Gravitational Waves

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Astroparticle School September 2025

Binary Merger Observations & Numerical Relativity

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Topics

1. Data Analysis Challenge
   - Characterising Noise
   - LIGO's Noise
   - Signals Hidden in Noise
   - Matched-Filter Searches
   - First Discovery
2. Interpreting the Signal
   - GW150914
   - Energy Carried by GWs
   - The Mass in GW150914
   - Binary Parameters
   - Extrinsic Parameters
   - Spins
   - Waveform Models
   - Parameter Estimation

Data Analysis Challenge

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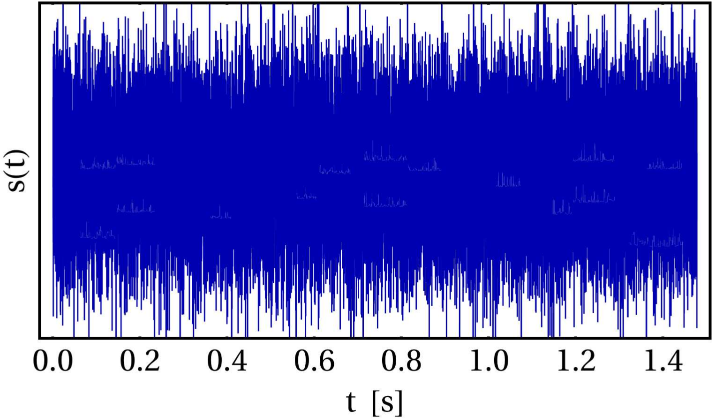
Typical Data Stream



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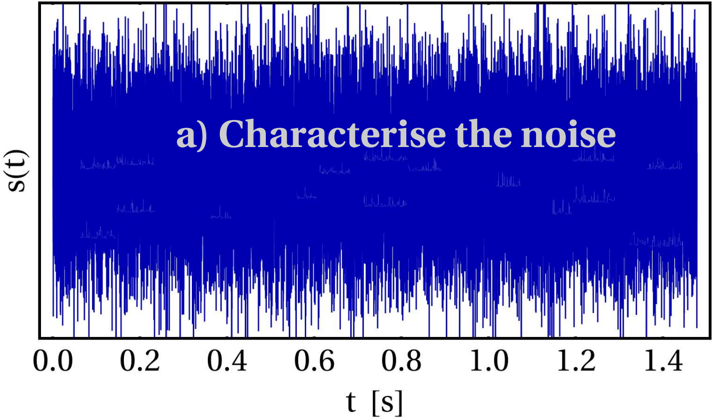

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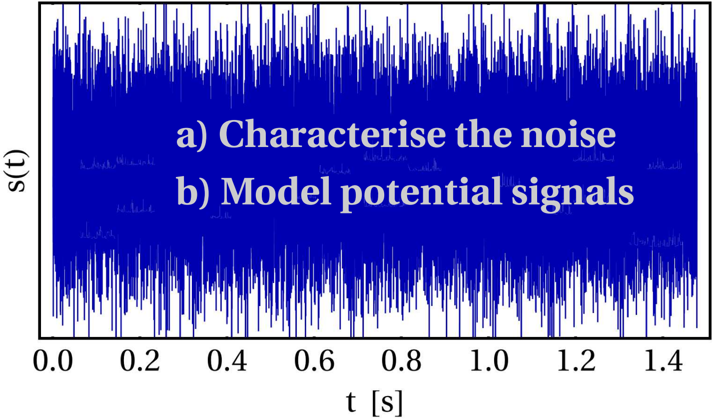

