Neutron star mergers
Lecture II

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Baernfels
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Plan of the lectures

- ✴Lecture I: **brief** introduction to numerical relativity
- ✴Lecture II: **brief** review dynamics of merging binaries
- ✴Lecture III: **brief** overview of EOS constraints from mergers

- ✴**L. Baiotti and L. Rezzolla, Rep. Prog. Phys. 80, 096901, 2017**
- ✴V. Paschalidis, Classical Quantum Gravity 34, 084002 2017
- ✴Rezzolla and Zanotti, *"Relativistic Hydrodynamics"*, Oxford University Press, 2013

Recap (I)

☑ The 3+1 splitting of the 4-dim spacetime represents an effective way to perform numerical solutions of the Einstein eqs.

☑ Such a splitting amounts to projecting all 4-dim. tensors either on spatial hypersurfaces or along directions orthogonal to such hypersurfaces.

☑ The 3-metric and the extrinsic curvature describe the properties of each slice.

☑ Two functions, the lapse and the shift, tell how to relate coordinates between two slices: the lapse measures the proper time, while the shift measures changes in the spatial coords.

Recap (II)

- ☑ A number of tensor differential identities allow to cast the Einstein equations in a 3+1 split: this is the ADM formulation.
- ☑ Einstein equations in the ADM formulation naturally split into evolution equations and constraint equations.
- ☑ This is not very different from Maxwell equations, where there are also evolution and constraint equations.
- ☑ The ADM eqs are ill posed and not suitable for numerics.
- ☑ Alternative formulations (BSSNOK, CCZ4, Z4c) have been developed that are strongly hyperbolic and hence well-posed.

Recap (III)

- Both CCZ4, Z4c formulations make use of the constraint equations and can use additional evolution equations to damp the violations

- The hyperbolic evolution eqs. to solve are: 6+6+(3+1) = 16. We also "compute" 3+1=4 elliptic constraint eqs

$$\mathcal{H} \equiv {}^{(3)}R + K^2 - K_{ij}K^{ij} = 0 \ , \quad \text{(Hamiltonian constraint)}$$

$$\mathcal{M}^i \equiv D_j(K^{ij} - g^{ij}K) = 0 \ , \quad \text{(momentum constraints)}$$

NOTE: these eqs are not solved but only monitored to verify

$$||\mathcal{H}|| \simeq ||\mathcal{M}^i|| < \varepsilon \sim 10^{-4} - 10^{-2}$$

- Four more equations are needed to set the gauges: lapse and shift

The two-body problem: Newton vs Einstein

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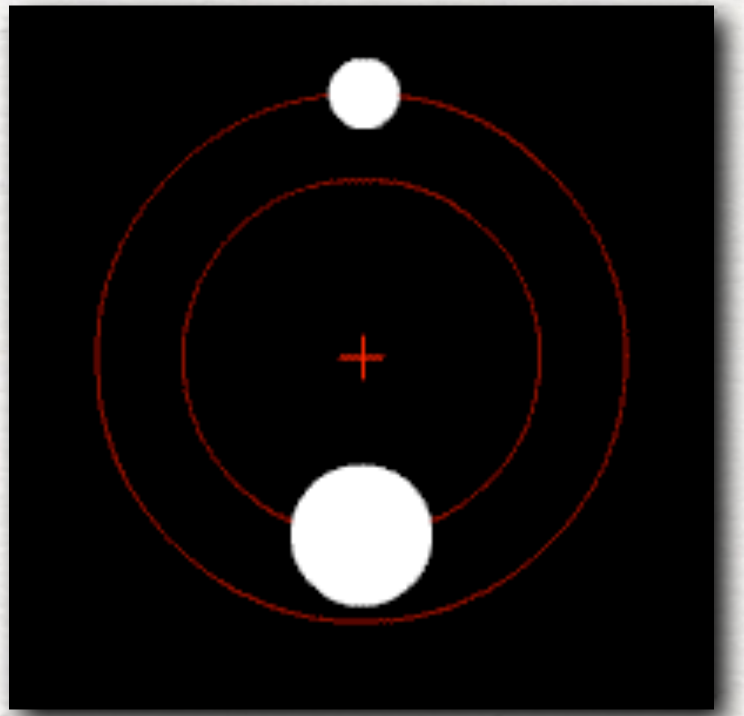


Take two objects of mass m_1 and m_2 interacting only gravitationally

In **Newtonian gravity** solution is analytic: there exist closed orbits (circular/elliptic) with

$$\ddot{\boldsymbol{r}} = -\frac{GM}{d_{12}^3}\boldsymbol{r}$$

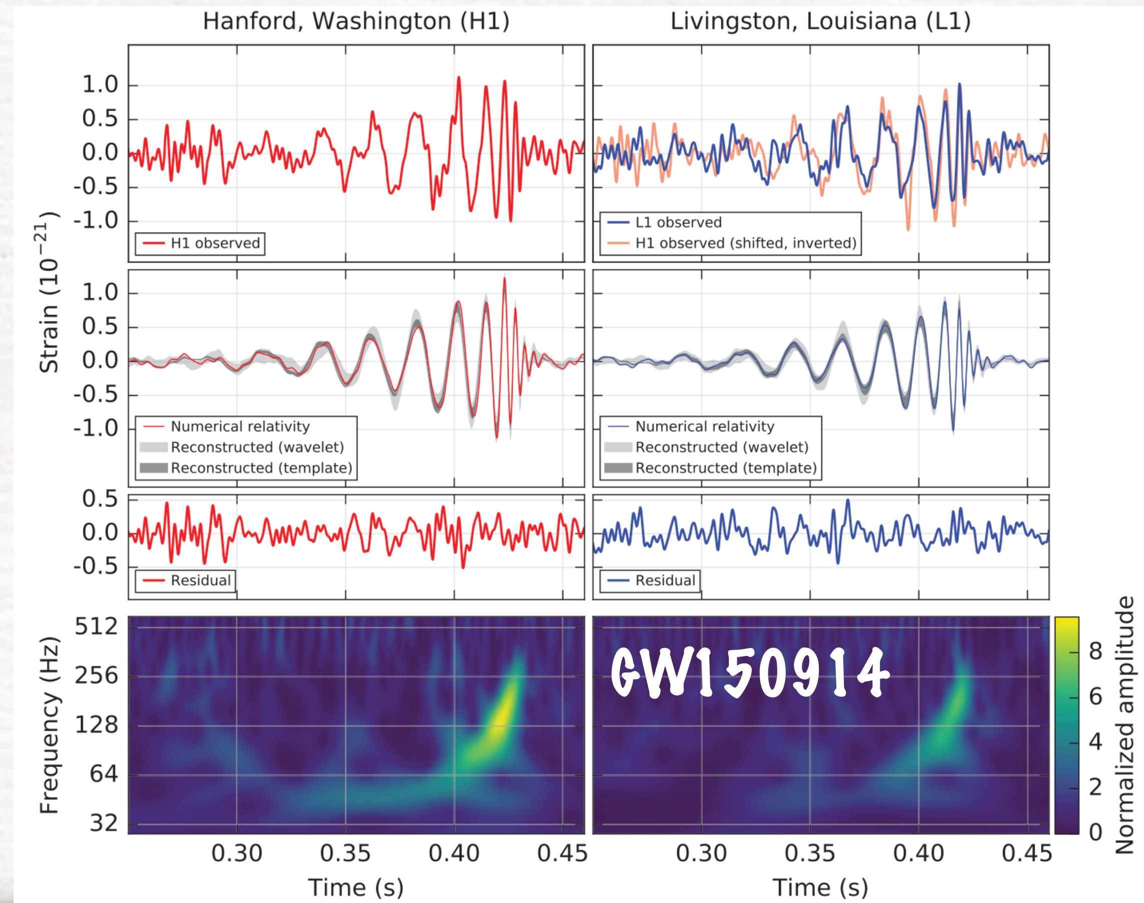
where $M \equiv m_1 + m_2\,, \boldsymbol{r} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2\,, d_{12} \equiv |\boldsymbol{r}_1 - \boldsymbol{r}_2|\,.$

