# Gravitational Waves

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Astroparticle School September 2025

Binary Merger Observations & Numerical Relativity

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# My Career in a Nutshell


- Diploma 2008
- Numerical Relativity
- “Small field,” more opportunities in gravitational waves


- PhD 2012 Signal Modelling
- Joining LIGO Collaboration
- Married


- 2012-2016
- Data Analysis
- First detection
- First child

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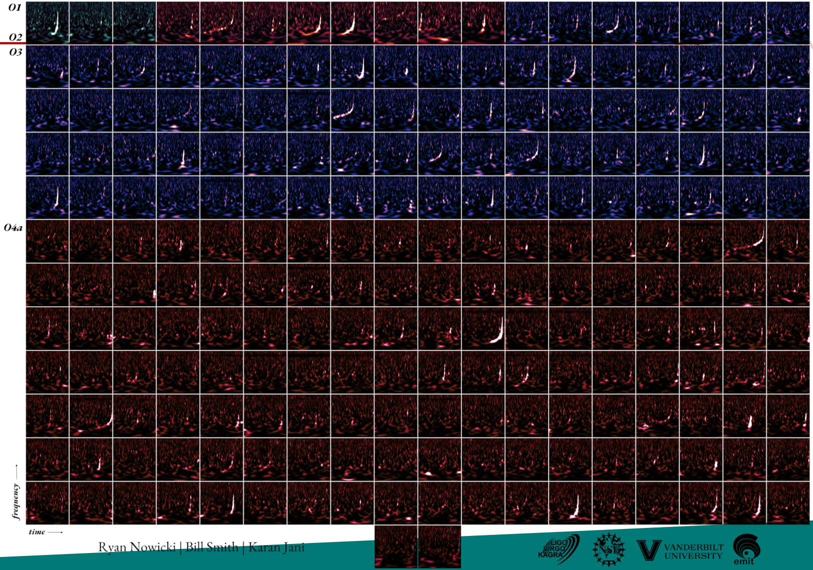
# Topics

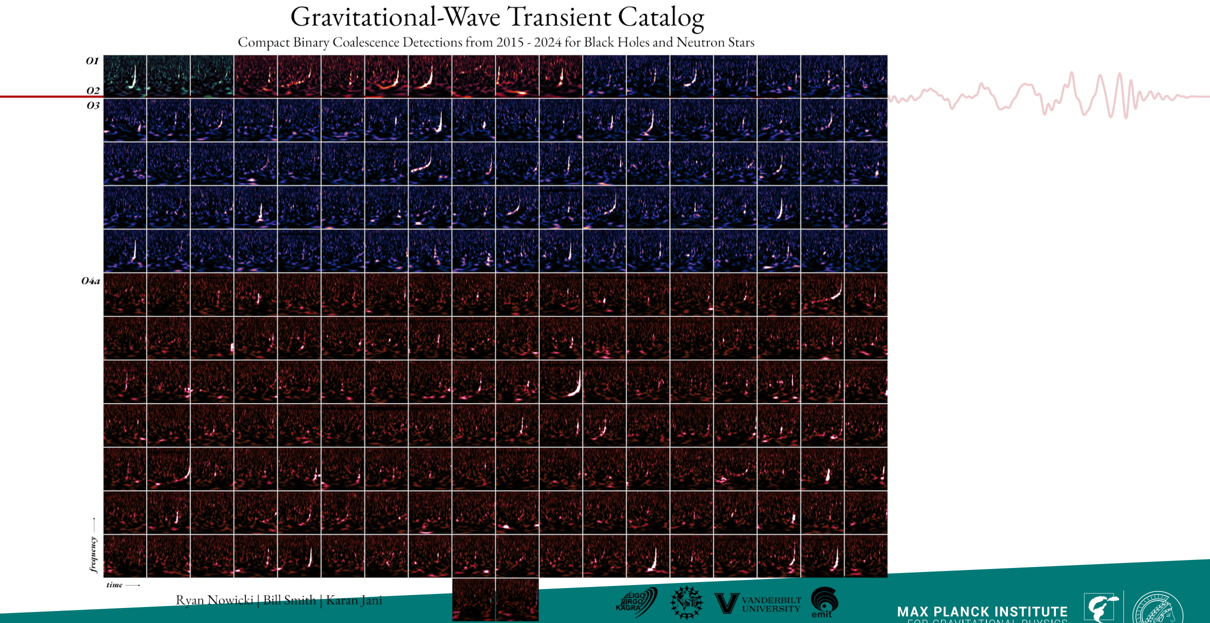
1. Gravity
   - Newton vs. Einstein
   - Curved Spacetime
   - Effect of Gravitational Waves
   - Binary System
2. Laser Interferometer Gravitational-wave Observatory (LIGO)
   - Basic Interferometry
   - Virtual Tour through LIGO
   - The GW Detector Network
3. Data Analysis Challenge
   - Characterising Noise
   - LIGO's Noise
   - Signals Hidden in Noise
   - Matched-Filter Searches
   - First Discovery
4. Interpreting the Signal
   - GW150914
   - Energy Carried by GWs
   - The Mass in GW150914
   - Waveform Models
   - Parameter Estimation

