

Gravitational Waves

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Astroparticle School September 2025

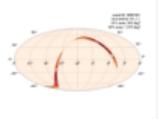
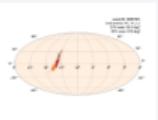
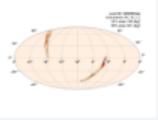
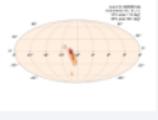


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FOR GRAVITATIONAL PHYSICS
(ALBERT EINSTEIN INSTITUTE)



Public Alerts

SORT: EVENT ID (A-Z) ▾

Event ID	Possible Source (Probability)	Significant	UTC	GCN	Location	FAR
S250904cv	BBH (>99%)	Yes	Sept. 4, 2025 13:49:52 UTC	GCN Circular Query Notices VOE		1 per 100.04 years
S250904br	BBH (>99%)	Yes	Sept. 4, 2025 10:22:08 UTC	GCN Circular Query Notices VOE		1 per 100.04 years
S250904ae	BBH (>99%)	Yes	Sept. 4, 2025 03:33:07 UTC	GCN Circular Query Notices VOE		1 per 100.04 years
S250901cb	BBH (>99%)	Yes	Sept. 1, 2025 18:59:41 UTC	GCN Circular Query Notices VOE		1 per 9.9253e+05 years

Topics



1. Interpreting the Signal

- The Inverse Problem
- Waveform Models
- Bayesian Parameter Estimation
- Stochastic Sampling

2. Gravitational-Wave Observations

- Observing Runs

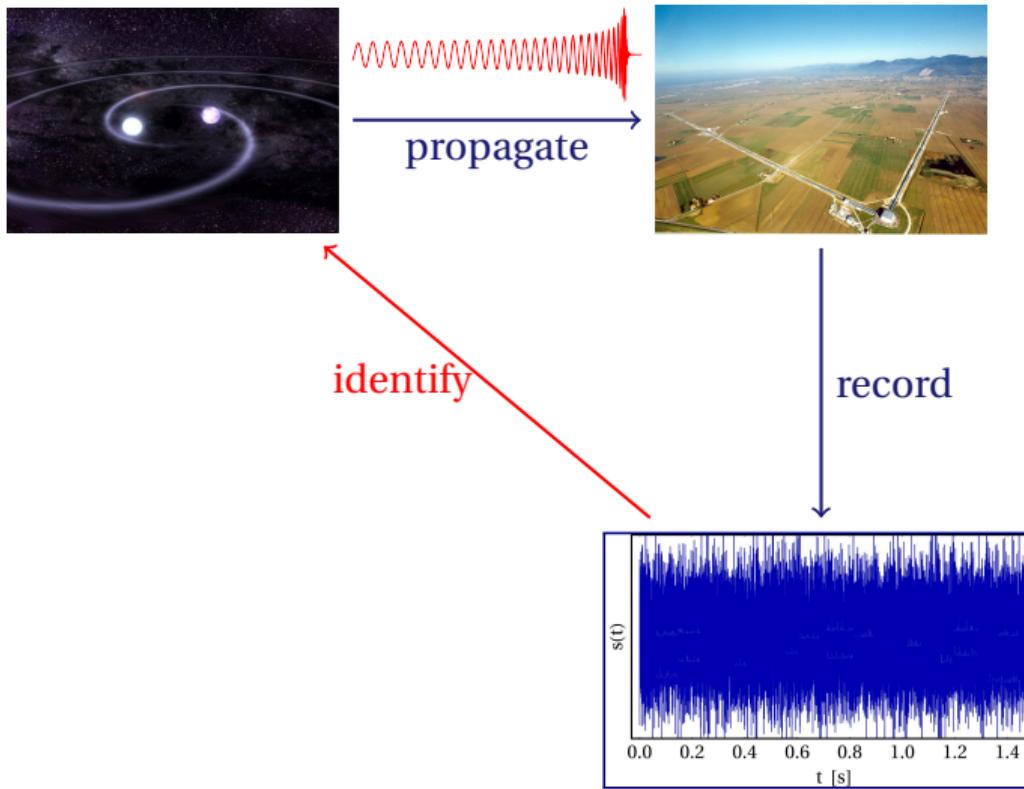
■ Masses and Spins

- Stellar Graveyard
- Select Highlights

3. The Future

- LISA
- Einstein Telescope
- Conclusions

Interpreting the Signal

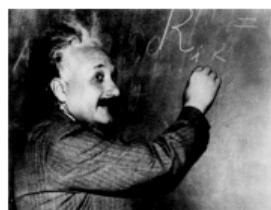




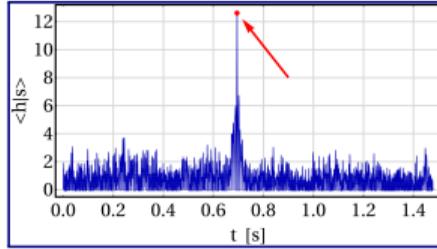
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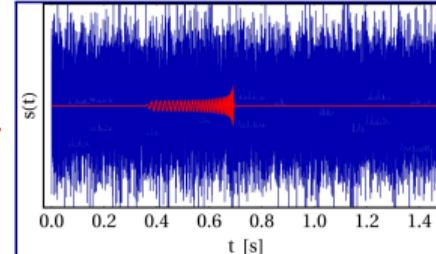
identify



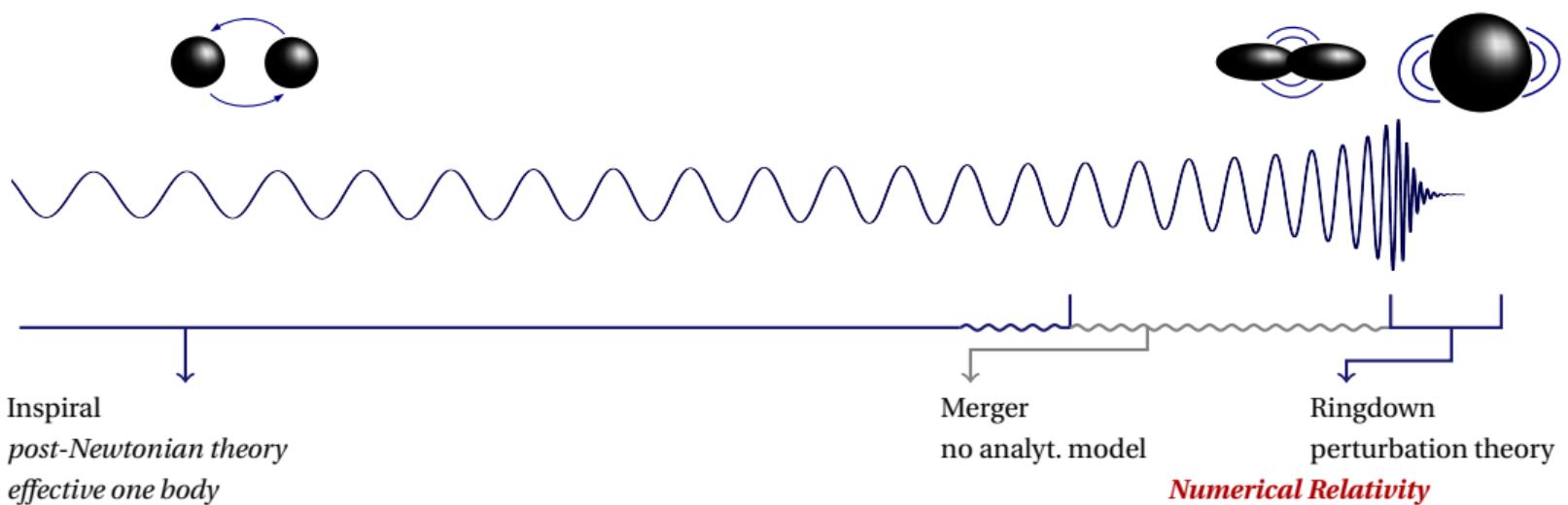
record



convolve

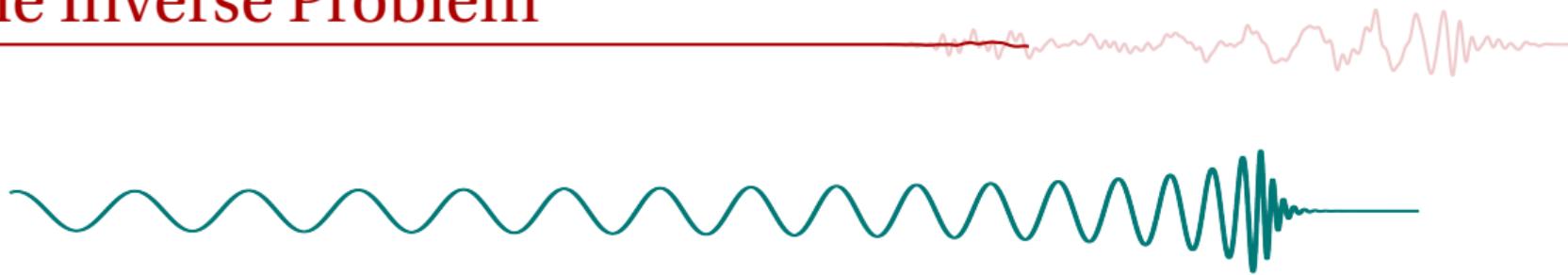


Modelling Binary Coalescences

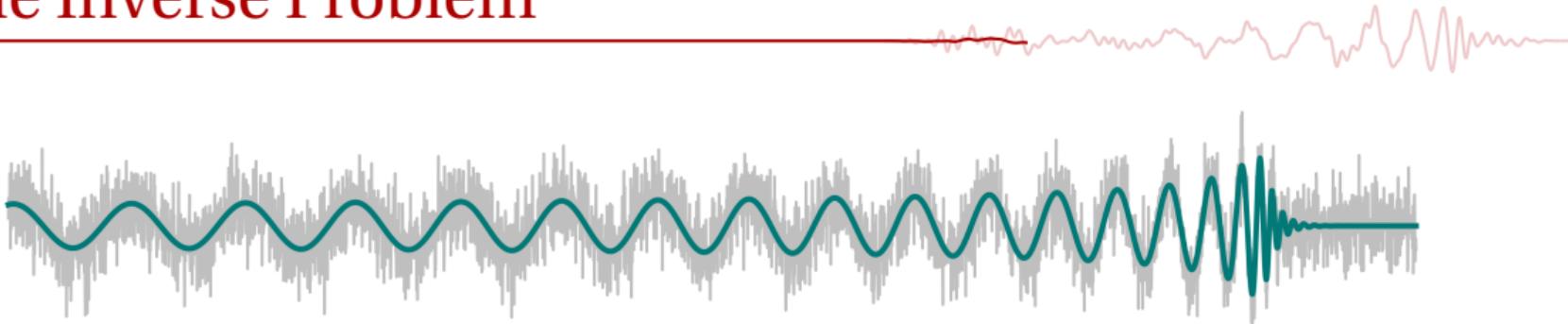


[FO, CQG 29 124002 (2012)]