In **Einstein's gravity** no analytic solution! No closed orbits: the system loses energy/angular momentum via gravitational waves.

The two-body problem in GR

- For BHs we know what to **expect**:

BH + BH → BH + GWs



Abbott+ 2016

The two-body problem in GR

- For BHs we know what to **expect**:

BH + BH ⟶ BH + GWs

- For NSs the question is more **subtle:** the merger leads to an hyper-massive neutron star (HMNS), ie a metastable equilibrium:

NS + NS ⟶ HMNS+**... ?** ⟶ BH+torus+**... ?** ⟶ BH + GWs

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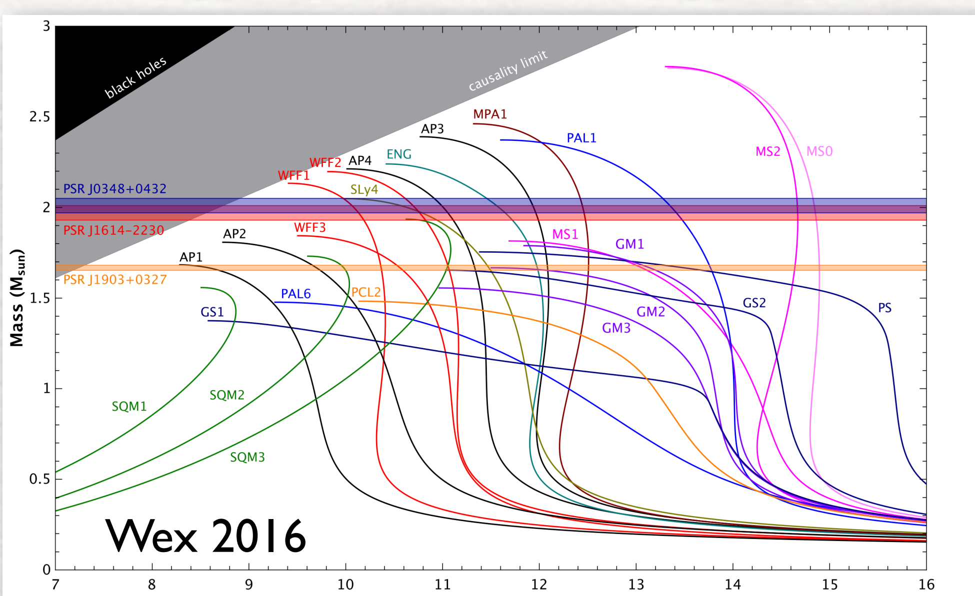
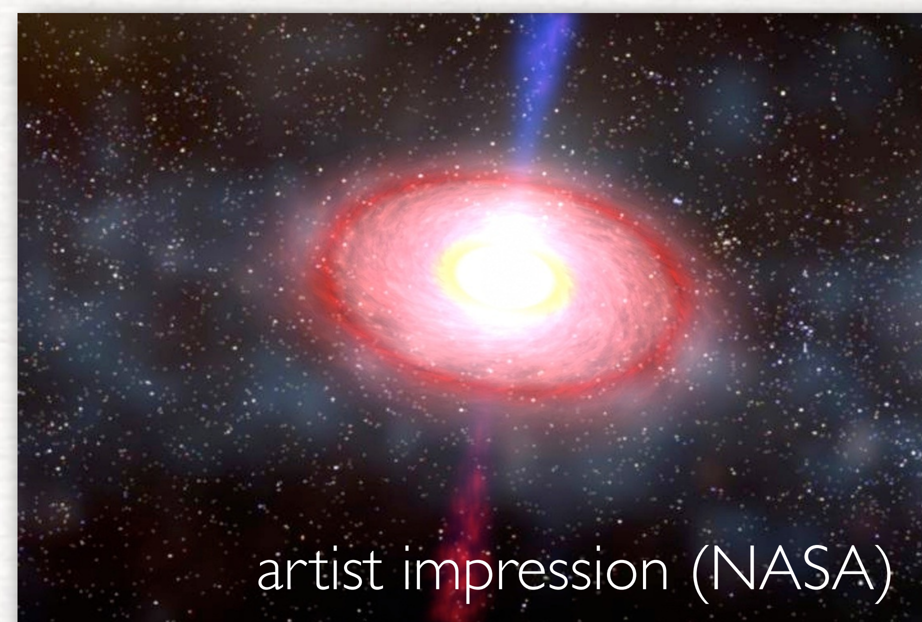
- **HMNS** phase can provide clear information on **EOS**





- **BH+torus** system may tell us on the central engine of **GRBs**

The two-body problem in GR

- For BHs we know what to **expect**:

BH + BH ⟶ BH + GWs

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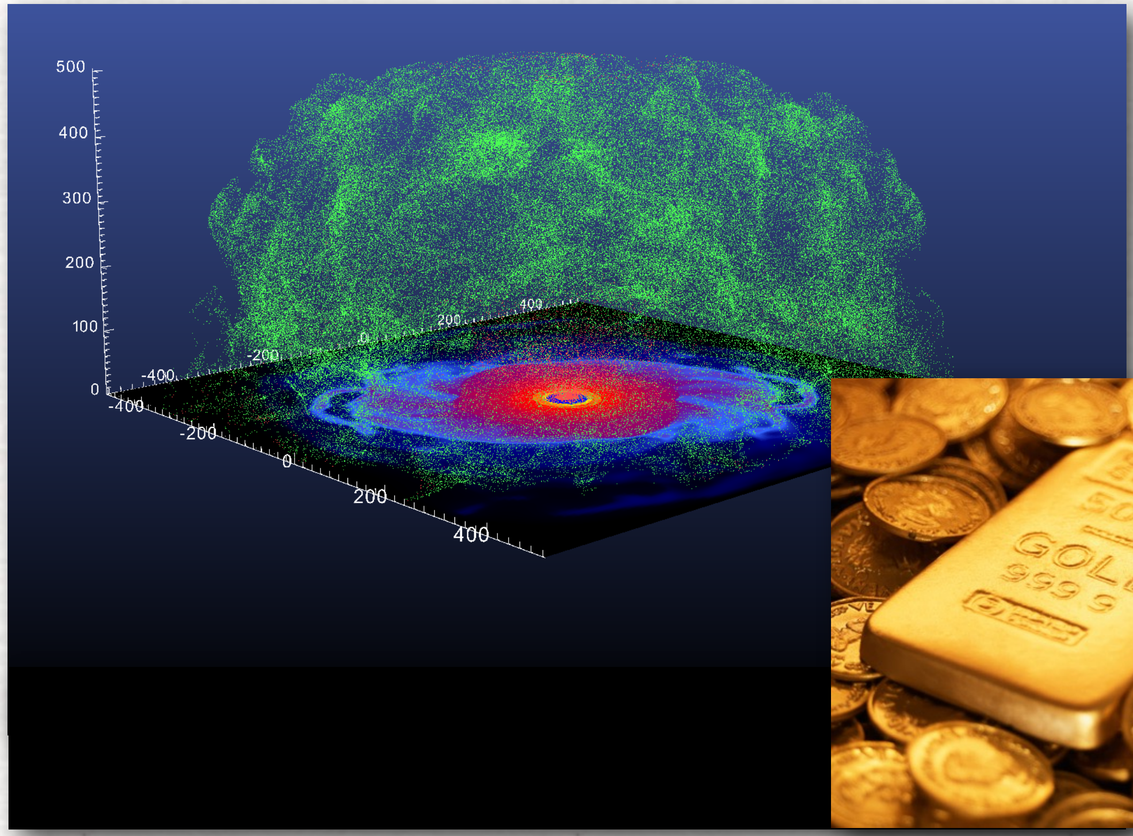
NS + NS ⟶ HMNS+**... ?** ⟶ BH+torus+**... ?** ⟶ BH + GWs

- **ejected matter** undergoes nucleosynthesis of heavy elements



The equations of numerical relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}\,, \qquad \text{(field equations)}$$

$$\nabla_\mu T^{\mu\nu} = 0\,, \qquad \text{(cons. energy/momentum)}$$

$$\nabla_\mu (\rho u^\mu) = 0\,, \qquad \text{(cons. rest mass)}$$

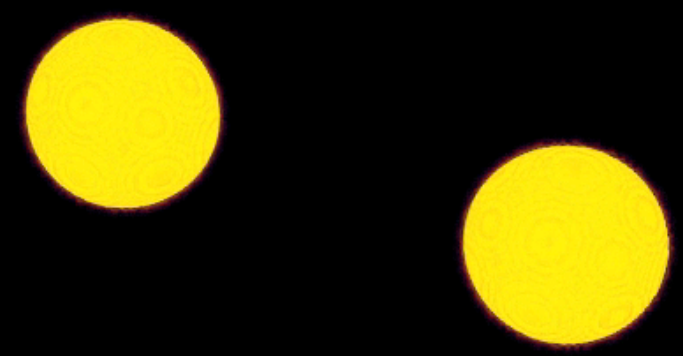
$$p = p(\rho, \epsilon, Y_e, \ldots)\,, \qquad \text{(equation of state)}$$

$$\nabla_\nu F^{\mu\nu} = I^\mu\,, \quad \nabla_\nu^* F^{\mu\nu} = 0\,, \qquad \text{(Maxwell equations)}$$

$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + T_{\mu\nu}^{\text{EM}} + \ldots \qquad \text{(energy – momentum tensor)}$$

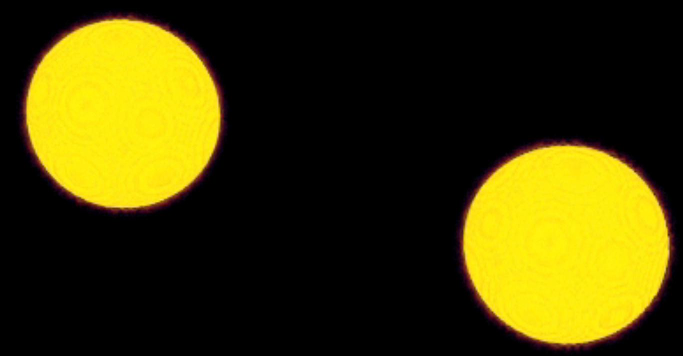
In GR these equations do not possess an analytic solution in the regimes we are interested in