Material to Download

https://s.gwdg.de/th635p

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Stationary Gaussian noise

- Assume the following noise properties:
  - Stationarity: invariant under time translation
  - Gaussianity with zero mean
- Fourier spectrum of the noise:

$$\tilde{n}_T(f) = \frac{1}{\sqrt{T}} \int_0^T n(t)\, e^{-i2\pi f\, t}\, dt$$

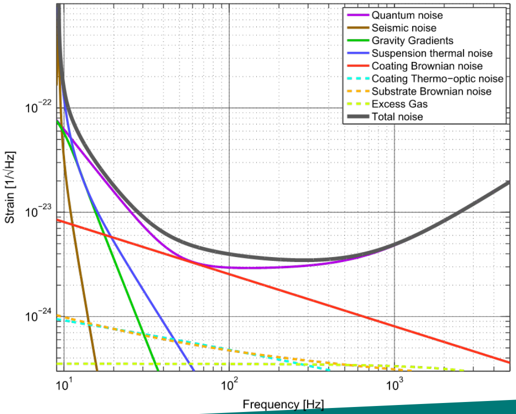
$$S_n(f) = \left\langle \lim_{T\to\infty} \left|\tilde{n}_T(f)\right|^2 \right\rangle$$

- Wiener-Khinchin theorem: S_n is the Fourier transform of the auto-correlation function

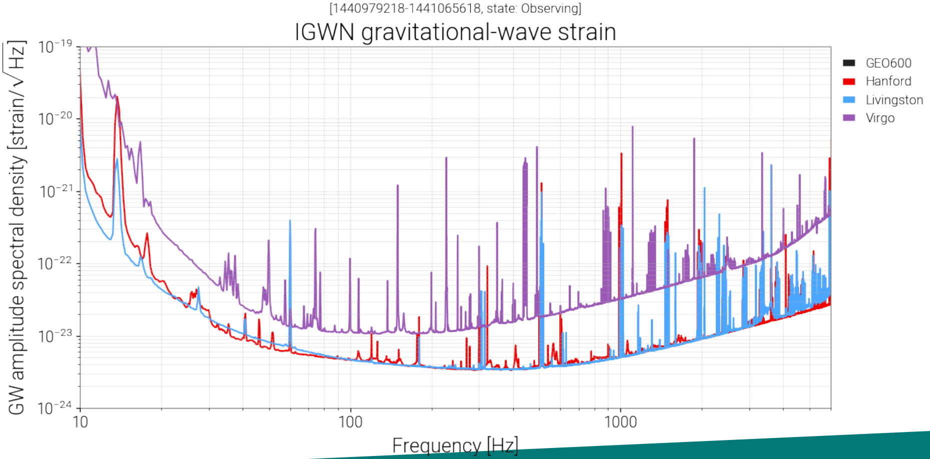
$$\Rightarrow \quad \langle \tilde{n}(f)\, \tilde{n}^*(f') \rangle = \frac{1}{2} S_n(f)\, \delta(f - f')$$

LIGO's Noise Spectral Density

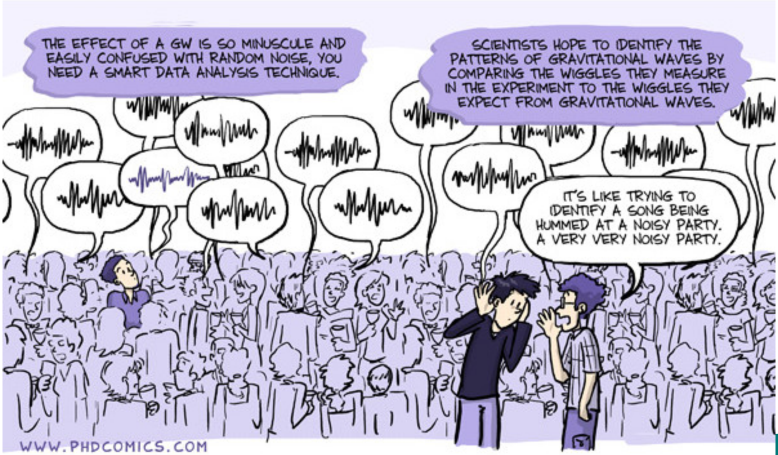
Class. Quantum Grav. **32** (2015) 074001 J Aasi *et al*



LIGO’s Noise on Sept 04



Signal Hidden in the Noise?