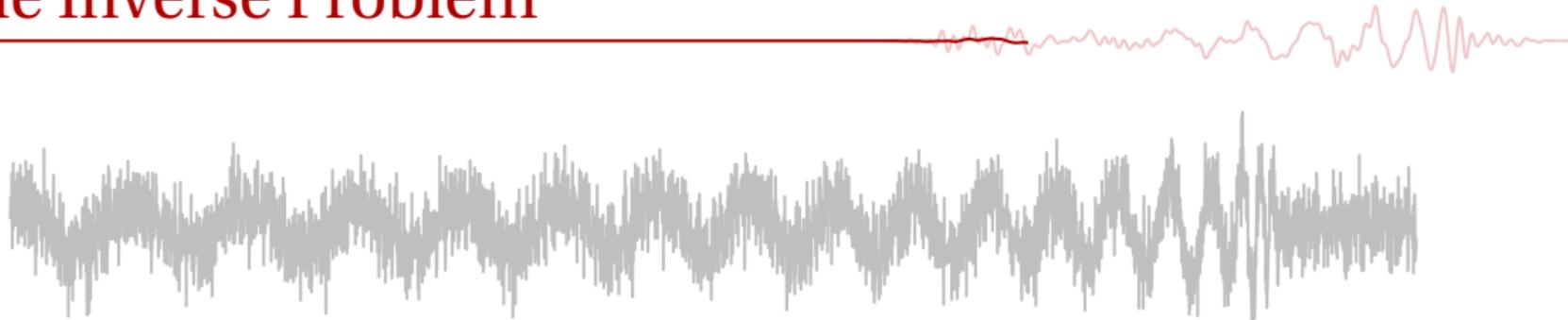
The Inverse Problem



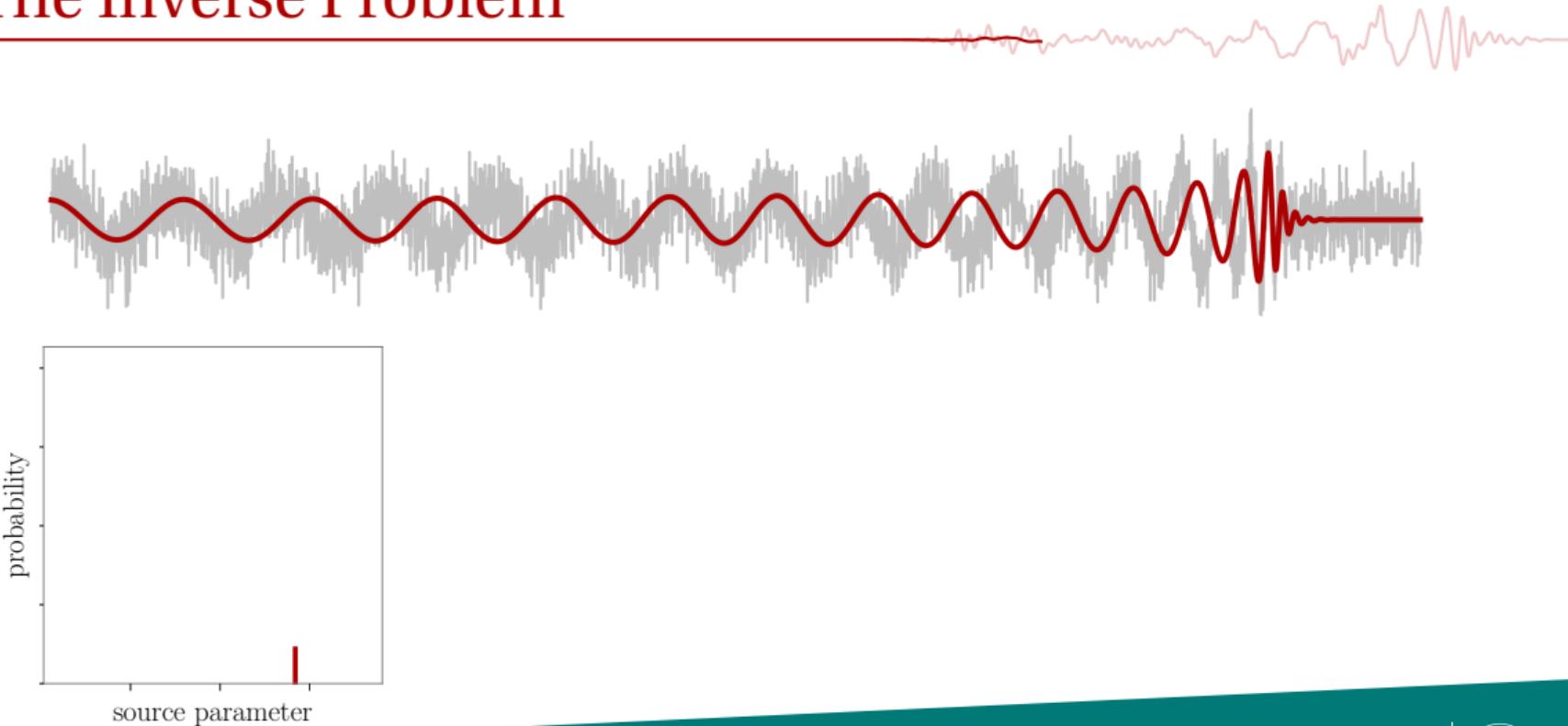
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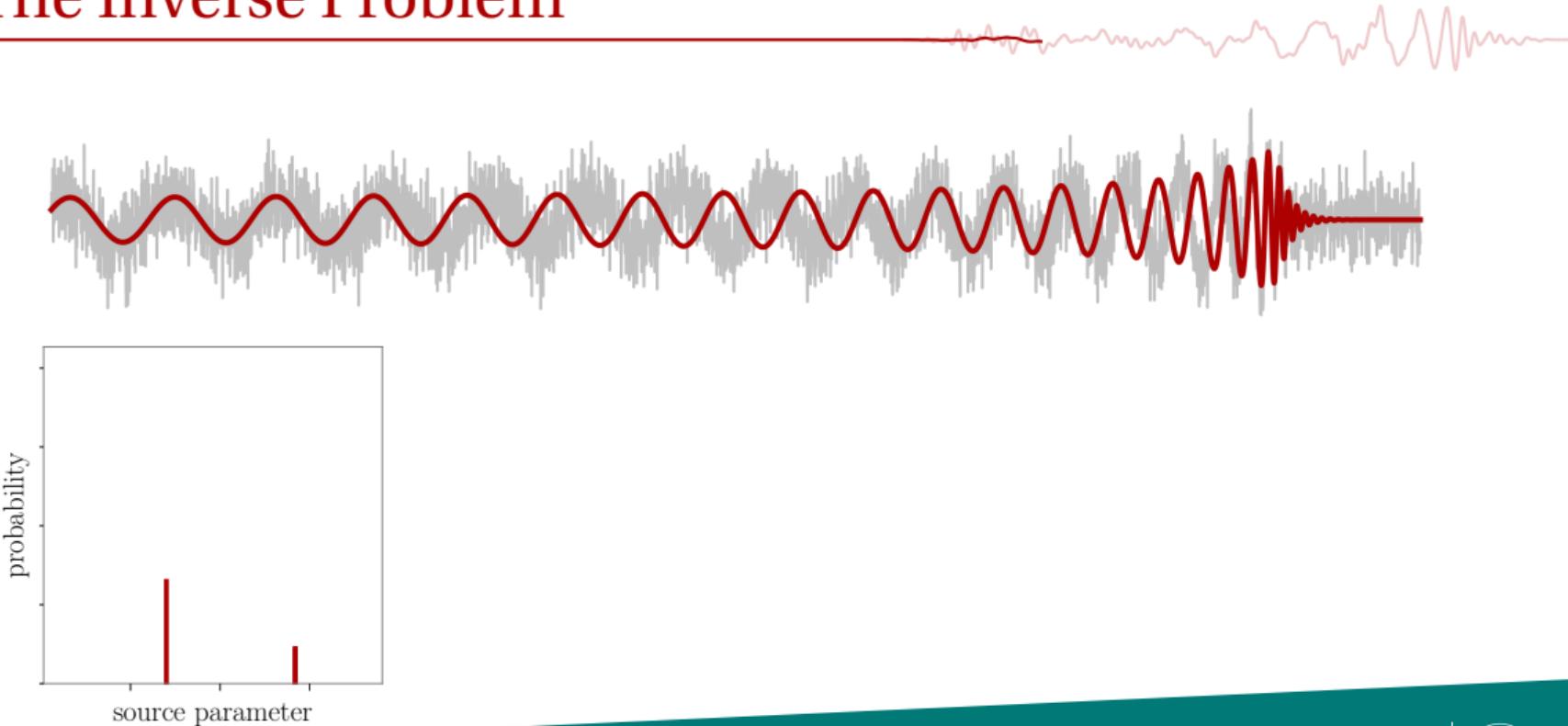
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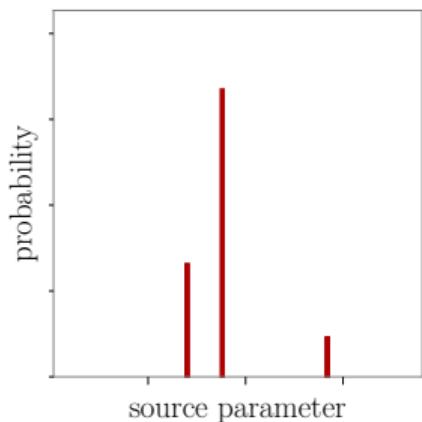
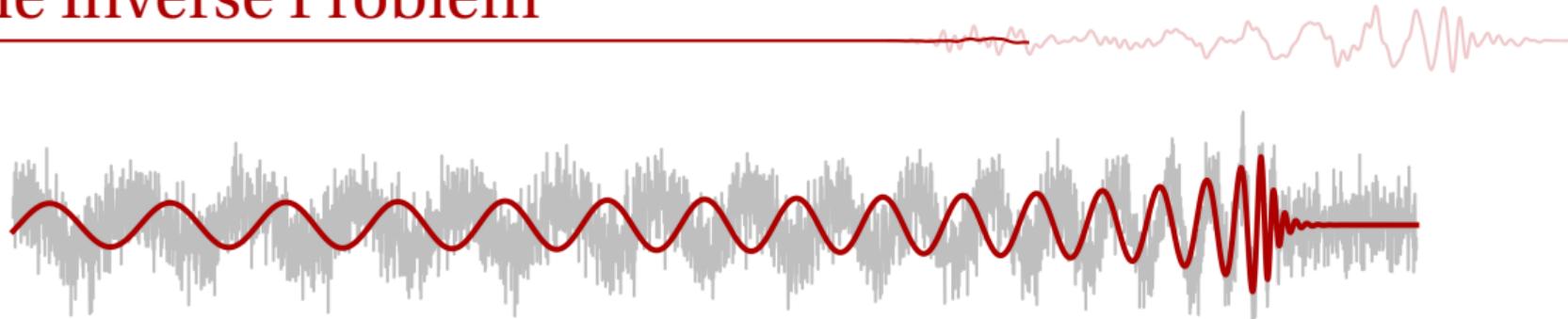
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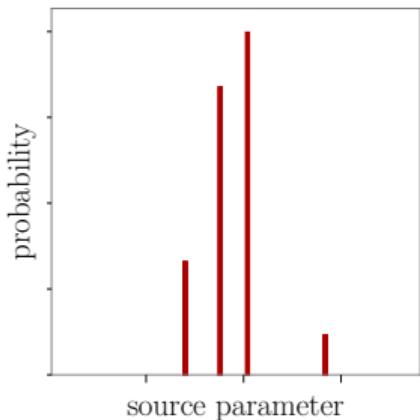
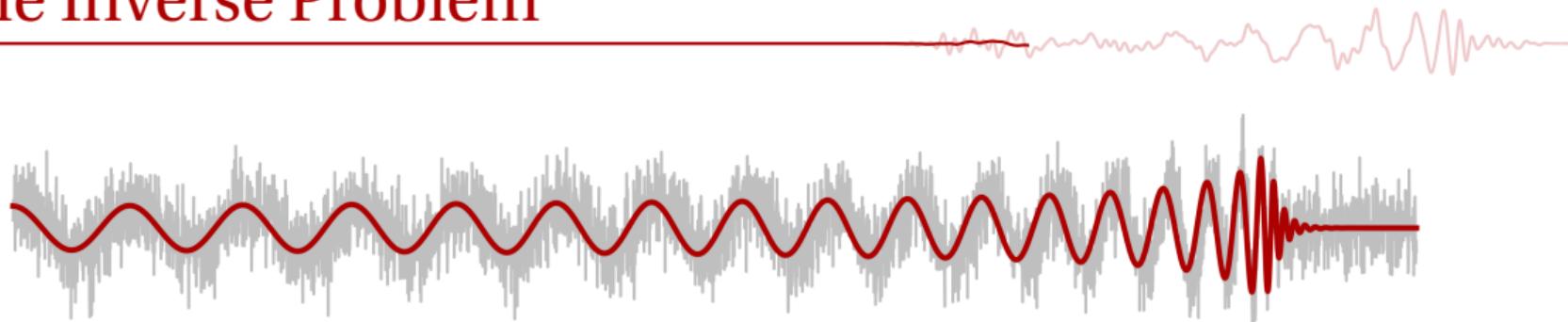
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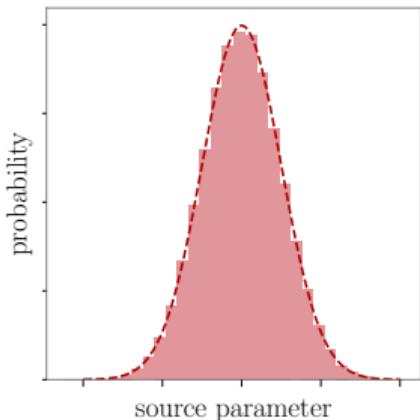
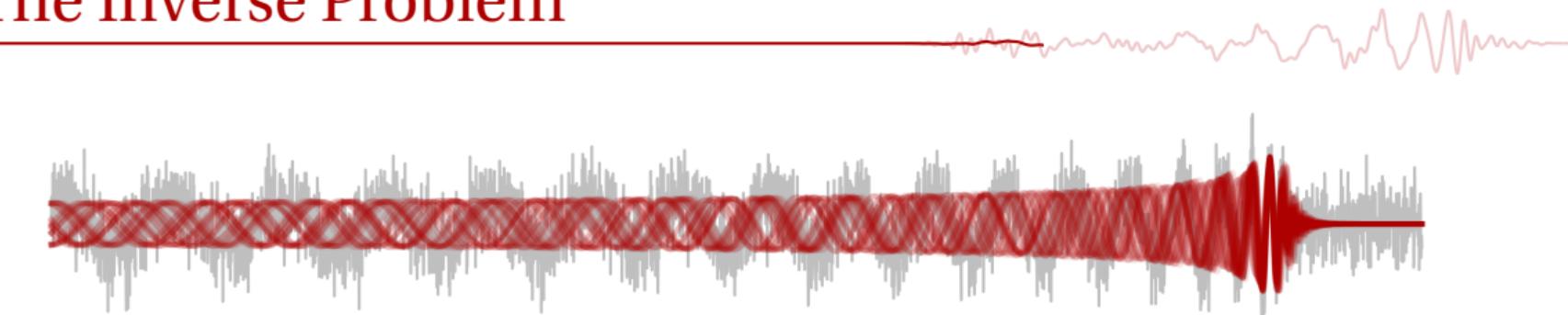
The Inverse Problem