# Gravitational-Wave Transient Catalog

Compact Binary Coalescence Detections from 2015 - 2024 for Black Holes and Neutron Stars

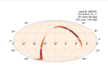
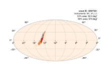
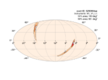
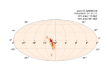



Ryan Nowicki | Bill Smith | Karan Jani


Gravitational-Wave Transient Catalog
Compact Binary Coalescence Detections from 2015 - 2024 for Black Holes and Neutron Stars
O1
O2
O3
O4a
frequency
time
Ryan Nowicki | Bill Smith | Karan Jani

# Public Alerts

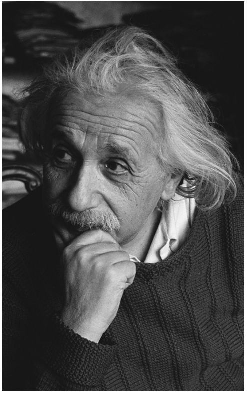
SORT: EVENT ID (A-Z)

| Event ID | Possible Source (Probability) | Significant | UTC | GCN | Location | FAR |
|---|---|---|---|---|---|---|
| S250904cv | BBH (>99%) | Yes | Sept. 4, 2025 13:49:52 UTC | GCN Circular Query Notices \| VOE |  | 1 per 100.04 years |
| S250904br | BBH (>99%) | Yes | Sept. 4, 2025 10:22:08 UTC | GCN Circular Query Notices \| VOE |  | 1 per 100.04 years |
| S250904ae | BBH (>99%) | Yes | Sept. 4, 2025 03:33:07 UTC | GCN Circular Query Notices \| VOE |  | 1 per 100.04 years |
| S250901cb | BBH (>99%) | Yes | Sept. 1, 2025 18:59:41 UTC | GCN Circular Query Notices \| VOE |  | 1 per 9.9253e+05 years |

# Gravity

# What is Gravity?

vs.





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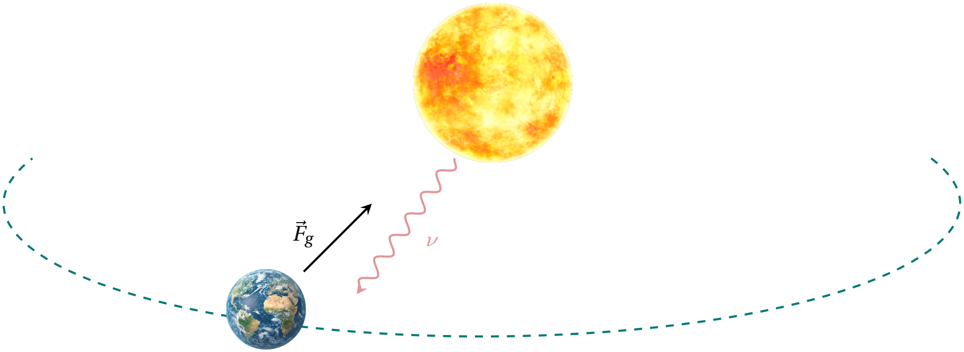
# Thought Experiment