Signal buried in the noise?

- How likely is a particular noise realisation?

$$p(n) \sim e^{-(n|n)/2},$$

with $(a|b) = 4\,\mathrm{Re} \int \frac{\tilde{a}(f)\,\tilde{b}^*(f)}{S_n(f)} df.$

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- Is it more likely that the observed data stream s contains a signal h, i.e., $s = n + h$?

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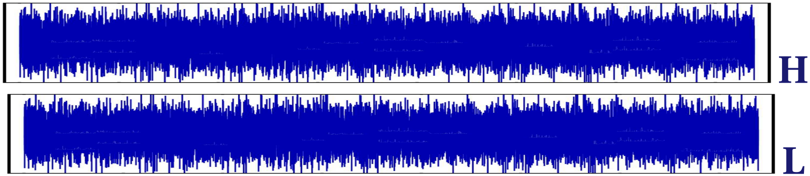
$$\ln \Lambda = (s|h) - \frac{(h|h)}{2} \leq \frac{1}{2} \frac{(s|h)}{\sqrt{(h|h)}} = \frac{1}{2}(s|\hat{h})$$

- $(s|\hat{h})$ is commonly referred to as the (recovered) signal-to-noise ratio

Real-life Searches

1. Need to know the signals we are looking for.
2. Filter data with ~~all~~ some of them.
3. Account for non-stationarity and non-Gaussianity of noise.
4. Quantify false alarm probability.



coherent signal?

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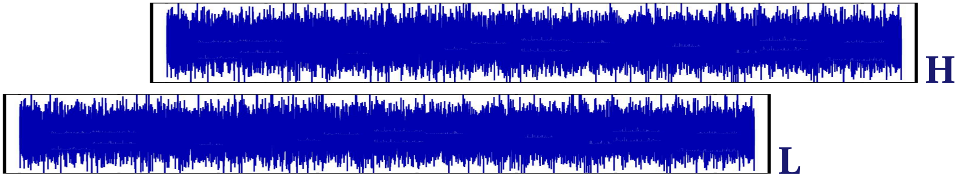
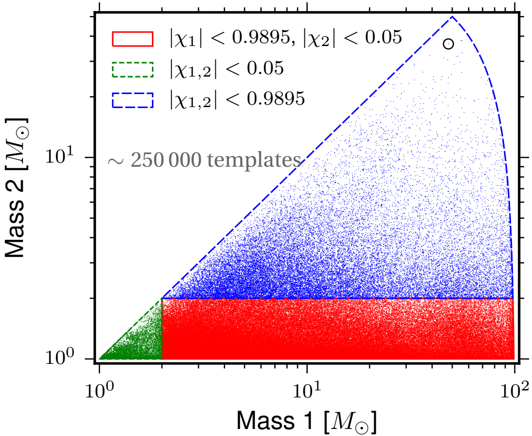


No coherent signal!