The Inverse Problem



The Inverse Problem



Likelihood & Posterior

Quantities d : data, h : GW model, θ : source parameters

$$\text{Inner Product } \langle d, h \rangle = 4 \operatorname{Re} \int \frac{\tilde{d}(f) \tilde{h}^*(f)}{S_n(f)} df$$

$$\text{Likelihood } \Lambda(d|h(\theta)) \sim \exp(-\|d - h(\theta)\|^2/2)$$

$$\text{Posterior } p(\theta|d) = \frac{\Lambda(d|\theta)\pi(\theta)}{p(d)}$$

Stochastic sampling



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- ▶ Integrals become simple operations on set Θ , e.g.,

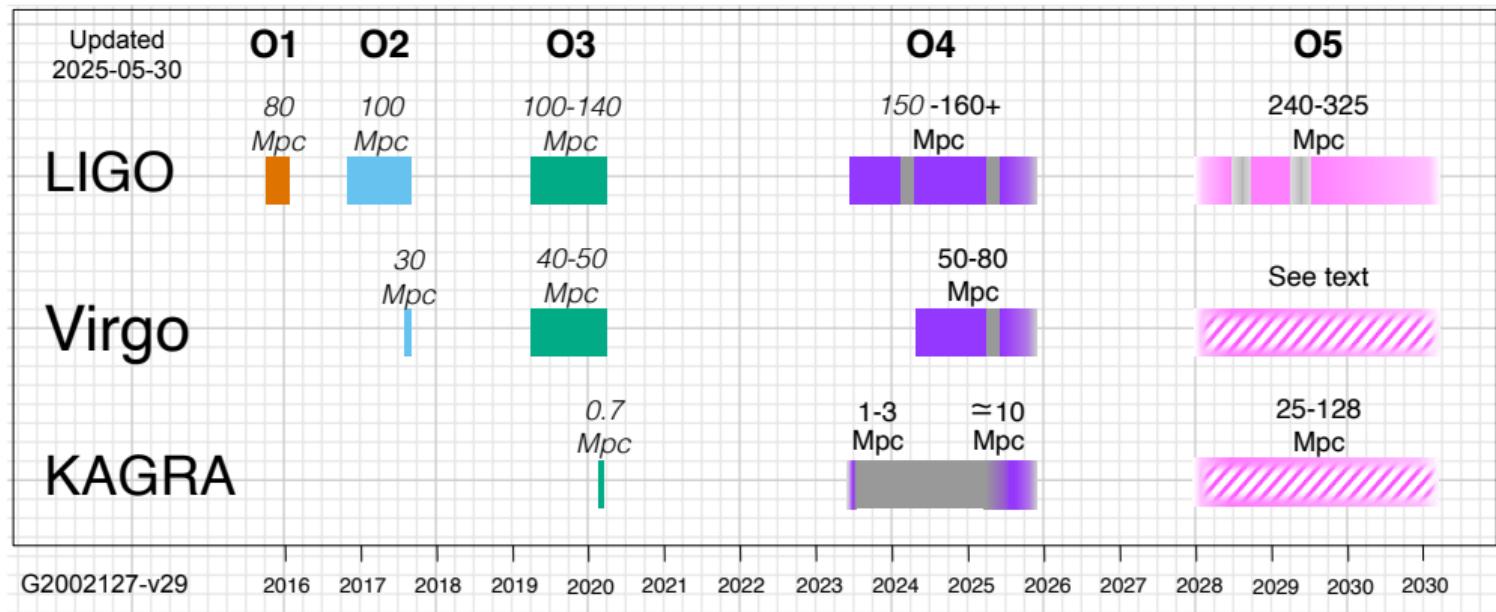
$$E[\theta_1] = \int \theta_1 p(\theta|s, I) d\vec{\theta} \approx \text{median}(\theta_1 \in \Theta)$$

Gravitational-Wave Observations

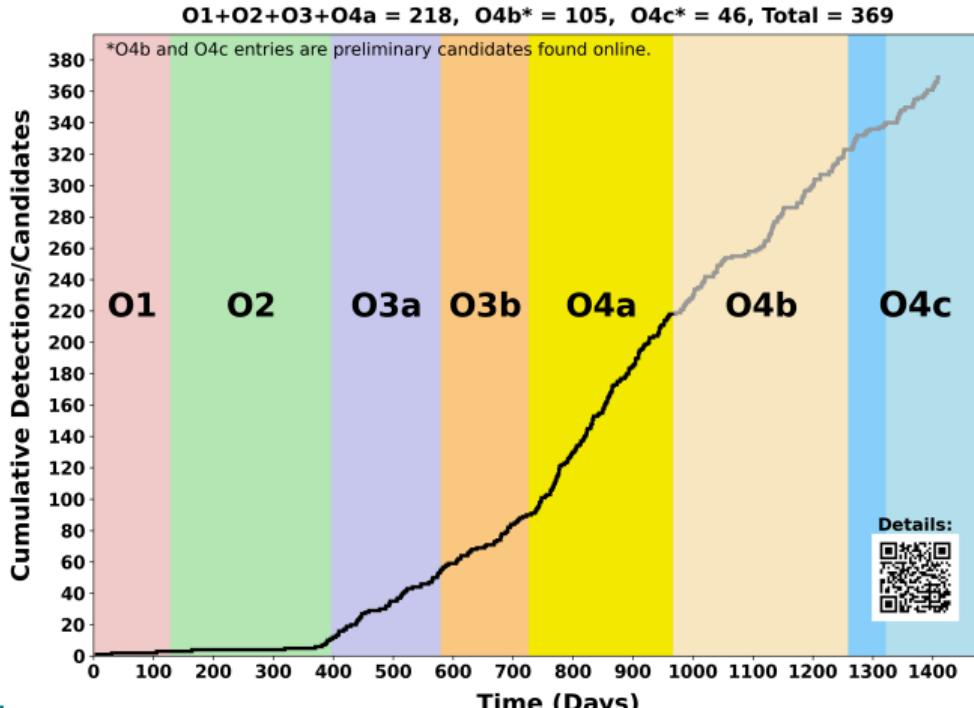


Timeline

[<https://observing.docs.ligo.org/plans/>]