Animations: Breu, Radice, LR



$M = 2 \times 1.35\, M_\odot$

LS220 EOS



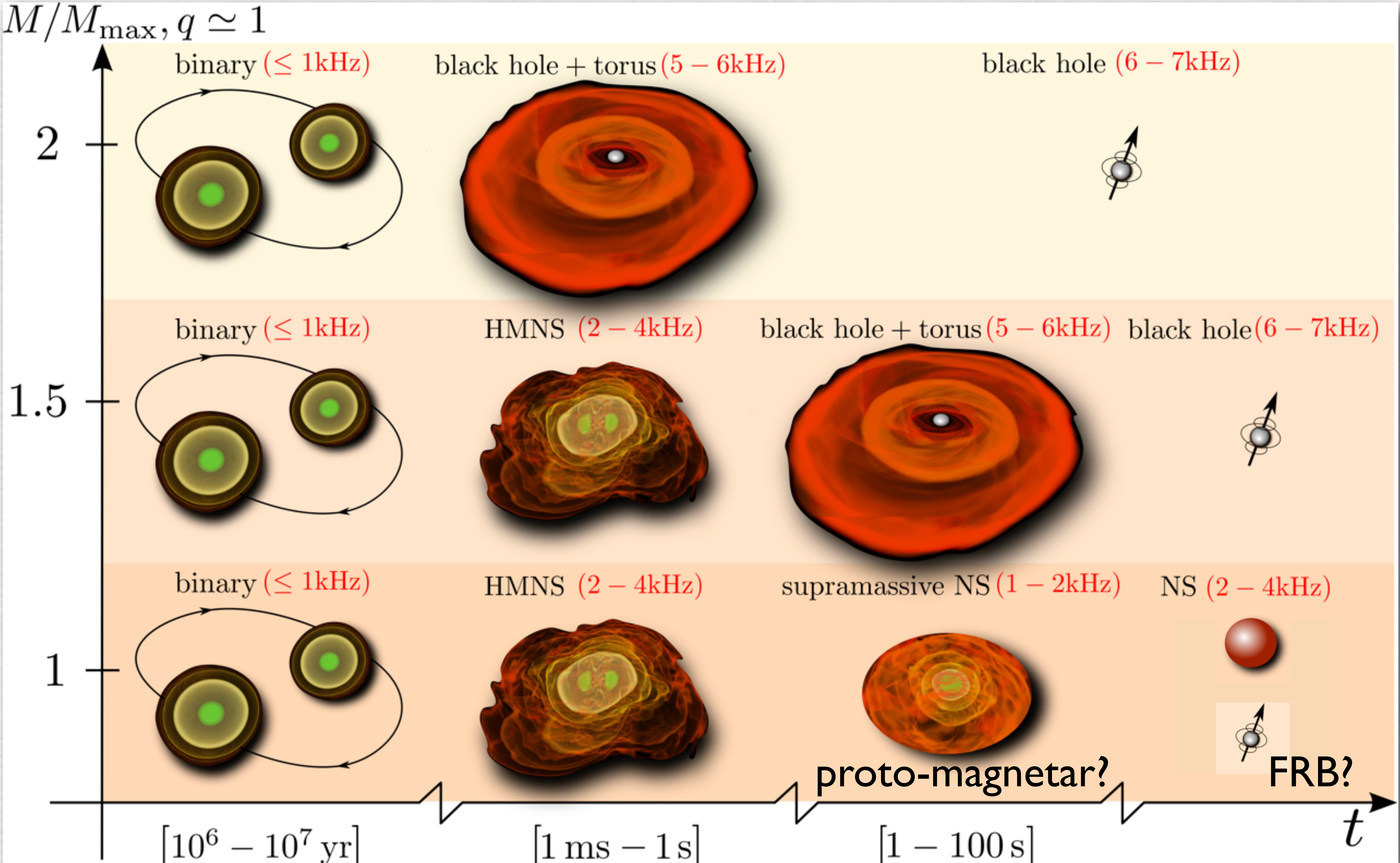
merger → HMNS → BH + torus

merger ⟶ HMNS ⟶ BH + torus

Quantitative differences are produced by:

- **total mass (prompt vs delayed collapse)**

Broadbrush picture



merger ⟶ HMNS ⟶ BH + torus

Quantitative differences are produced by:

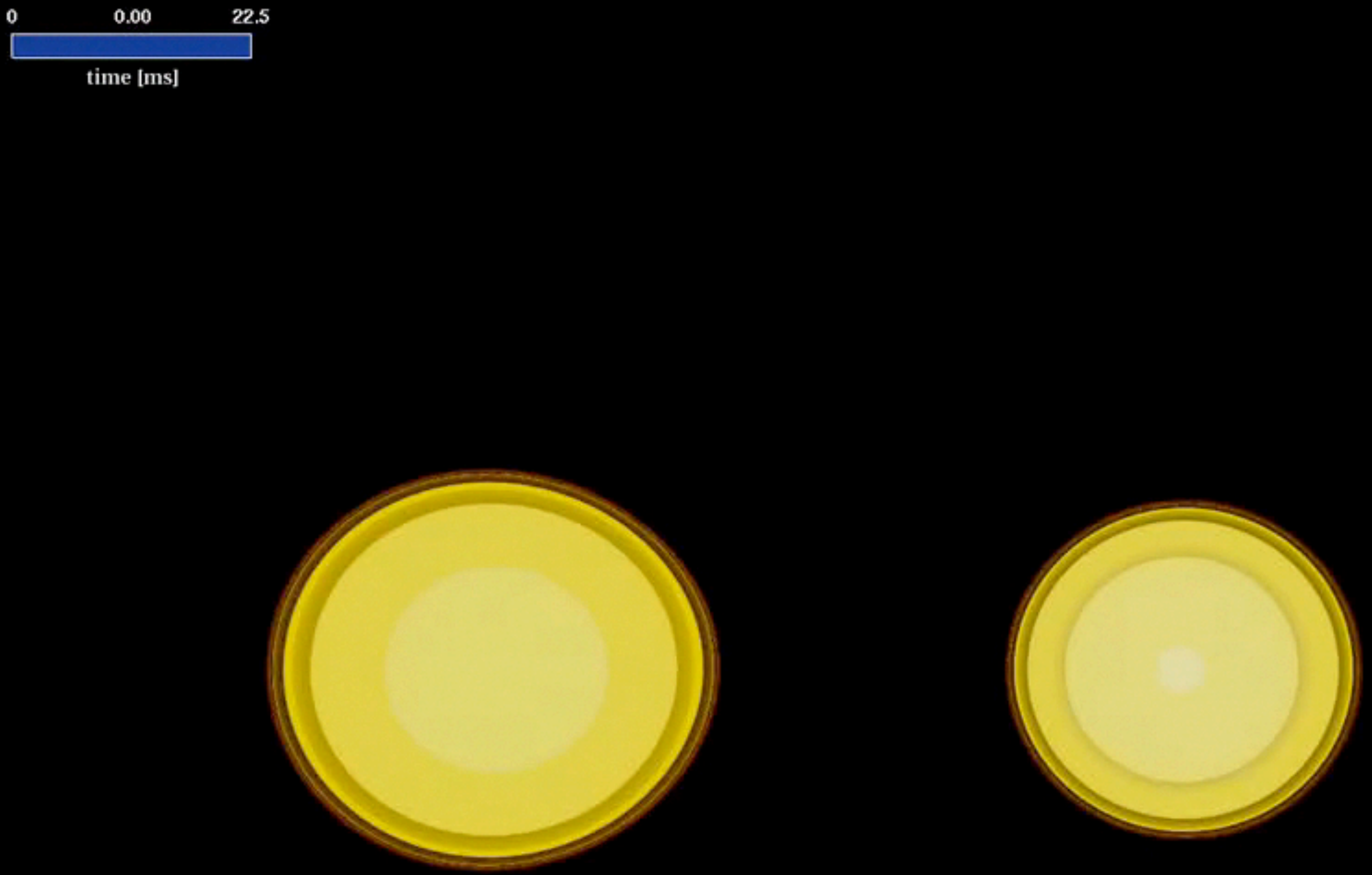
- total mass (prompt vs delayed collapse)
- mass asymmetries (HMNS and torus)

0 0.00 22.5
time [ms]

Animations: Giacomazzo, Koppitz, LR

Total mass : $3.37\,M_{\odot}$; mass ratio :0.80;

9 15
log(rho)[g/cm^3]