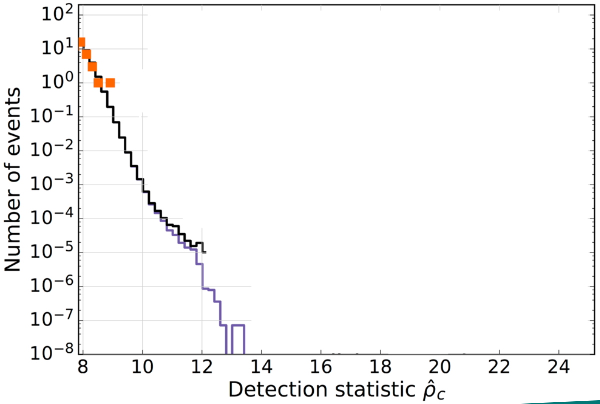
Template Bank



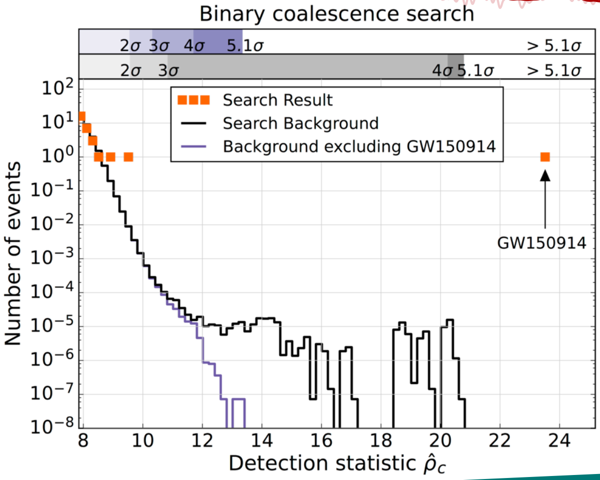
[PRD93, 122003 (2016)]

Search results

Binary coalescence search



Search results

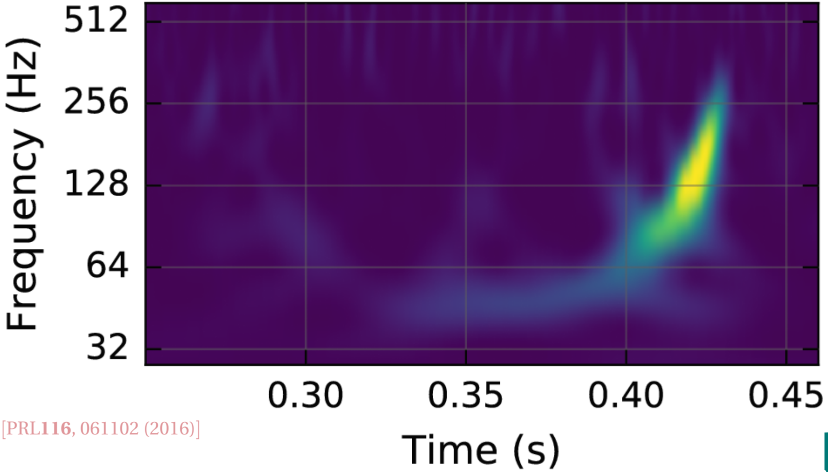


First Discovery


theguardian
So it turns out Einstein was right all along ...
The New York Times
Clinton Paints Sanders Plans As Unrealistic
WITH FAINT CHIRP, SCIENTISTS PROVE EINSTEIN CORRECT
A RIPPLE IN SPACE-TIME
Long in Clinton's Corner, Blacks Notice Sanders
Last Occupier in Rural Oregon
Frankfurter Allgemeine
ZEITUNG FÜR DEUTSCHLAND
Ayrault wird Außenminister Frankreichs
Einstein hat wieder mal recht
Einsteins Beben
天文學可能發生革命性變化
Gras fressen: Effenberg als Trainer in Paderborn › Die Seite Drei
Süddeutsche Zeitung
NEUESTE NACHRICHTEN AUS POLITIK, KULTUR, WIRTSCHAFT UND SPORT
Das Streiflicht
Der Beweis
Koalition einigt sich auf Asylpaket
EDIÇÃO LISBOA SEX 12 FEV 2016
publico.pt
Simulação de dois buracos negros em colisão, provocando ondas gravitacionais