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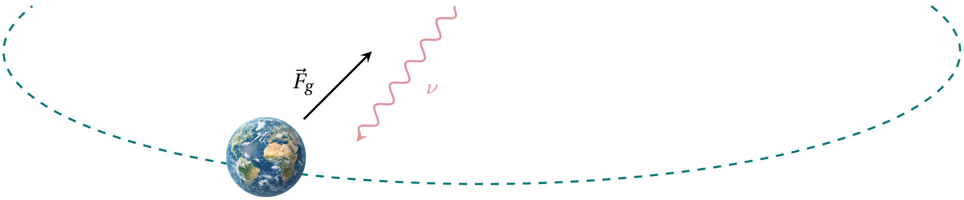

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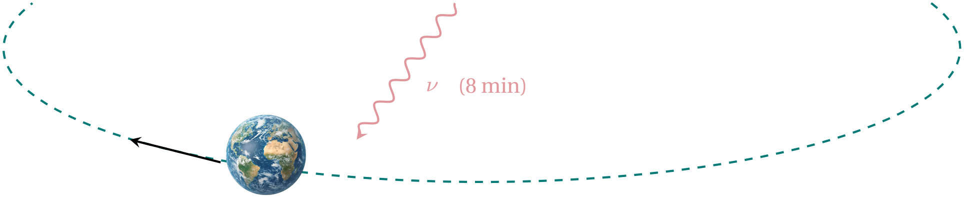

# Gravity is Special

$$F = m_i a = -m_g g$$

- $F$: Force
- $a$: acceleration, change of velocity
- $g$: acceleration, gravitational field
- $m_i$: inertial mass
- $m_g$: gravitational mass


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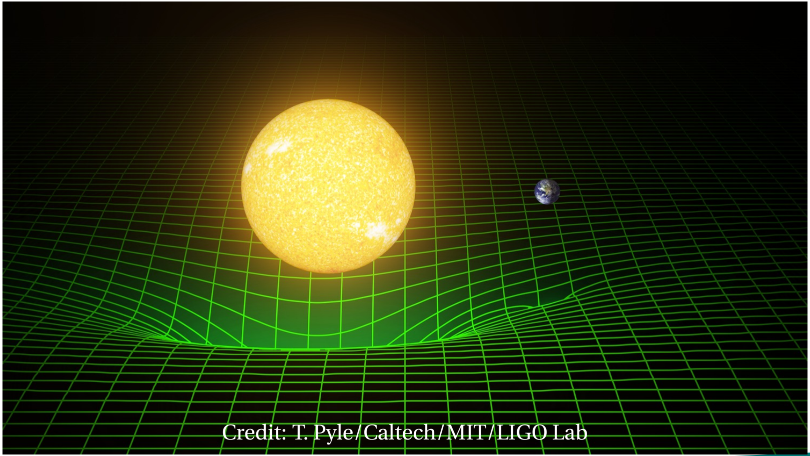
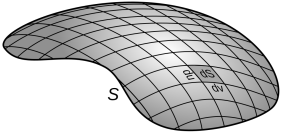
# Gravity = Curved Spacetime



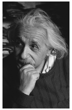
# Curvature $\leftrightarrow$ Separation




Properties of spacetime encoded in **metric** $g_{\mu\nu}$

Assume small perturbation of flat spacetime,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad |h_{\mu\nu}| \ll 1$$



