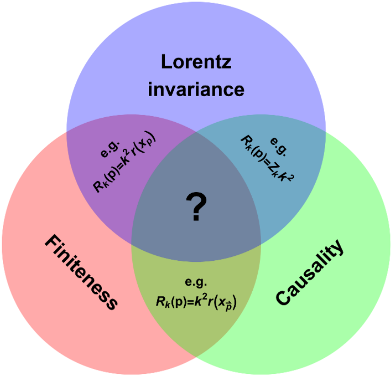
b) adding non-linear terms to split reorganizes vertex structure

[F.S., J. Wang, in preparation]

$\Longrightarrow$ Lorentz invariant flows for 2-point functions

Let's talk about: regulators $R_k(p^2)$

[J. Braun, et. al., arXiv:2206.10232]



- impossible to have all 3 criteria at once?
- regulators satisfying Finiteness & Causality less stable?

! warning stopped !

technical interlude done

flow of the graviton 2-point function

fixed points, phase space, graviton mass

Closure of the flow equation

ansatz for Γ_k (gravity + scalar matter + $U(1)$-gauge fields):

$$\Gamma_k \simeq \Gamma_k^{\rm EH} + \Gamma_k^{\rm gf} + S^{\rm ghost} + S^{\rm scalar} + S^{\rm vector} \, .$$

where

$$\Gamma_k^{\rm EH} = \frac{1}{16\pi G_k} \int dt d^3y N \sqrt{\sigma} \left(K^{ij} K_{ij} - K^2 -^{(3)} R + 2\Lambda_k \right)$$

$$S^{\rm scalar} = \frac{1}{2} \sum_{n=1}^{N_s} \int dt d^3y N \sqrt{\sigma} \, g^{\mu\nu} \left(\partial_\mu \phi\right) \left(\partial_\nu \phi\right)$$

$$S^{\rm vector} = \frac{1}{4} \sum_{n=1}^{N_v} \int dt d^3y \, N \sqrt{\sigma} \, F_{\mu\nu} F^{\alpha\beta} + S^{\rm gauge-fixing} + S^{\rm ghosts}$$

- generates: graviton 2-point function + 3- and 4-point vertices

Beta functions and interacting fixed points

introduce dimensionless couplings

$$g_k = k^2 G_k \, , \qquad \lambda_k = k^{-2} \Lambda_k \, , \qquad \mu_k^2 = -2k^2 \lambda_k$$

beta functions from evaluating the projected flow equation

$$k\partial_k \lambda_k = \beta_\lambda(g_k, \lambda_k; N_s, N_v) \, , \qquad k\partial_k g_k = \beta_g(g_k, \lambda_k; N_s, N_v)$$

- depend on the number of matter fields N_s, N_v
- very complicated expressions

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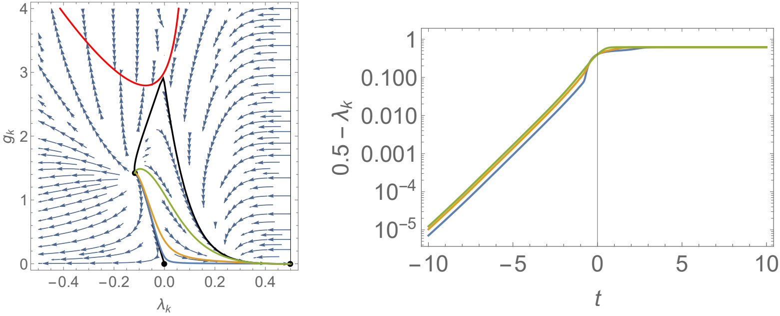
special interest: non-Gaussian fixed points (NGFPs)

$$\beta_g(g_*, \lambda_*) = 0 \, , \quad \beta_\lambda(g_*, \lambda_*) = 0$$

- allow to take $k \to \infty$ in a controlled way

Phase diagram I: pure gravity



- RG flow: dominated by NGFP, GFP, IR-FP

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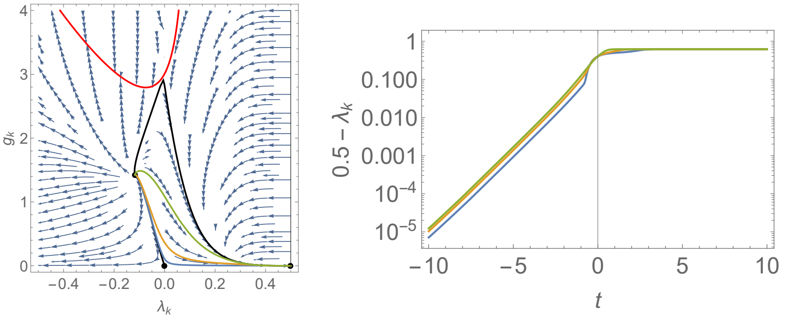
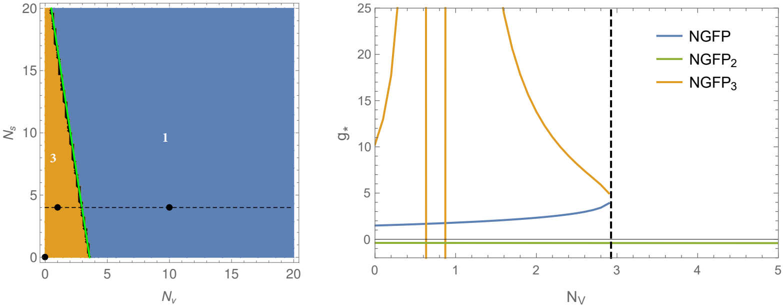