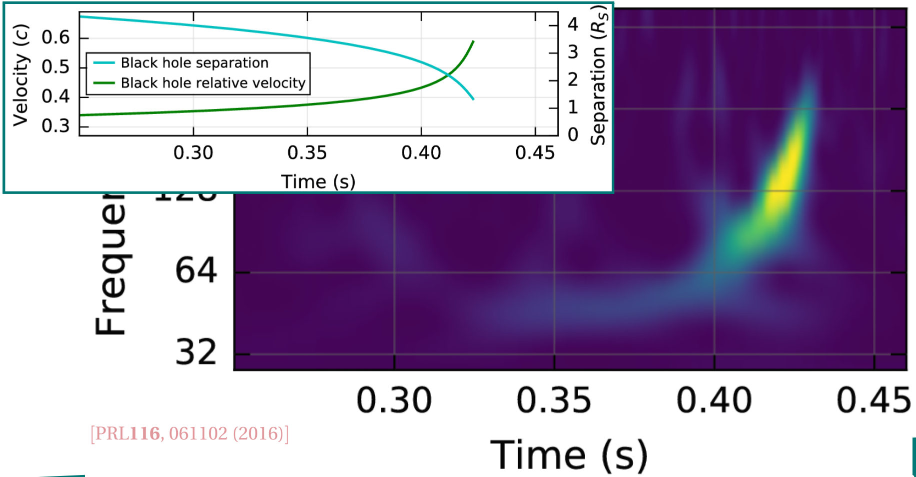
Interpreting the Signal

GW150914



[PRL **116**, 061102 (2016)]

GW150914



[PRL **116**, 061102 (2016)]

Binary phase evolution

Newtonian binary (arbitrary masses)

$$h(t) = \mathcal{A}\frac{4M\,\eta\,v(t)^2}{D}\cos(2\phi_{\rm orb}(t)+\phi_0)$$

$$E(t) = -\frac{M\eta}{2}v(t)^2$$

$$\mathcal{L}(t) = \frac{c^5}{G}\frac{32}{5}\eta^2\left(\frac{v(t)}{c}\right)^{10}$$

$$v(t) = \omega(t)\,r(t)$$

$$\eta = \frac{m_1\,m_2}{M^2}$$

Keppler's law:

$$v^2(t) = \frac{GM}{r(t)}$$

1. From the energy-balance law, $dE/dt = -\mathcal{L}$, derive a differential equation for the binary's velocity $v(t)$.
2. Calculate $v(t)$ and $\omega(t)$. Integrate $\omega(t)$ to obtain $\phi_{\rm orb}(t)$.

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Solution

$$\frac{dv}{dt}(t) = -\frac{\mathcal{L}(t)}{dE(v)/dv} = \frac{1}{Gc^5}\frac{32}{5M}\eta\, v^9(t)$$

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$$M_c = M\,\eta^{3/5}$$

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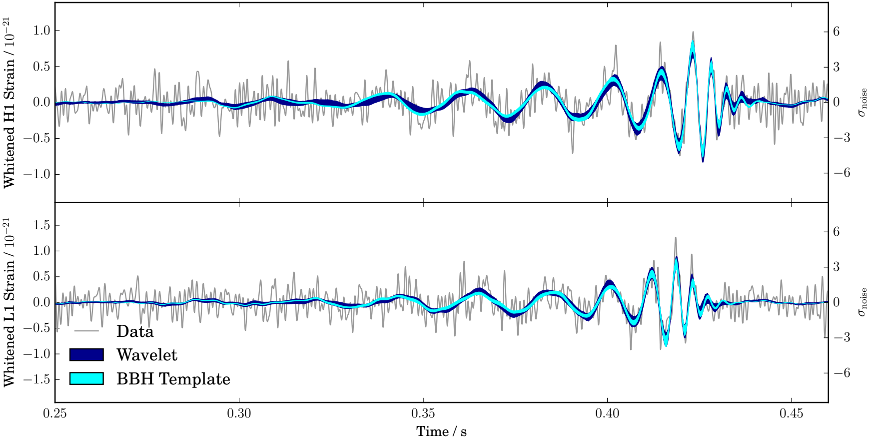
$$\phi_{\rm orb}(t) = \int \omega(t)\,dt = \phi_c - \left(\frac{c^3}{G}\frac{t_c - t}{5M_c}\right)^{5/8}$$

GW150914 – How much mass?