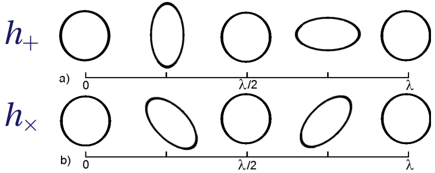
Einstein's Eq. $\rightarrow \Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$

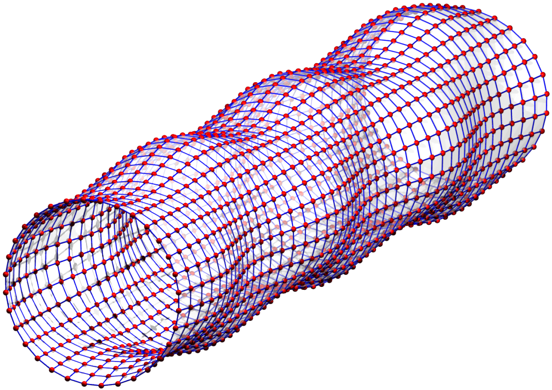
$$\bar{h}^{ij}(t, \vec{x}) = \frac{G}{c^4} \frac{2}{|\vec{x}|} \frac{d^2}{dt^2} \int y^i y^j \rho(t - |\vec{x}|/c, \vec{y}) d^3 y$$

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow$$ *Gravitational waves*

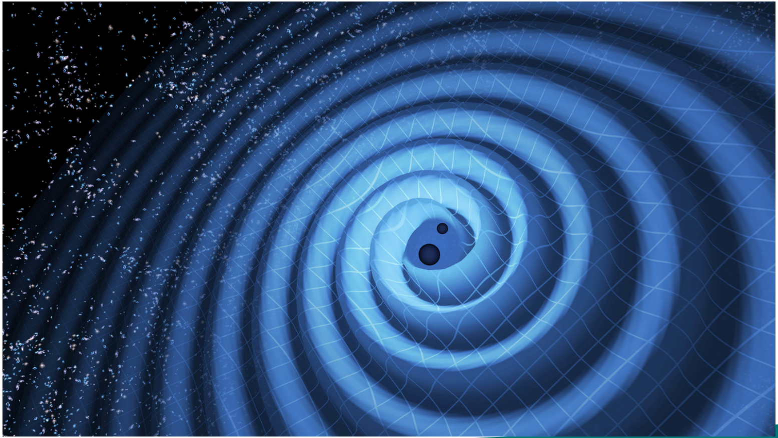
# The Effect of Gravitational Waves


$$\frac{\delta L}{L} \approx \frac{h}{2}$$



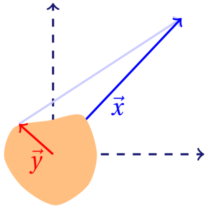
# Merging Black Holes



# Newtonian Binary

$$\bar{h}_{ij}(t,\vec{x}) = \frac{4G}{c^4}\int \frac{T_{ij}(t-|\vec{x}-\vec{y}|,\vec{y})}{|\vec{x}-\vec{y}|}\, d^3y$$

$$\approx \frac{G}{c^4}\frac{2}{|\vec{x}|}\frac{d^2}{dt^2}\int y^i y^j \rho(t-|\vec{x}|,\vec{y})\, d^3y$$




(for $|\vec{x}| \gg |\vec{y}|$, finite-size, non-relativistic sources)

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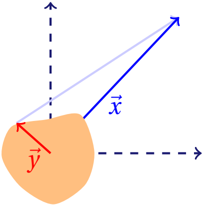


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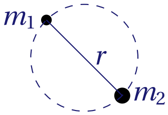


Assume the perfect circular motion of two point particles $m_1 = m_2$ with a constant orbital frequency $\omega$.

1. Parameterize the position of both particles $\vec{x}_1(t), \vec{x}_2(t)$.
2. Use $\rho(t,\vec{y}) = m_1\delta(\vec{y}-\vec{x}_1) + m_2\delta(\vec{y}-\vec{x}_2)$ to calculate $\bar{h}_{ij}$.