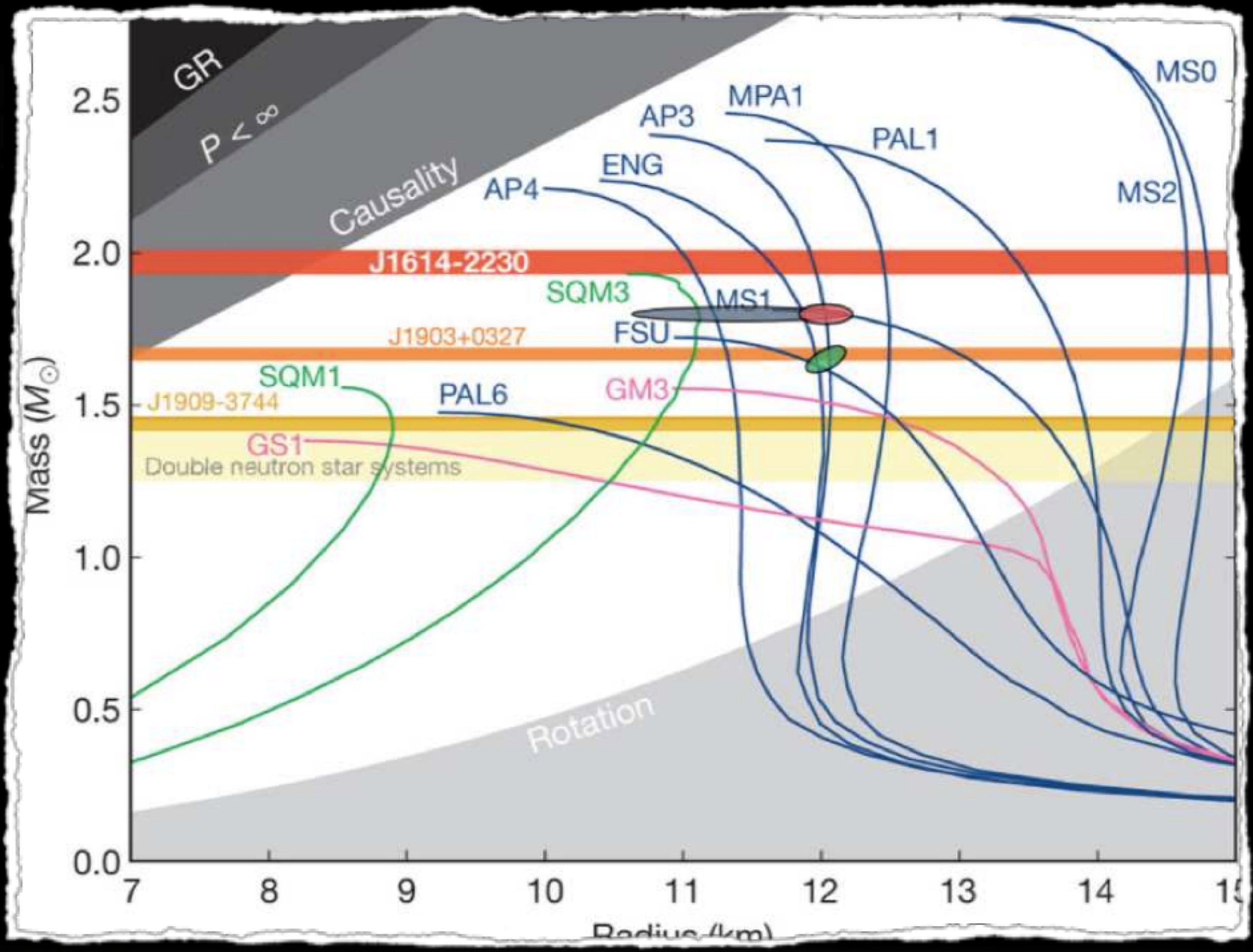
- the torii are generically more massive
- the torii are generically more extended
- the torii tend to stable quasi-Keplerian configurations
- overall unequal-mass systems have all the ingredients needed to create a GRB

merger ⟶ HMNS ⟶ BH + torus

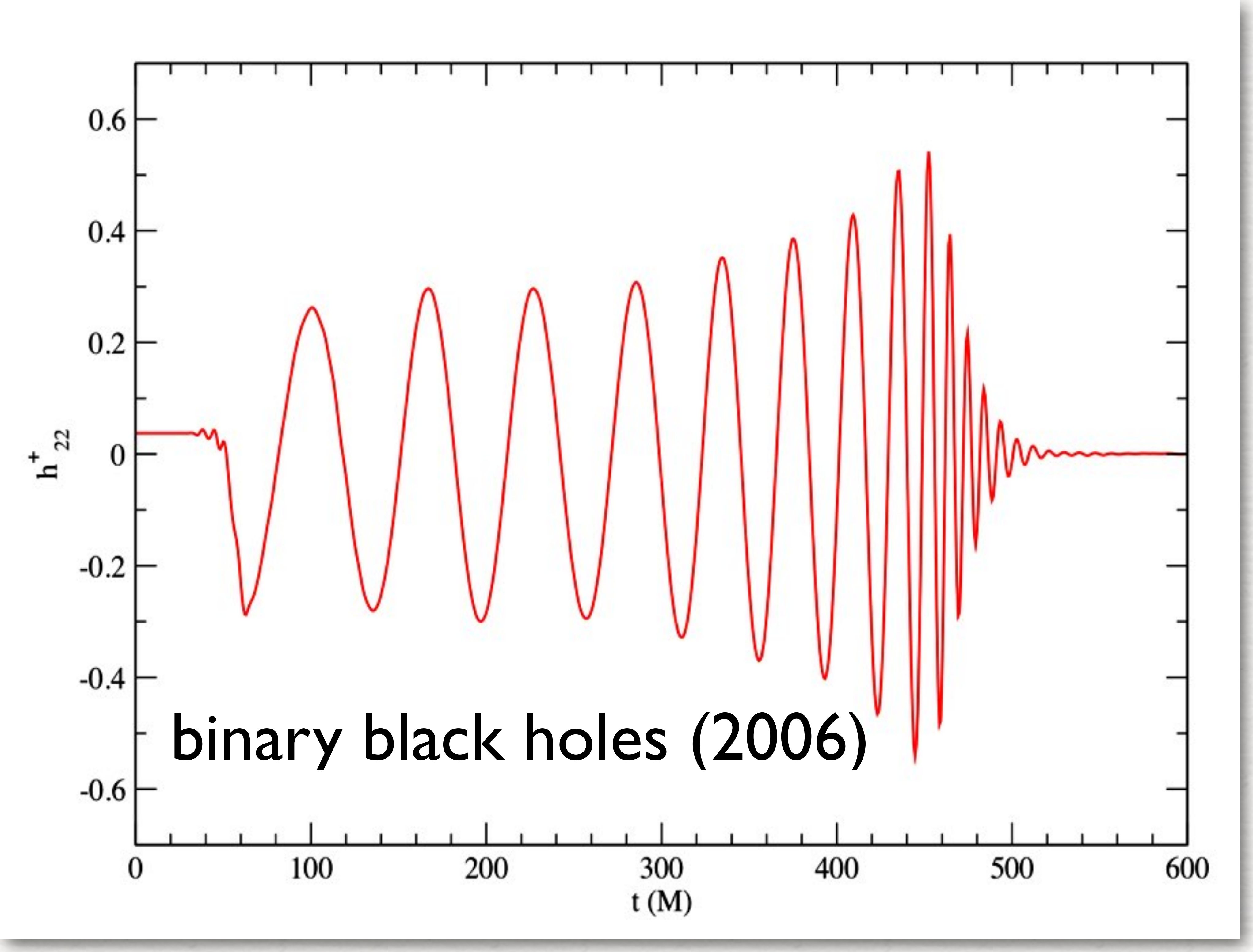
Quantitative differences are produced by:

- total mass (prompt vs delayed collapse)
- mass asymmetries (HMNS and torus)
- soft/stiff EOS (inspiral and post-merger)
- magnetic fields (equil. and EM emission)
- radiative losses (equil. and nucleosynthesis)

How to constrain the EOS from the GWs



Anatomy of the GW signal



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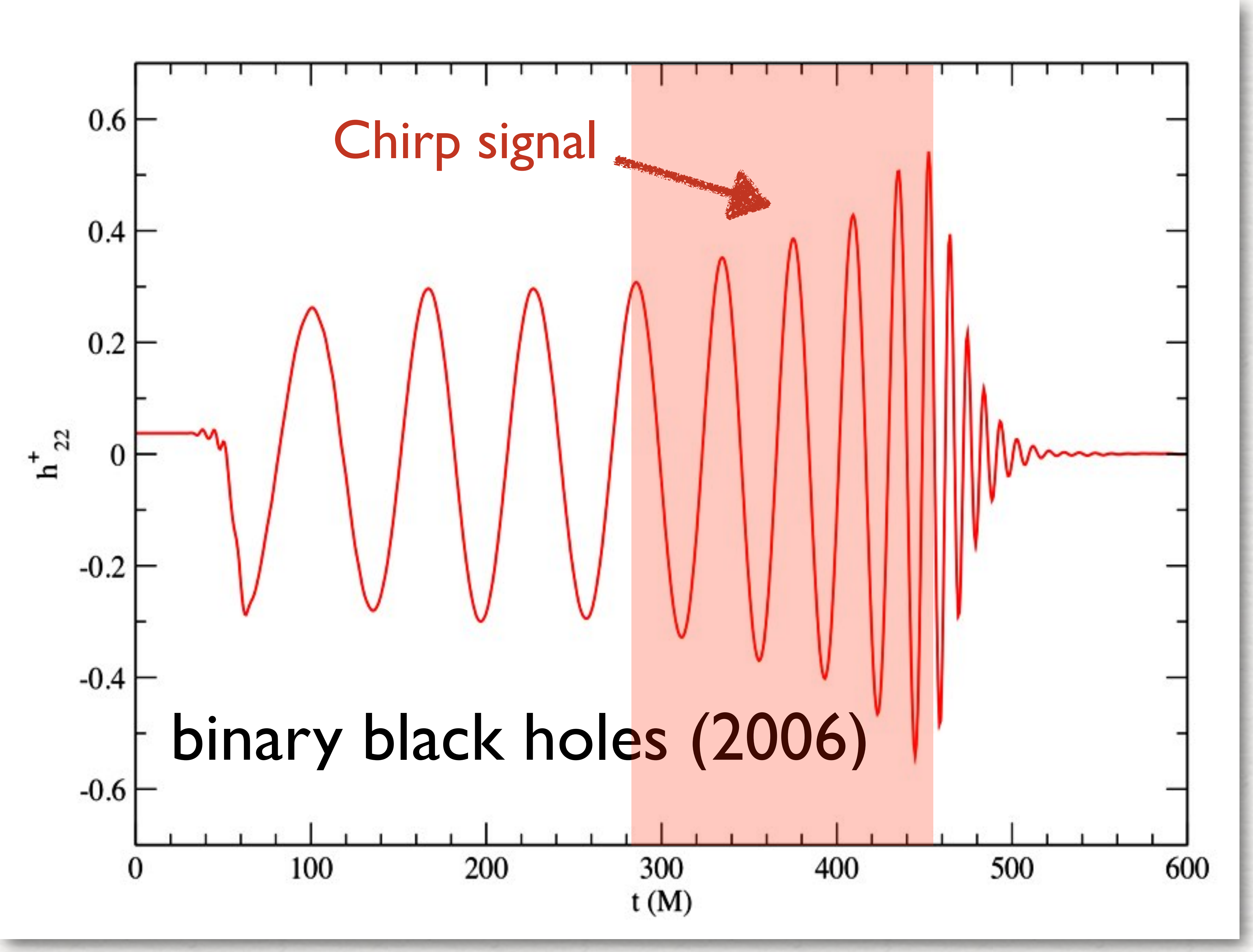

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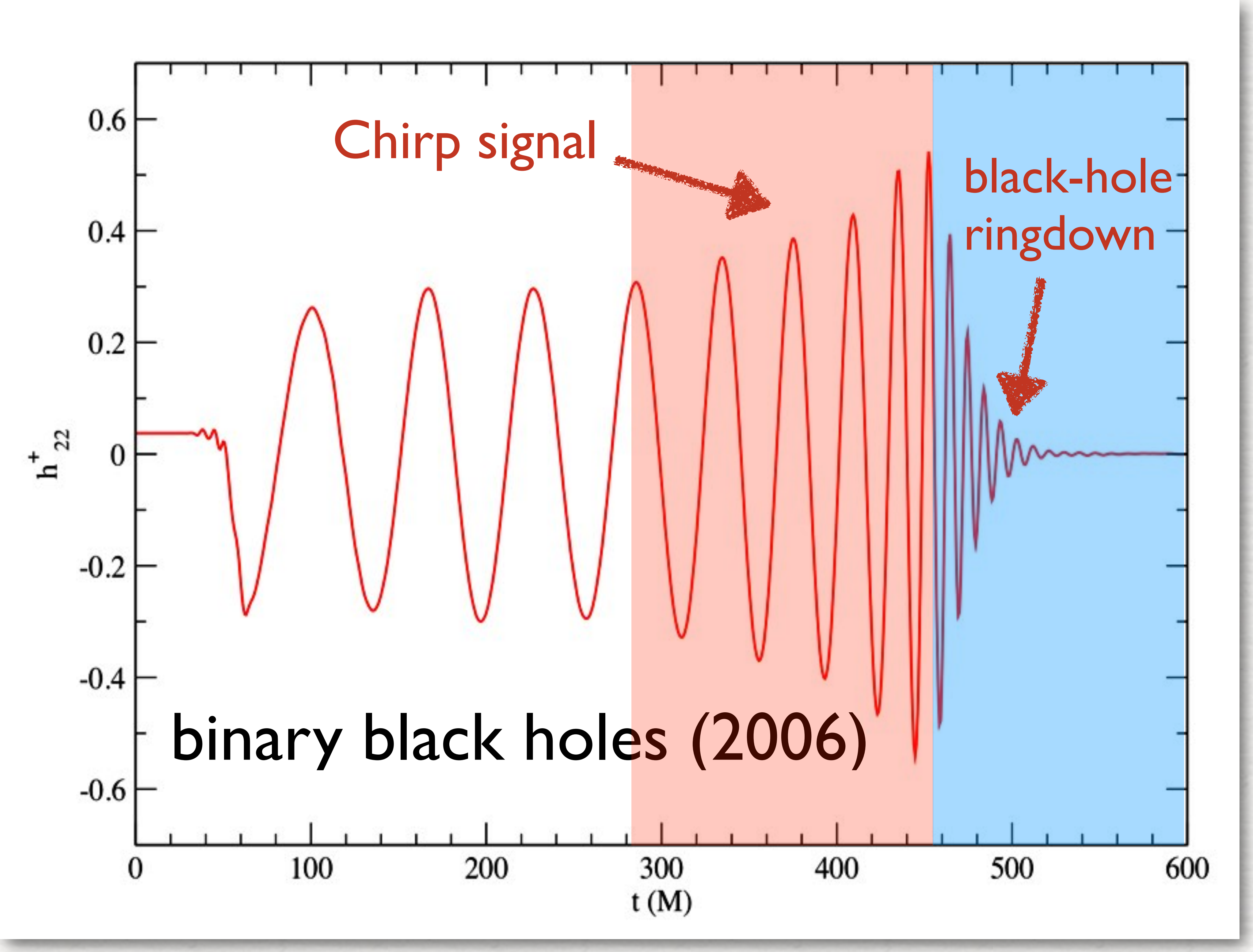