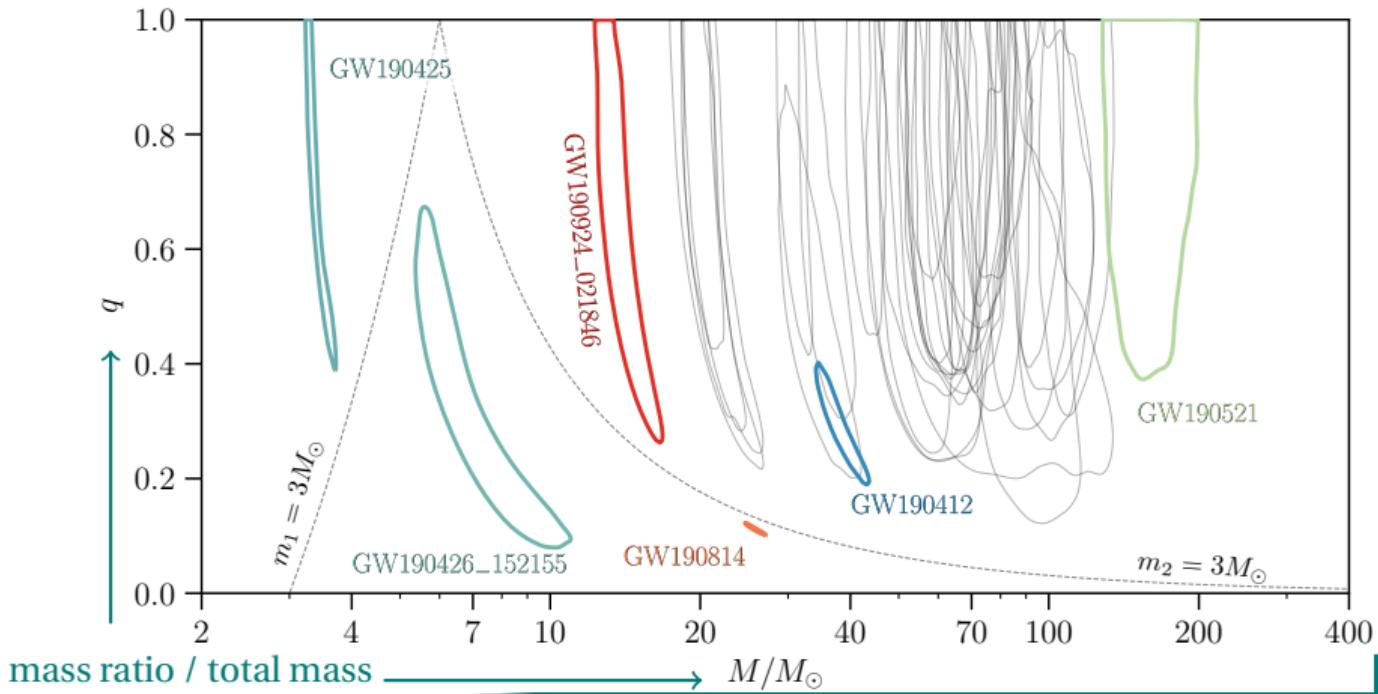


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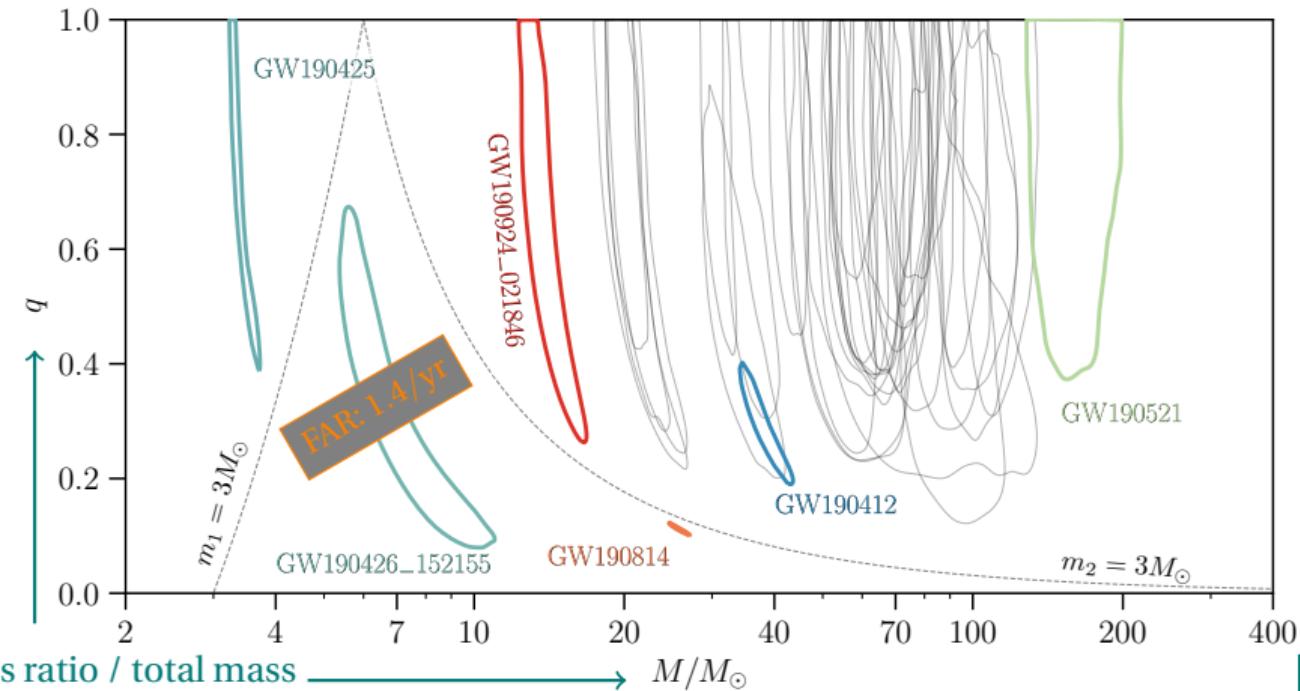


Mass Posteriors

[GWTC-2 2010.14527]

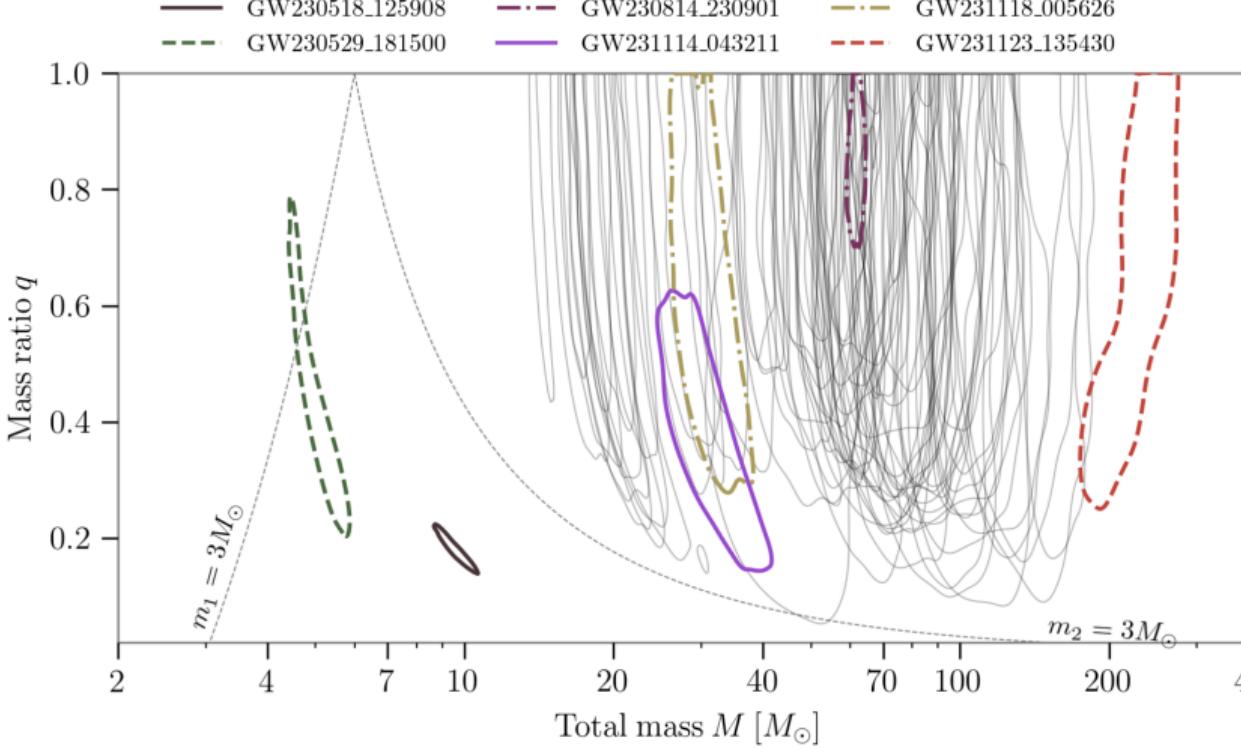


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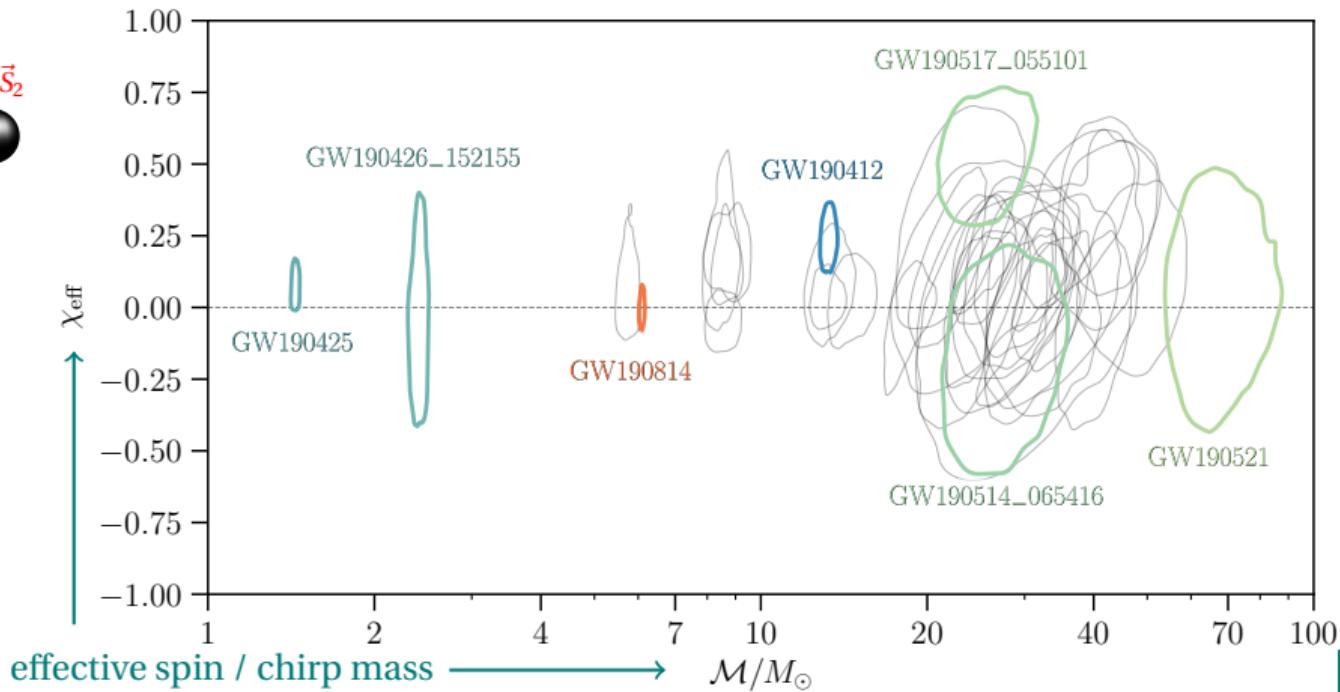
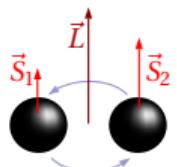
[GWTC-2 2010.14527]

Mass Posteriors



[GWTC-4 2508.18082]