# Solution

Centre-of-mass coordinate system:

$$\vec{x}_1 = \left[\frac{r}{2}\cos(\omega t), \frac{r}{2}\sin(\omega t)\right] = -\vec{x}_2$$



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$$\int y^1\, y^1\, \rho(t,\vec{y})\, d^3y = m_1 \frac{r^2}{4}\cos^2(\omega t) + m_2 \frac{r^2}{4}\cos^2(\omega t) = \frac{Mr^2}{4}\cos^2(\omega t)$$

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$$= \frac{Mr^2}{8}\begin{pmatrix} 1+\cos(2\omega t) & \sin(2\omega t) \\ \sin(2\omega t) & 1-\cos(2\omega t) \end{pmatrix}$$


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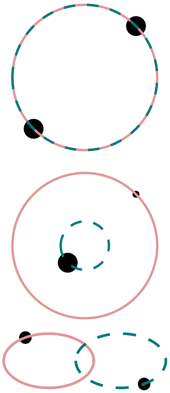
$$\bar{h}_{ij}(t,\vec{x}) = -\frac{G}{c^4}\frac{Mr^2\omega^2}{|\vec{x}|}\begin{pmatrix} \cos(2\omega t') & \sin(2\omega t') \\ \sin(2\omega t') & -\cos(2\omega t') \end{pmatrix}, \qquad t' = t - |\vec{x}|$$

# Compact Binaries

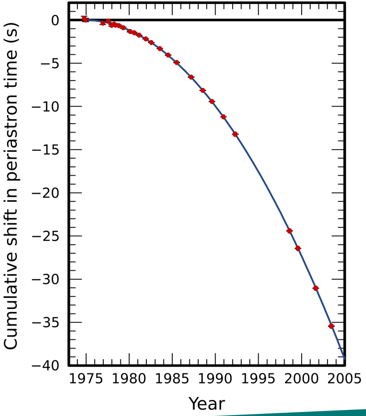
## What we learned

- Amplitude scales with $\frac{G}{c^4}\frac{Mr^2\omega^2}{D}$, frequency goes as $2\omega$
- Order of magnitude: $Gc^{-4} \approx 10^{-44}\mathrm{s}^2/(\mathrm{kg\,m}) \approx 5\times 10^{-20}\frac{\mathrm{Mpc}}{M_\odot\,c^2}$
- Unequal masses: $M \mapsto 4\mu = 4m_1m_2/M$
- separation $r$ and orbital frequency $\omega$ not independent
  Kepler's law: $\omega^2 = \frac{GM}{r^3}$
- 4 scales of the binary: $m_1$, $m_2$, $D$, $r$
- Eccentric systems more complex