[PRL**116**, 241102 (2016)]

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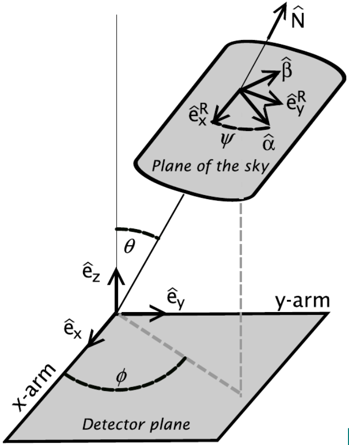
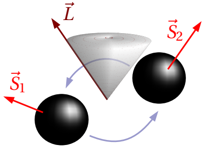
Binary parameters

- 2 masses (m_1, m_2 / M_c, q)
- 6 spin vector parameters
- reference time and phase
- 2 eccentricity parameters
- tidal parameters (for neutron stars)
- 2 angles for sky location
- inclination angle (ι)
- polarization angle (ψ)
- distance (D)



[Sathyaprakash, Schutz, LRR]



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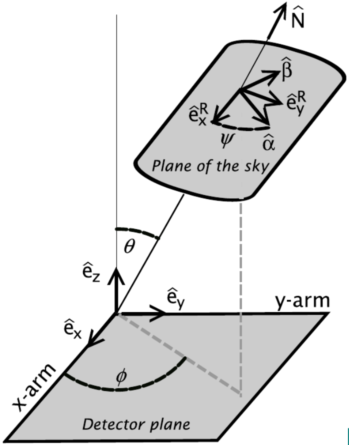

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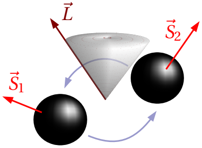

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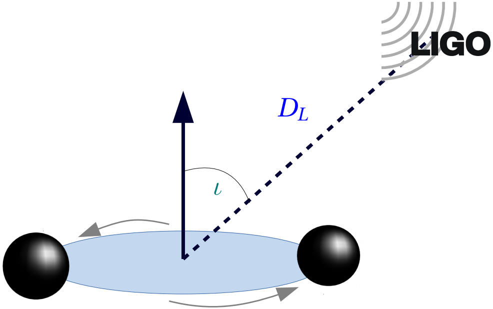
Distance and Inclination

$$h_+ \approx 2\left(1 + \cos^2 \iota\right) \frac{M\eta v(t)^2}{D_L} \cos\left[2\phi_{\mathrm{orb}}(t)\right]$$



M: total mass, $M = m_1 + m_2$

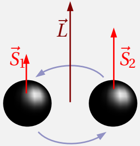
η: symmetric mass ratio, $\eta = m_1 m_2 / M^2$

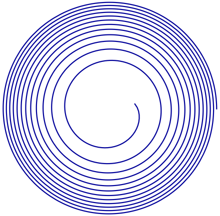
v: orbital velocity

ϕ_{orb}: orbital phase

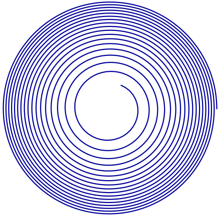
Non-precessing systems

- Parameters: $M,\ \eta,\ \chi_i = \dfrac{\vec{S}_i \cdot \hat{L}}{m_i^2}$
- Reduced spin: $\chi_{\rm eff} = \dfrac{m_1\chi_1 + m_2\chi_2}{M}$