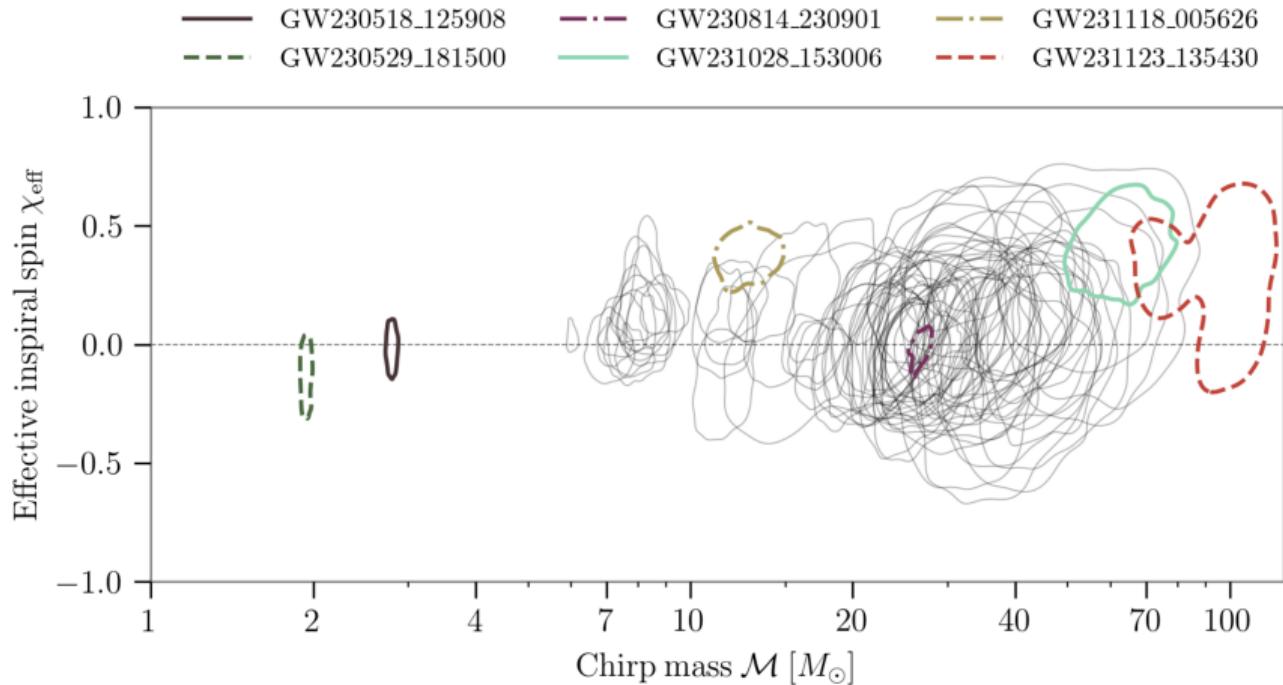
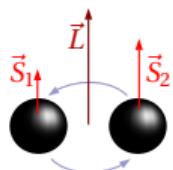
Spins



[GWTC-2 2010.14527]



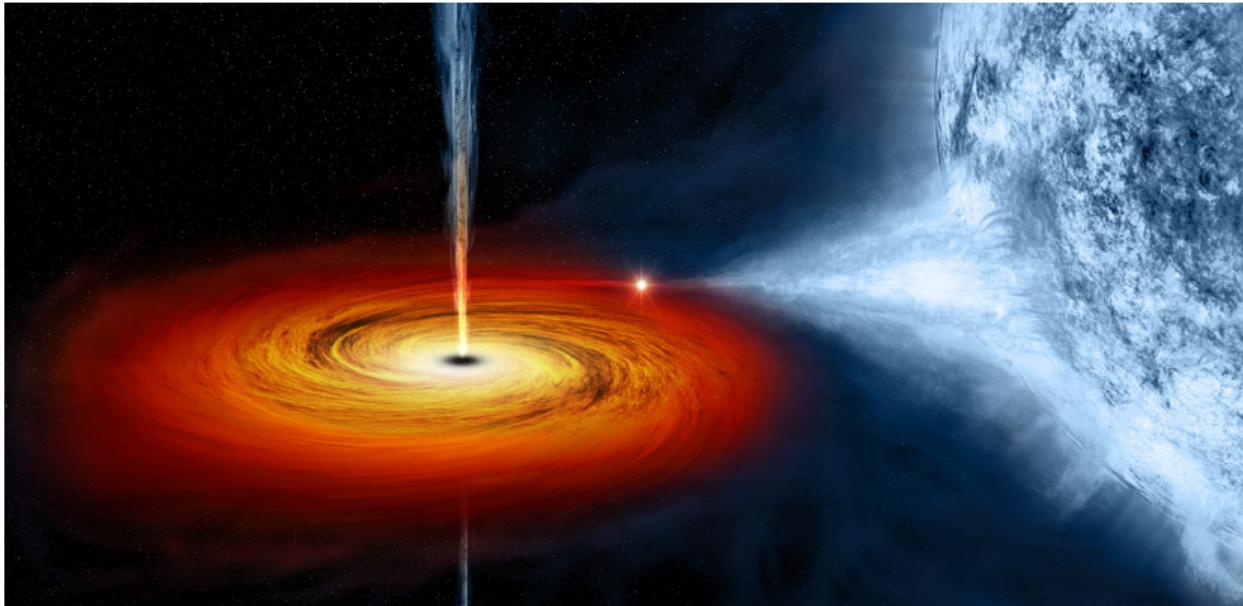
Spins



[GWTC-4 2508.18082]