# Observational evidence

Hulse-Taylor pulsar 1978, Nobel prize 1993




- $m_1 = 1.441 M_\odot$
- $m_2 = 1.387 M_\odot$

$\Rightarrow$ $M = 2.828 M_\odot$

- $D \approx 6.4\,\text{Mpc}$
- $P \approx 7.75\,\text{hr}$

$\Rightarrow$ $\omega_{\text{orb}} \approx 2.25 \times 10^{-4}\,\text{s}^{-1}$

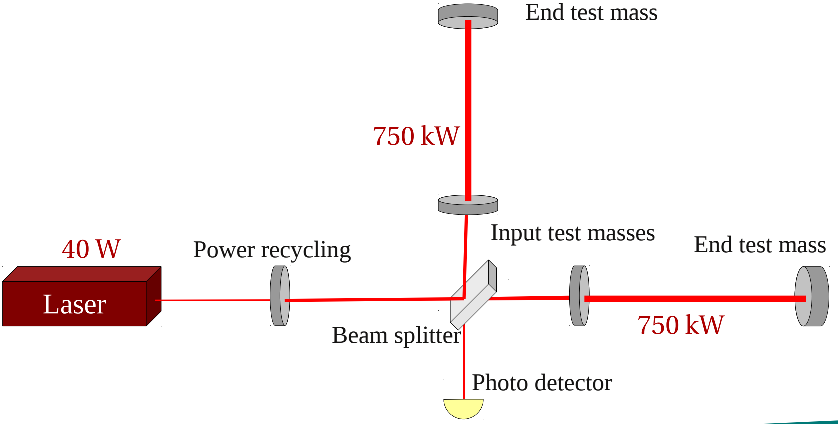
- $r \sim 10^6\,\text{km}$

$$\Rightarrow \quad h \sim \mathcal{O}(10^{-26})$$

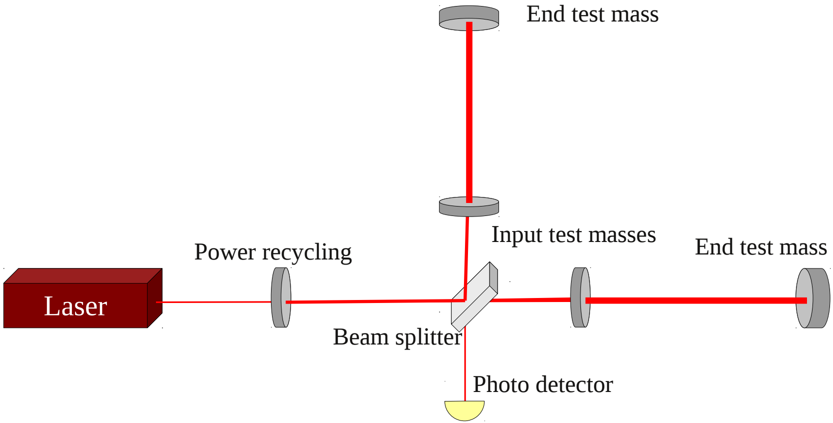
[for comparison (in metres): Carbon atom radius $\mathcal{O}(10^{-10})$, proton radius $\mathcal{O}(10^{-15})$, earth-moon $\mathcal{O}(10^8)$, earth-sun $\mathcal{O}(10^{11})$]