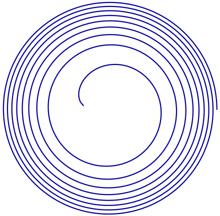




$q = 1,\ \ \chi_1 = \chi_2 = 0$

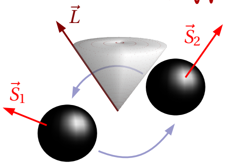


$q = 1,\ \ \chi_1 = \chi_2 = 0.99$

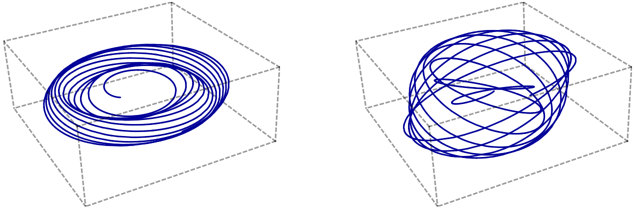


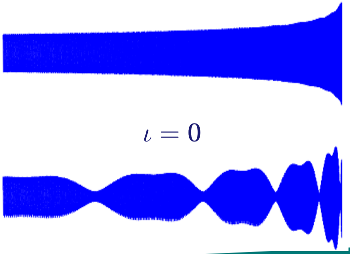
$q = 1,\ \ \chi_1 = \chi_2 = -0.99$

Generic spins: precession



- $\eta, \vec{S}_1^{\perp}, \vec{S}_2^{\perp} \mapsto \eta, \chi_p$ [Schmidt, FO, Hannam 1408.1810]
- Waveform depends non-trivially on inclination ι

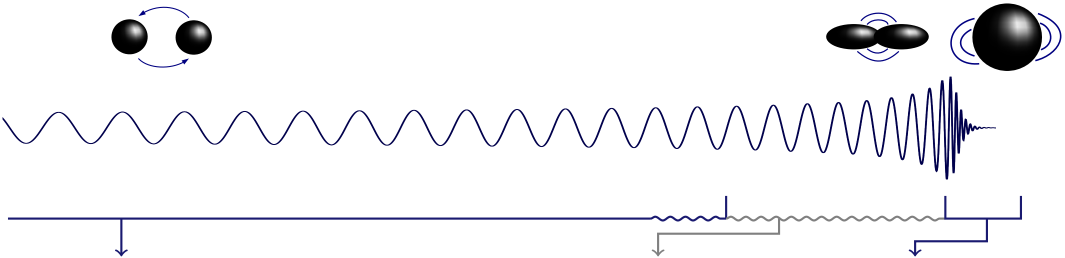




Binary Black Hole Explorer

Binary Black Hole Explorer

Modelling Binary Coalescences



Inspiral
post-Newtonian theory
effective one body

Merger
no analyt. model

Ringdown
perturbation theory

Numerical Relativity

[*FO*, CQG 29 124002 (2012)]

The Inverse Problem

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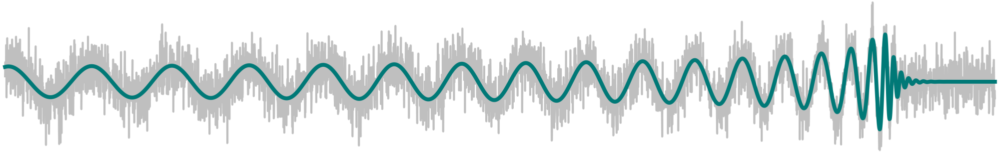

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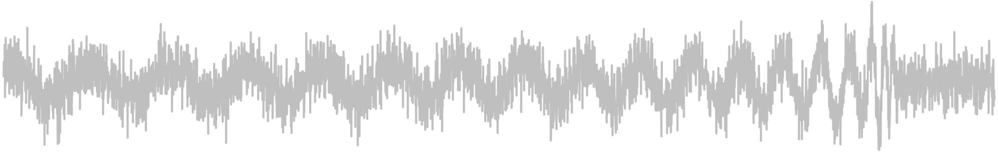

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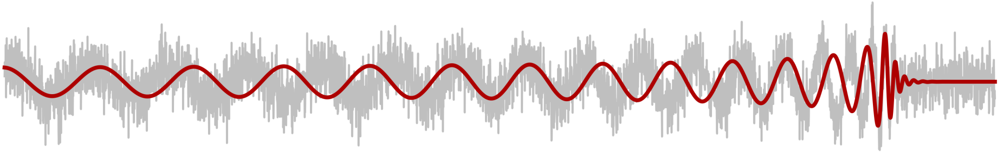