Binary Evolution

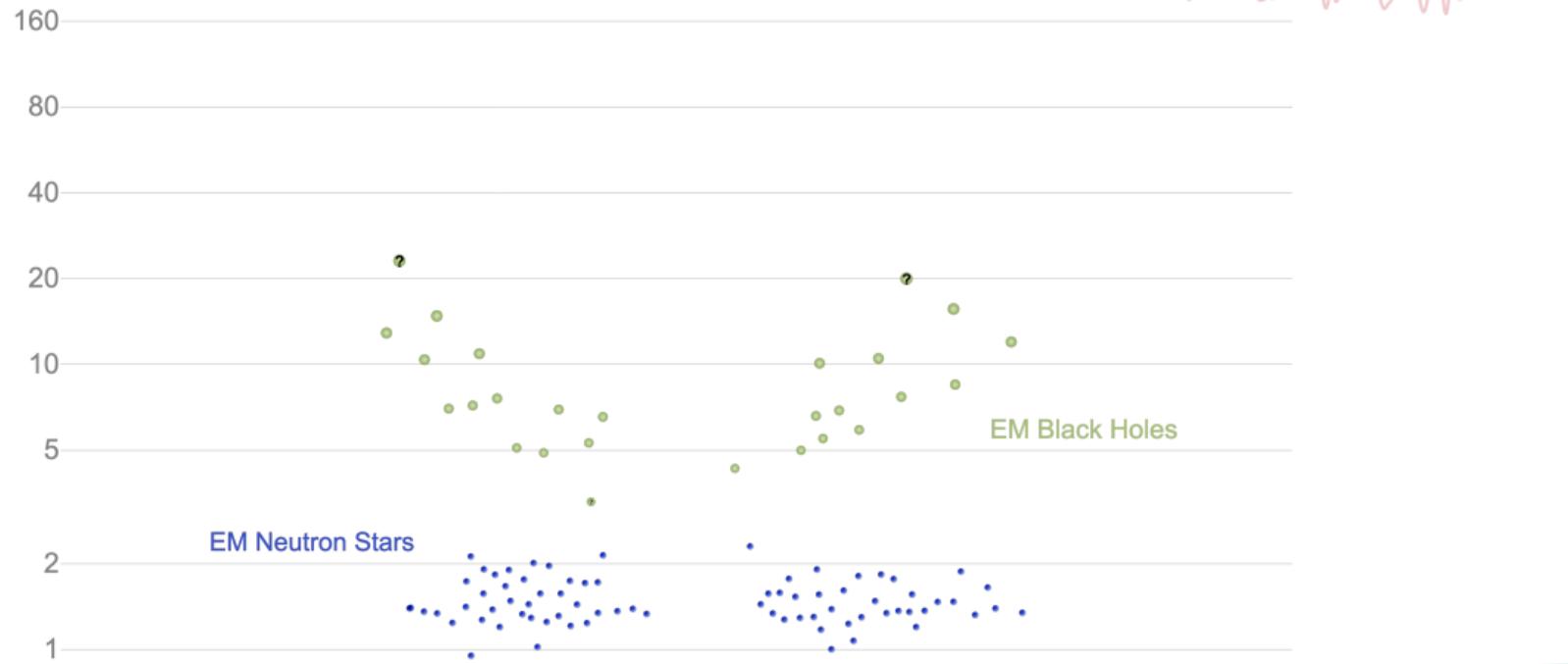


Material to Download



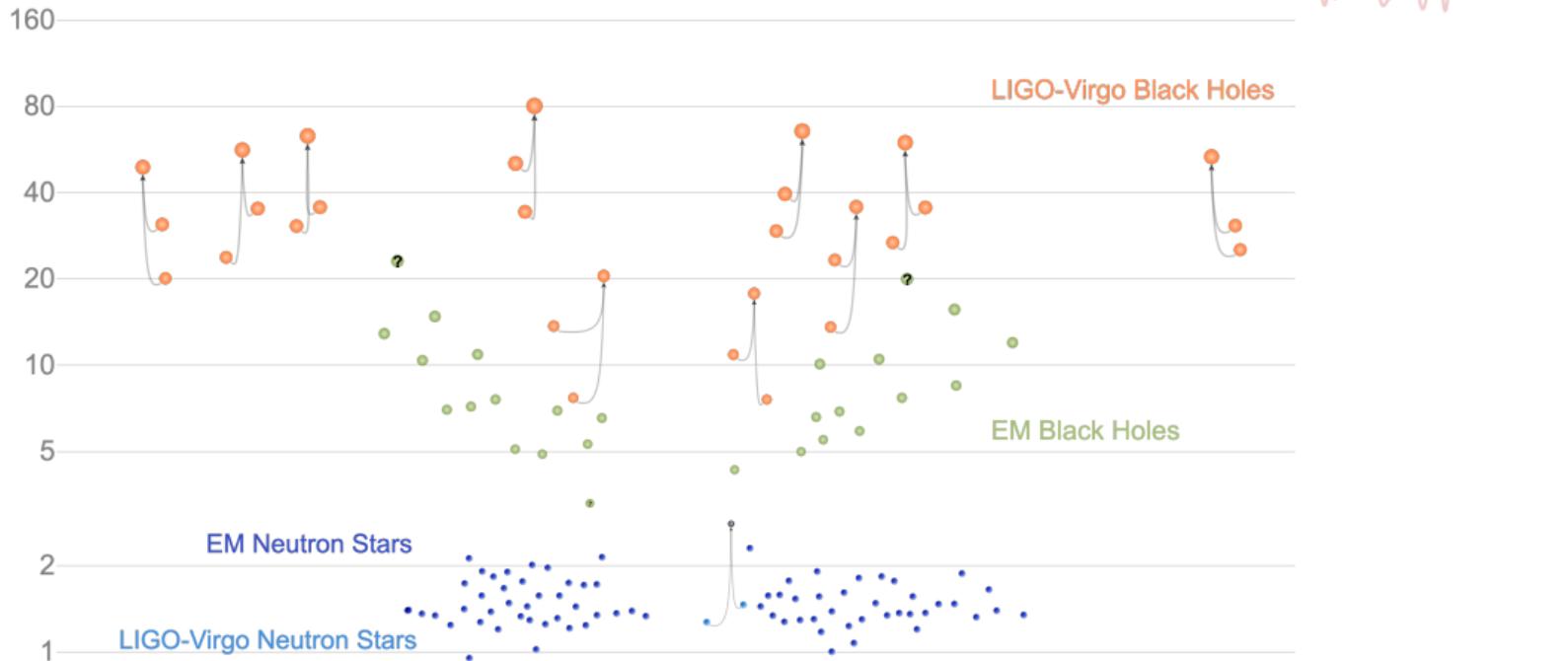
<https://s.gwdg.de/th635p>

Masses in the Stellar Graveyard



GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern
15/24

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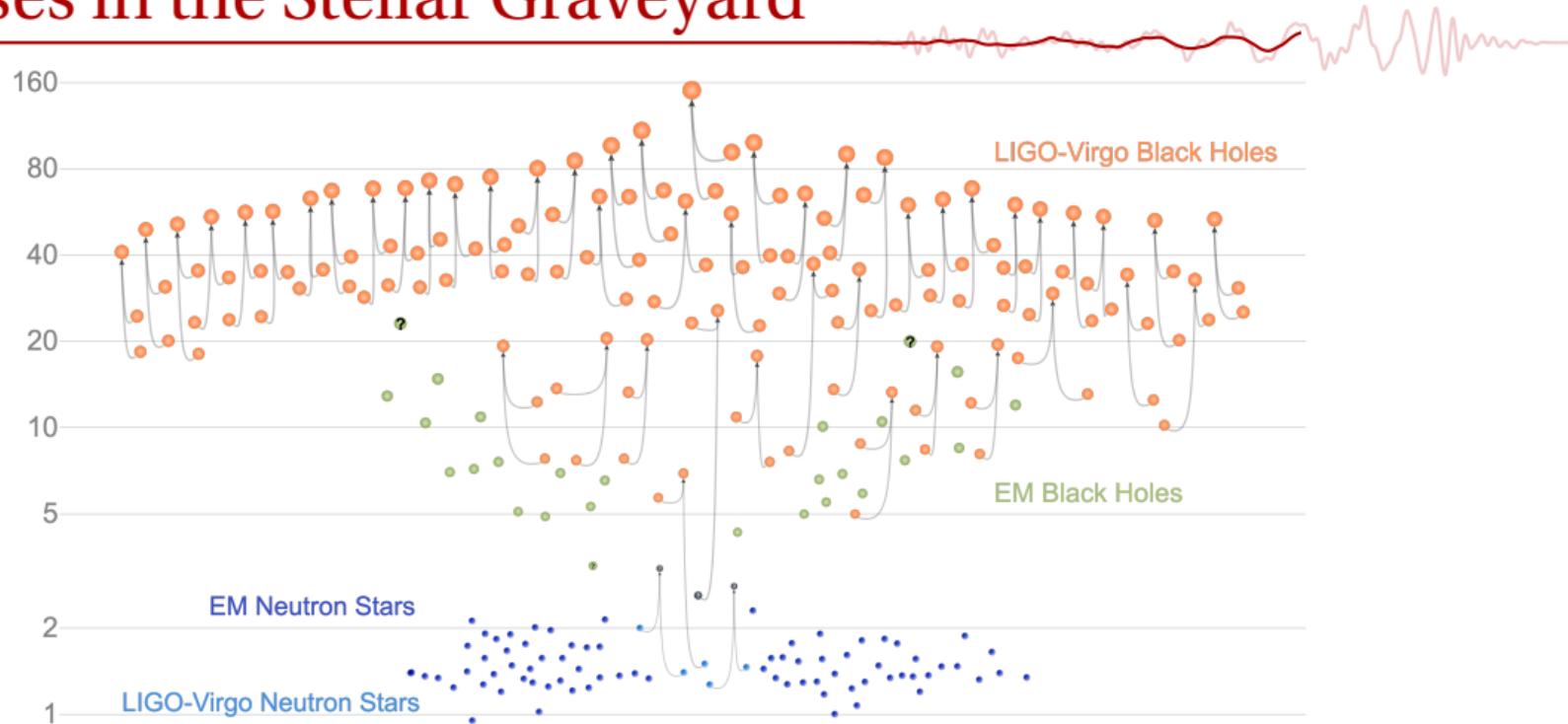


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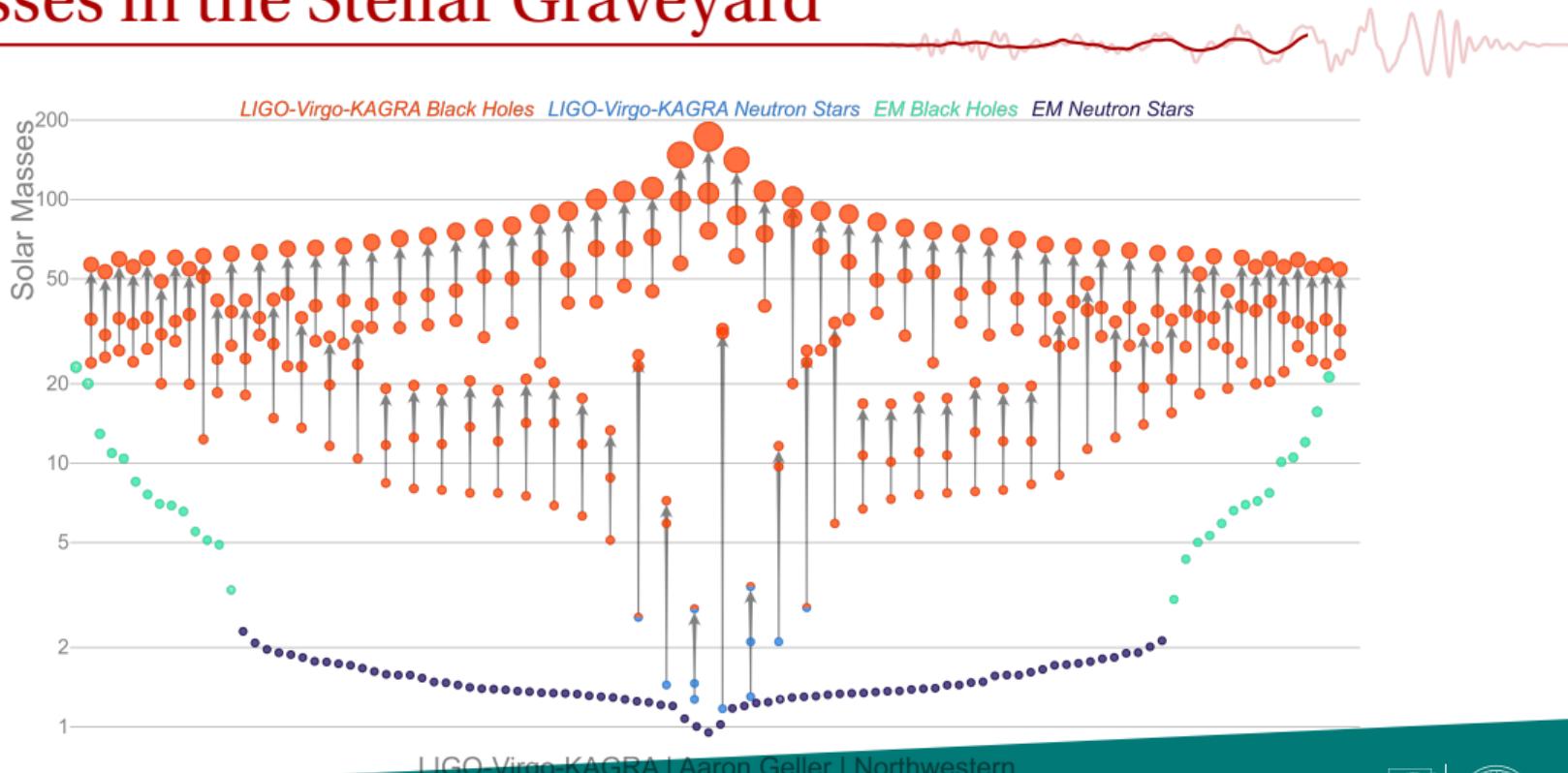


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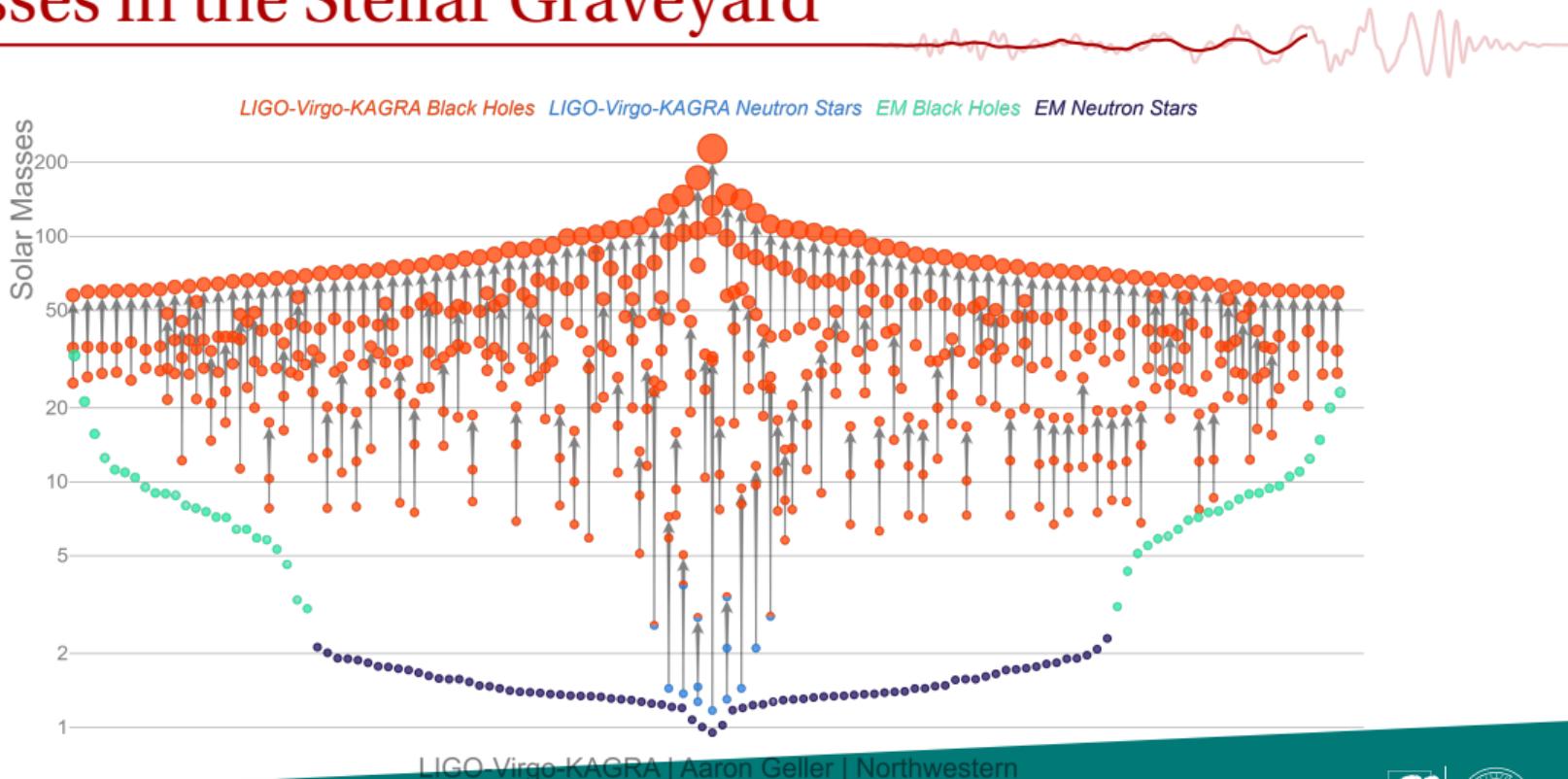
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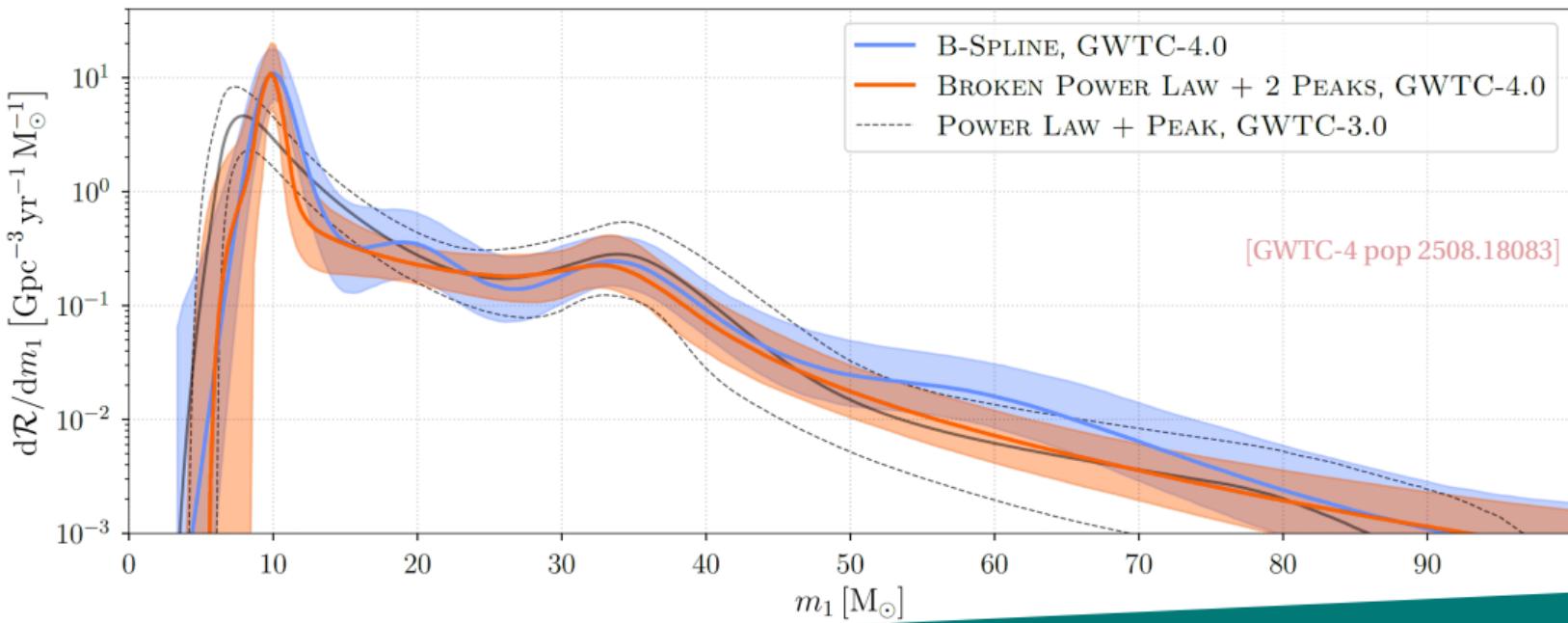
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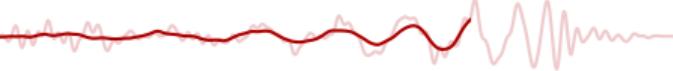
Masses in the Stellar Graveyard