# Laser Interferometer Gravitational-wave Observatory (LIGO)

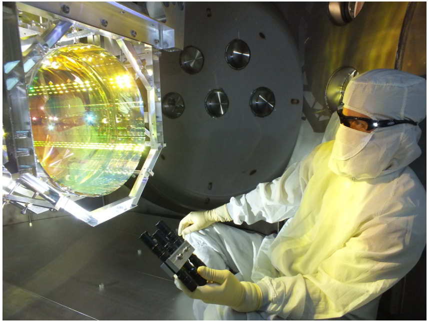
# Most precise length measurements

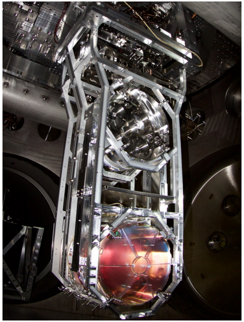


# Virtual Tour through LIGO



# Impressions



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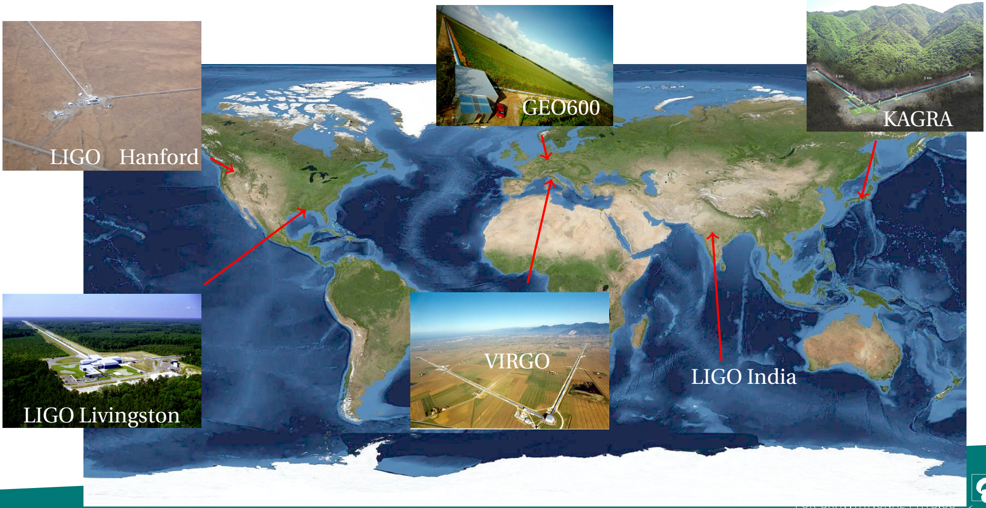
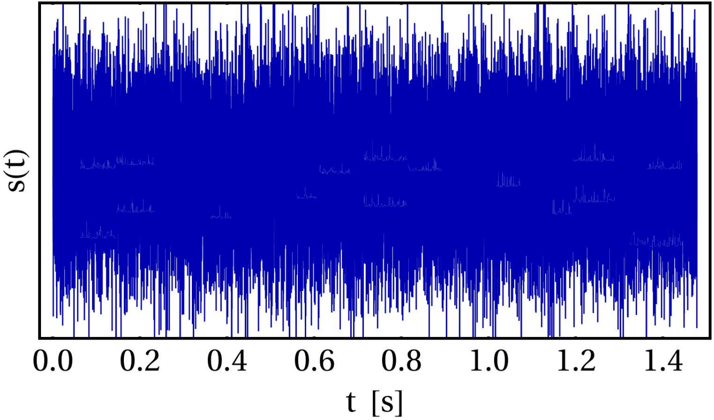
# A Global Network



# Data Analysis Challenge

# Typical Data Stream



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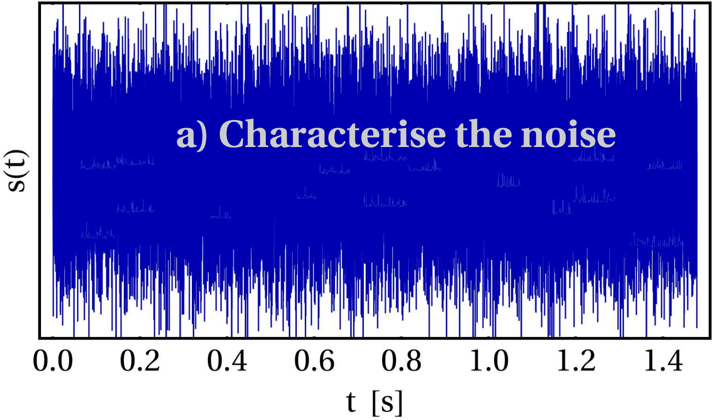

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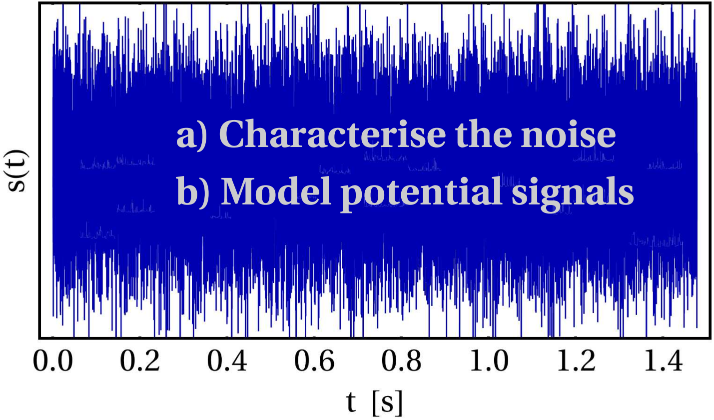



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