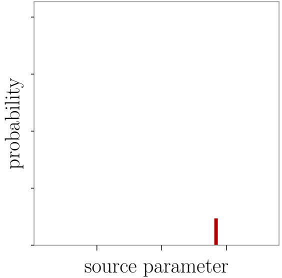

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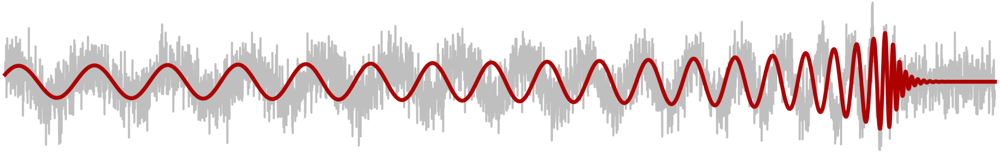

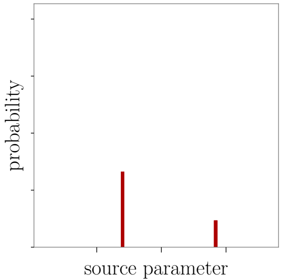

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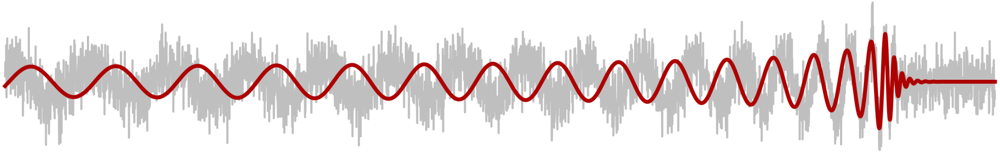

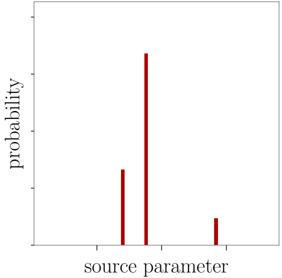

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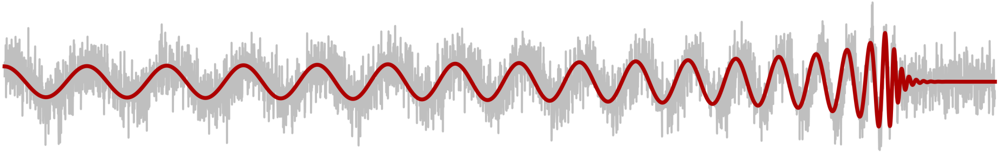

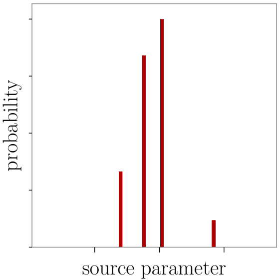

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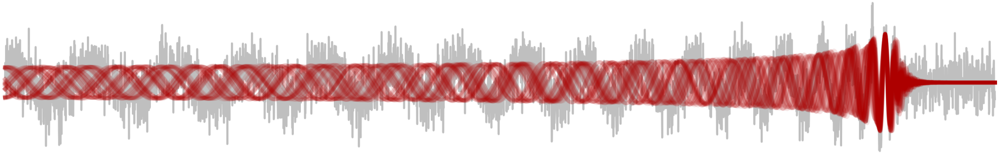

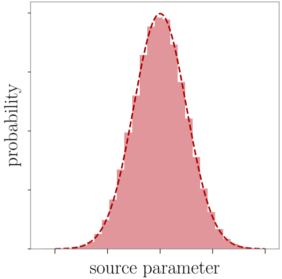

Likelihood & Posterior

Quantities d : data, h : GW model, θ : source parameters

Inner Product $\langle d, h \rangle = 4\,\mathrm{Re} \int \frac{\tilde{d}(f)\tilde{h}^*(f)}{S_n(f)} df$

Likelihood $\Lambda(d|h(\theta)) \sim \exp(-\|d - h(\theta)\|^2/2)$

Posterior $p(\theta|d) = \frac{\Lambda(d|\theta)\pi(\theta)}{p(d)}$