Population Properties



My Personal Highlights



GW150914 The first GW signal. Loud, but now considered typical.

GW170817 The first (and only) GW-multi-messenger event. Merger of two neutron stars.
Largest coordinated global observation campaign across the EM spectrum.

GW190412 Black holes of unequal masses. First higher harmonics.

GW190521 Black holes with masses that shouldn't have formed in a supernova. Where did
they come from?

GW190814 Very unequal masses. The less massive object is a ...?

GW200129 Precessing black holes and potentially a remnant with a kick.

GW200115_042309 A black holes swallowing (or disrupting) a neutron star.

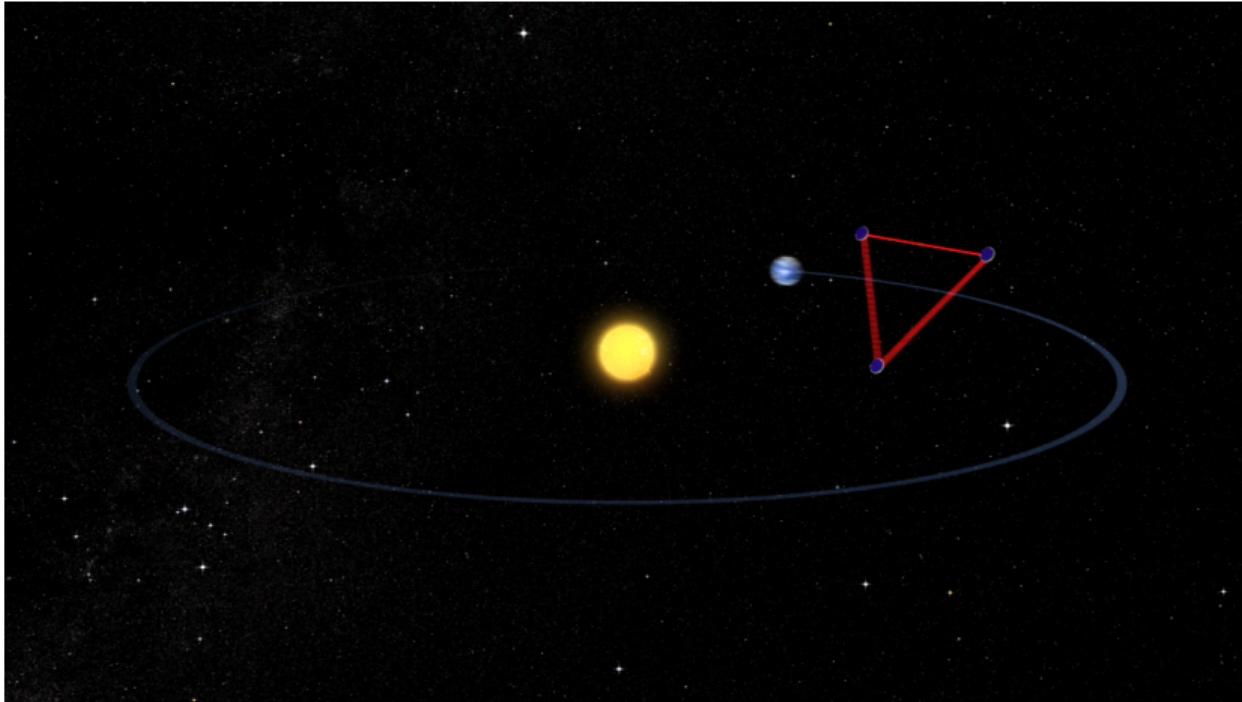
GW25xxxx



The Future



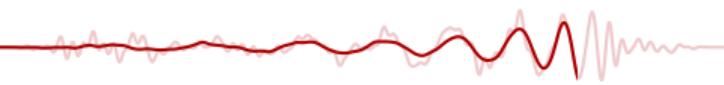
LISA



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LISA



LISA - LASER INTERFEROMETER SPACE ANTENNA

Gravitational waves are ripples in spacetime that alter the distances between objects. LISA will detect them by measuring subtle changes in the distances between **free-floating cubes** nestled within its three spacecraft.

③ identical spacecraft exchange **laser beams**. Gravitational waves change the distance between the **free-floating cubes** in the different spacecraft. This tiny change will be measured by the laser beams.

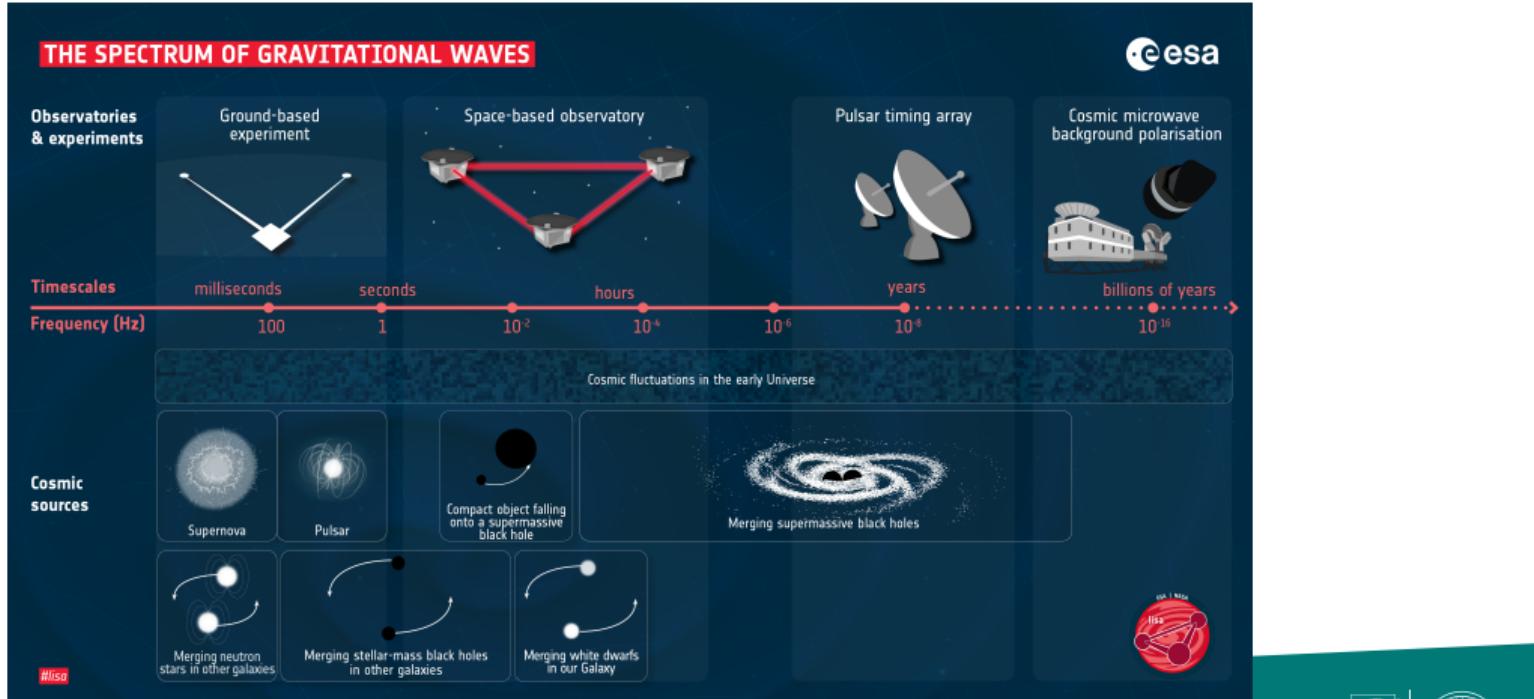
* Changes in distances travelled by the laser beams are not to scale and extremely exaggerated

Powerful events such as colliding black holes shake the fabric of spacetime and cause gravitational waves.