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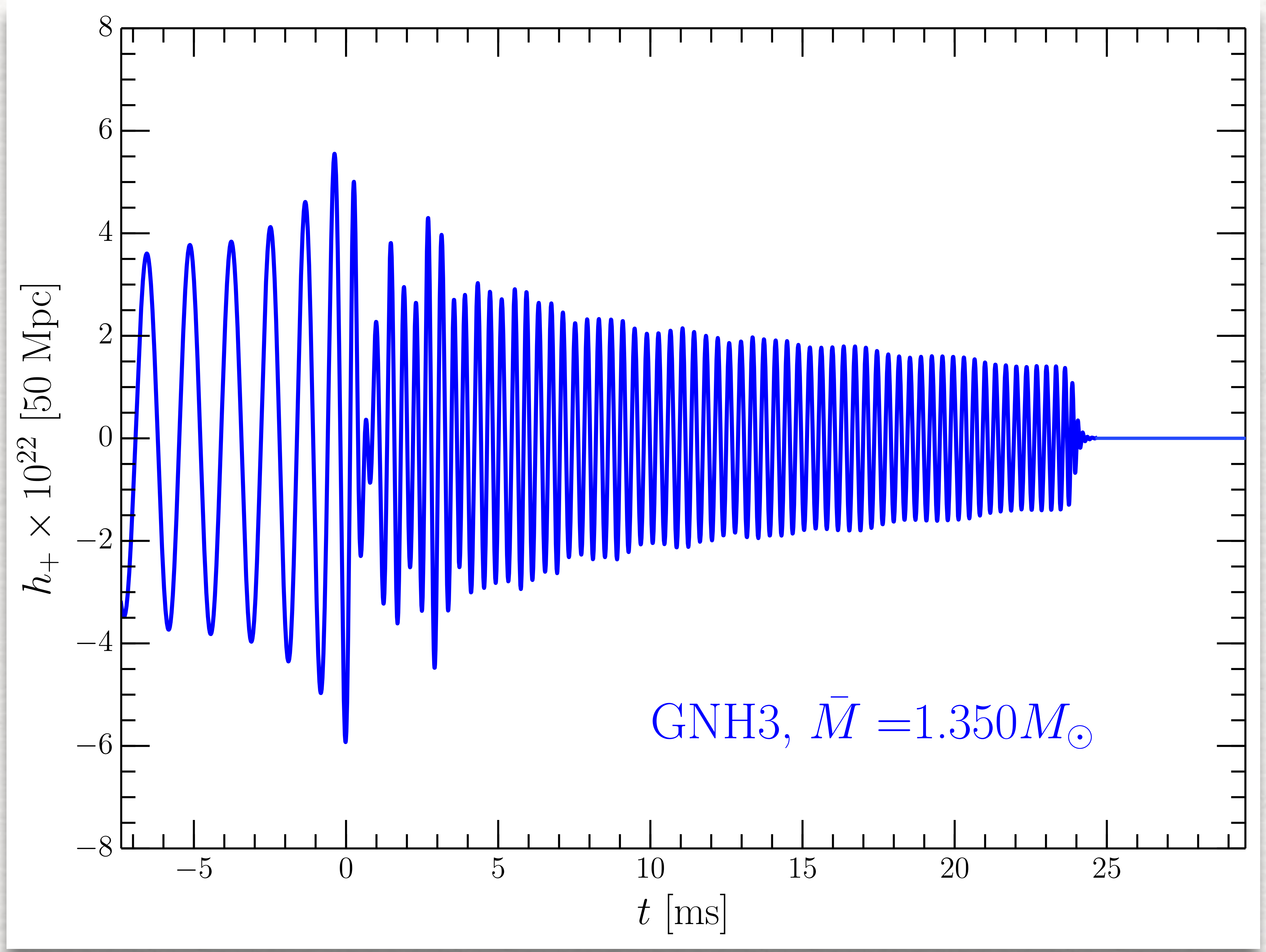