- RG flow: dominated by NGFP, GFP, IR-FP
- IR-FP is an attractor for $k \to 0$

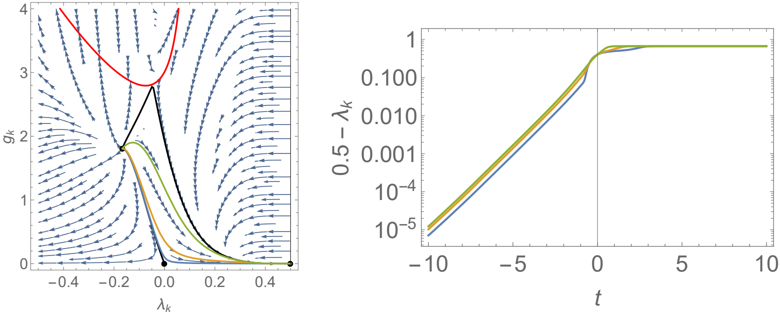
$\Longrightarrow$ zero renormalized graviton mass!

Fixed points in the presence of matter fields



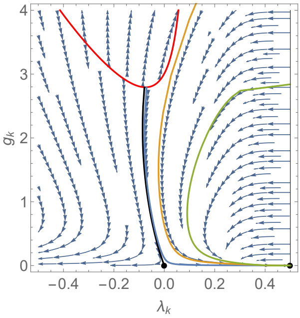
- NGFP (Asymptotic Safety): exists in the orange region
- green line: NGFP annihilates into the complex plain

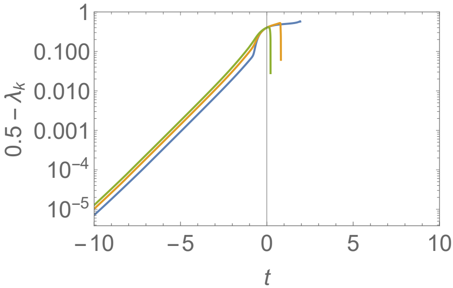
Phase diagram II: $N_s = 4$, $N_v = 1$



- qualitatively identical to the pure gravity case!

Phase diagram III: $N_s = 4$, $N_v = 10$





- NGFP has annihilated
- IR-FP persists also in the absence of a UV-completion
  - IR-FP: effective field theory effect

The transit to Lorentzian signature

Lorentzian Signature RG flows

[J. Wang, F.S., Phys.Rev.D 111 (2025) 106007, arXiv:2501.03752]

properties of the ADM-decomposition

- the foliation allows to decompose $p^\mu \mapsto \{ \, p^0 \, , \, \vec{p} \, \}$
- we can use regulators $\mathcal{R}_k(\vec{p}^{\,2})$
  - pro: does not modify causal structure of propagators
  - pro: traces are finite
  - con: regularization is not Lorentz covariant

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analytic continuation of the Lapse function (no complex geometry)

$$N(t, \vec{y}) \mapsto iN(t, \vec{y})$$

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! beta functions for both signatures are identical !

Lorentzian Signature RG flows: phase diagram

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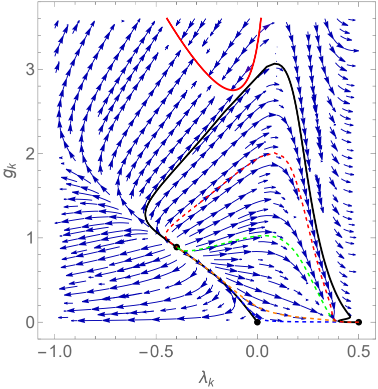

- RG flow: dominated by UV-FP, GFP, IR-FP
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Summary and Outlook

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graviton 2-point function in the ADM-formalism:

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remarkable properties of the RG flow:

- non-Gaussian fixed point: (high-energy completion)
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    - generalizes to gravity-matter systems (mild conditions)
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! identical results for Lorentzian signature !

Outlook

phenomenology of cosmological observables:

- flat background $\Longrightarrow$ cosmological background
- generalize momentum dependence of 2-point function
- 2-point correlators $\Longrightarrow$ 3-point correlators (non-Gaussianities)

new effects appearing at Lorentzian signature:

- study vacuum dependence of the Lorentzian flow equation

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Questions?