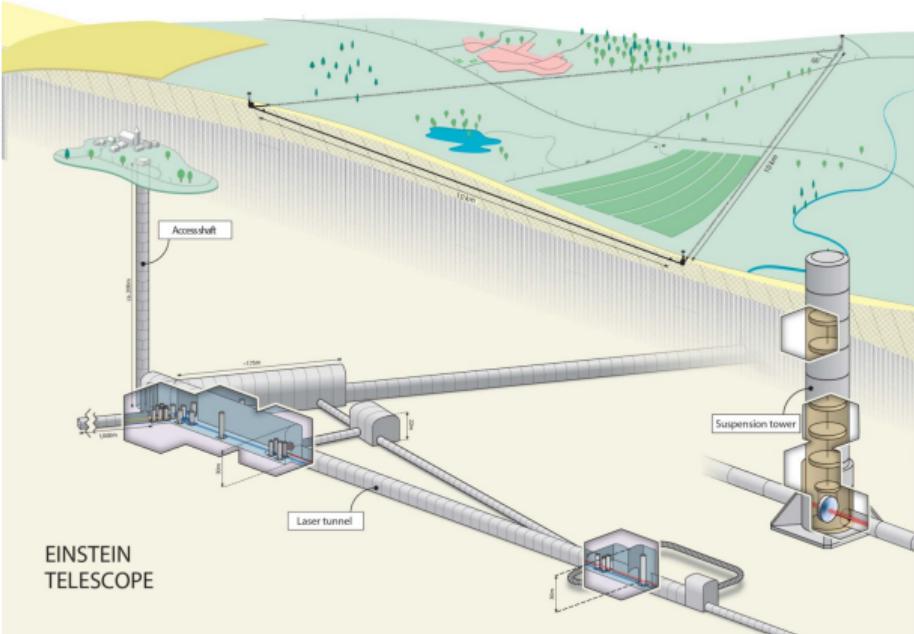
Free-floating golden cubes

The diagram illustrates the LISA mission's three-spacecraft interferometer. Three white rectangular boxes represent the spacecraft, arranged in a triangle. Red lines labeled "laser beams" connect the top and bottom spacecraft. A yellow circle labeled "Sun" is at the bottom left, and a small blue Earth-like sphere is labeled "Earth" at the top left. A dashed line from the Sun indicates a 20-degree angle. A callout box shows a close-up of one spacecraft's interior with four "Free-floating golden cubes". A red wavy line at the top represents gravitational waves passing through the interferometer. The esa logo is in the top right corner.

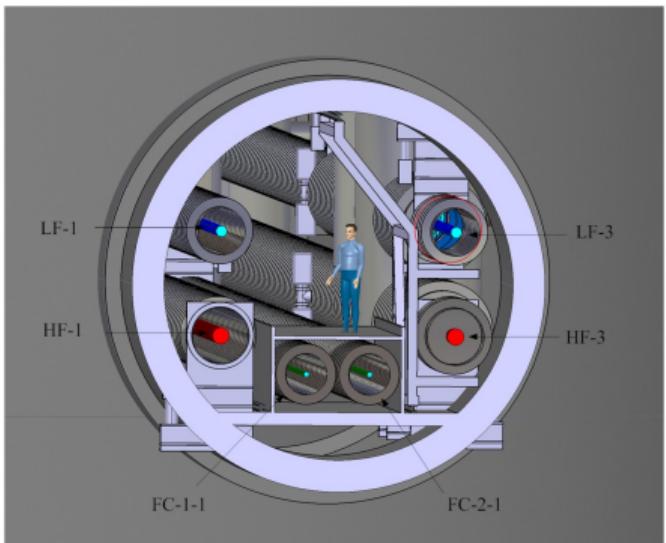
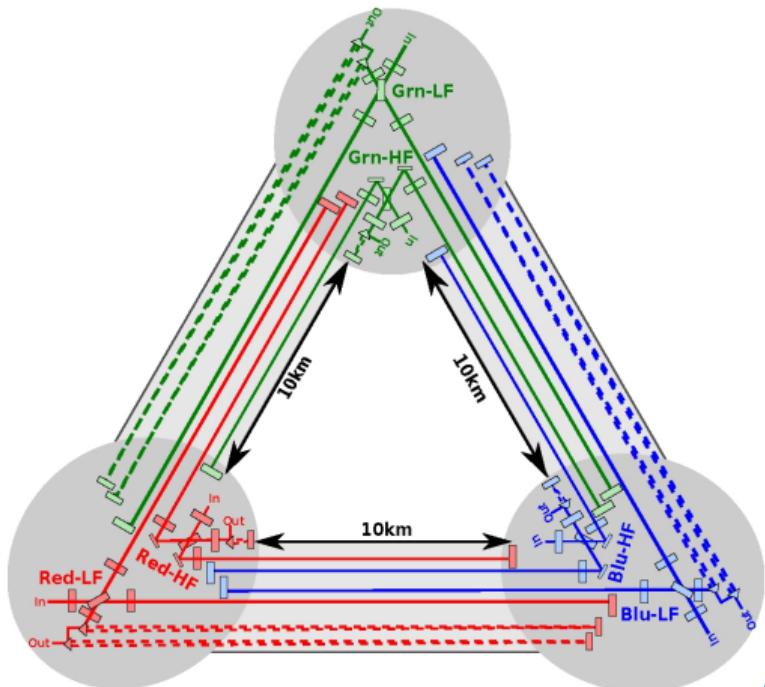
The Gravitational-Wave Spectrum



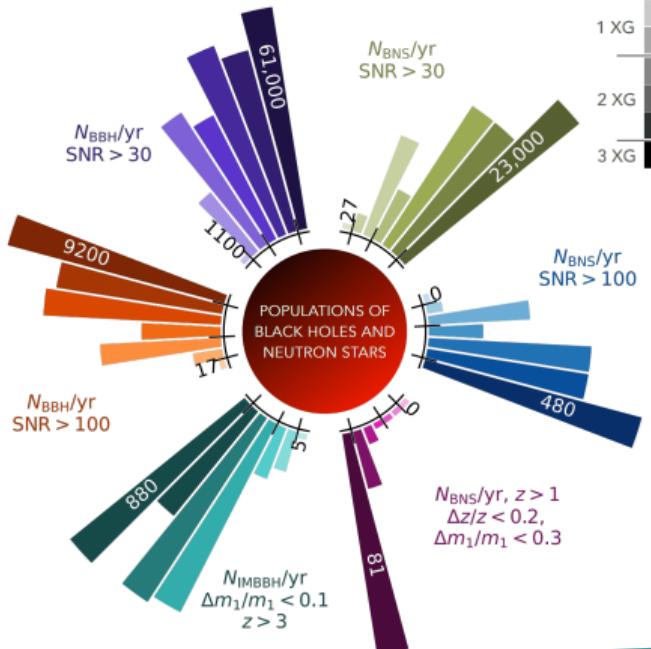
Einstein Telescope



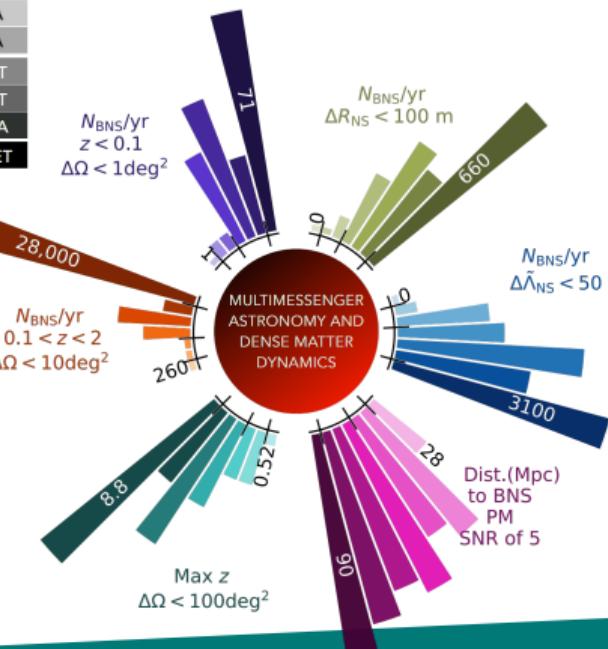
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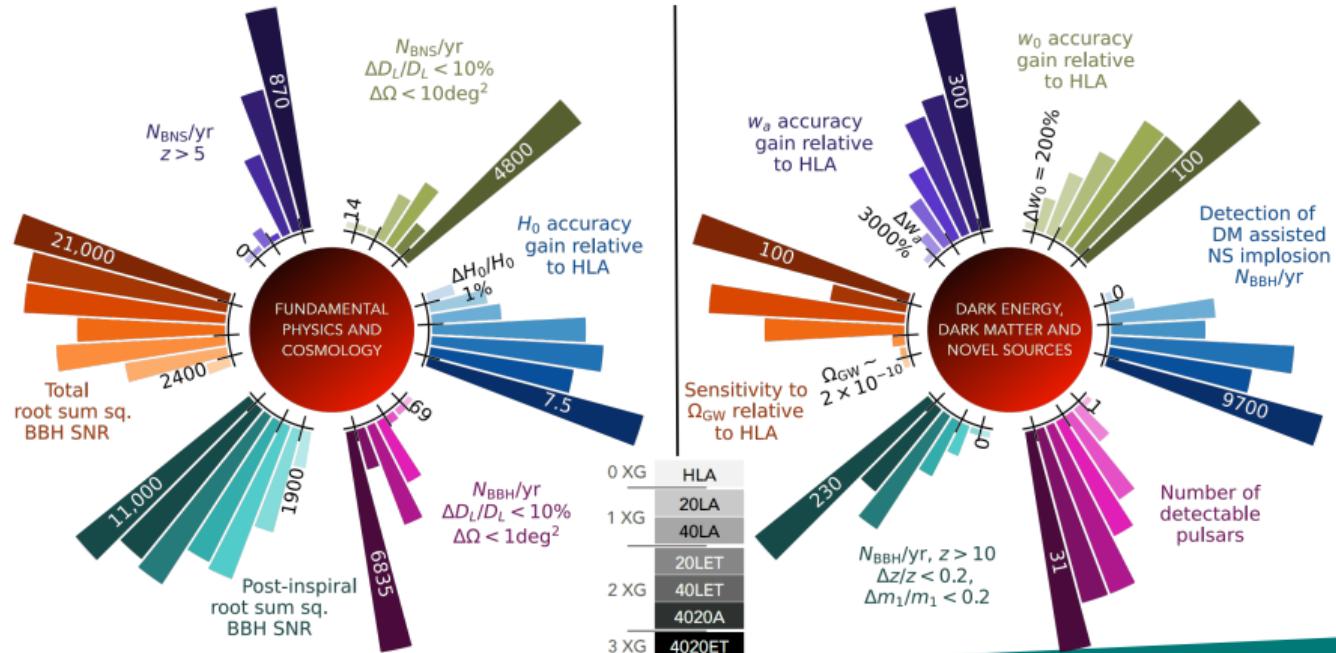
Science Prospects



0 XG	HLA
1 XG	20LA 40LA
2 XG	20LET 40LET 4020A
3 XG	4020ET



Science Prospects



Conclusions



- ▶ Gravitational-wave astronomy is reality
- ▶ The Universe is full of black holes with (surprisingly) diverse properties
→ diverse formation channels?
- ▶ More observations to come and needed to unravel big questions
(formation, environment, evolution, expansion, early Universe, ...)
- ▶ So far, no signs of Einstein's theory failing
- ▶ Continued work needed on possible systematics in the models and analyses
- ▶ Exciting future: LIGO India, LISA, Einstein Telescope
- ▶ Multimessenger astronomy will become the norm, not the exception