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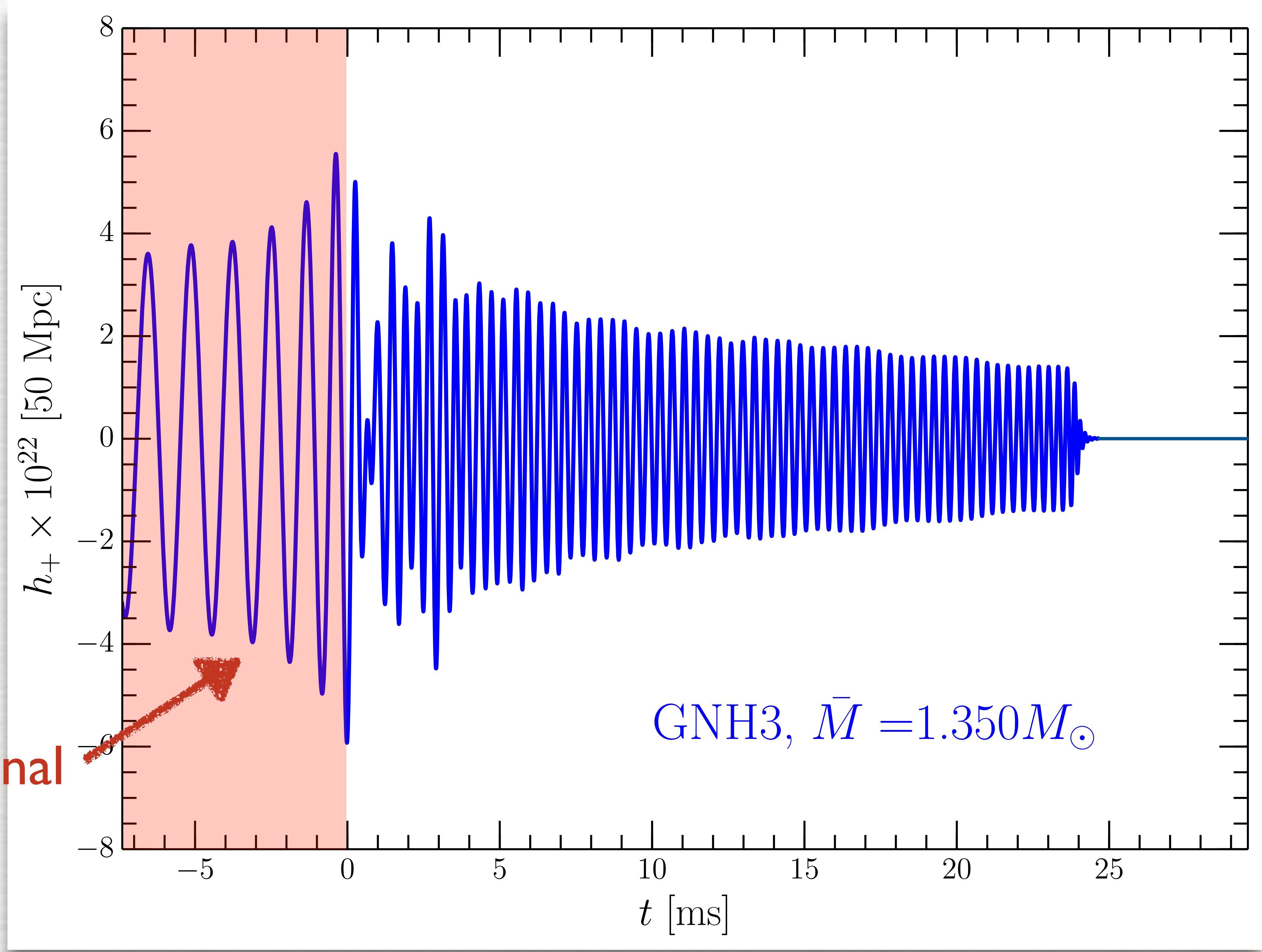
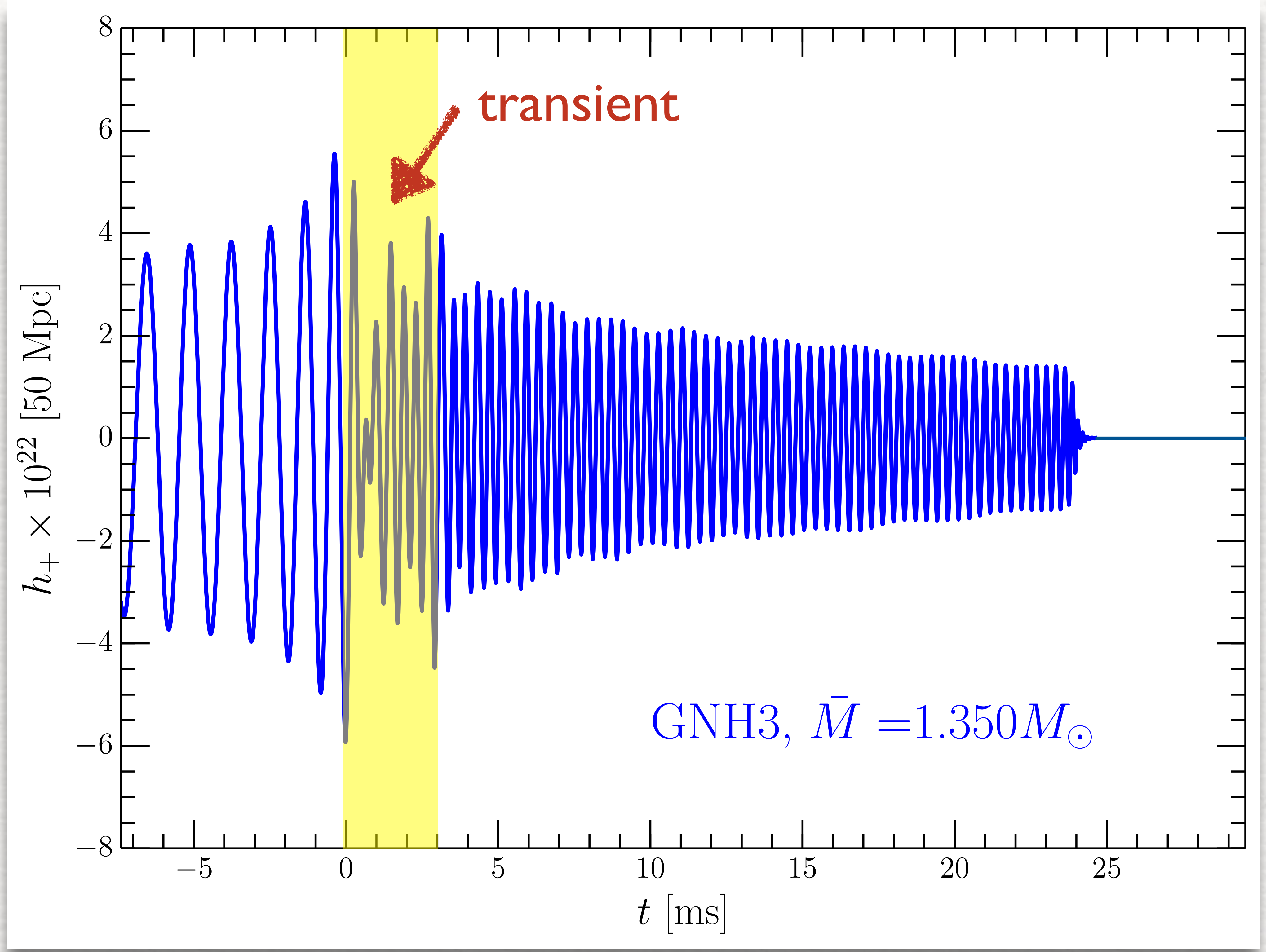


Chirp signal

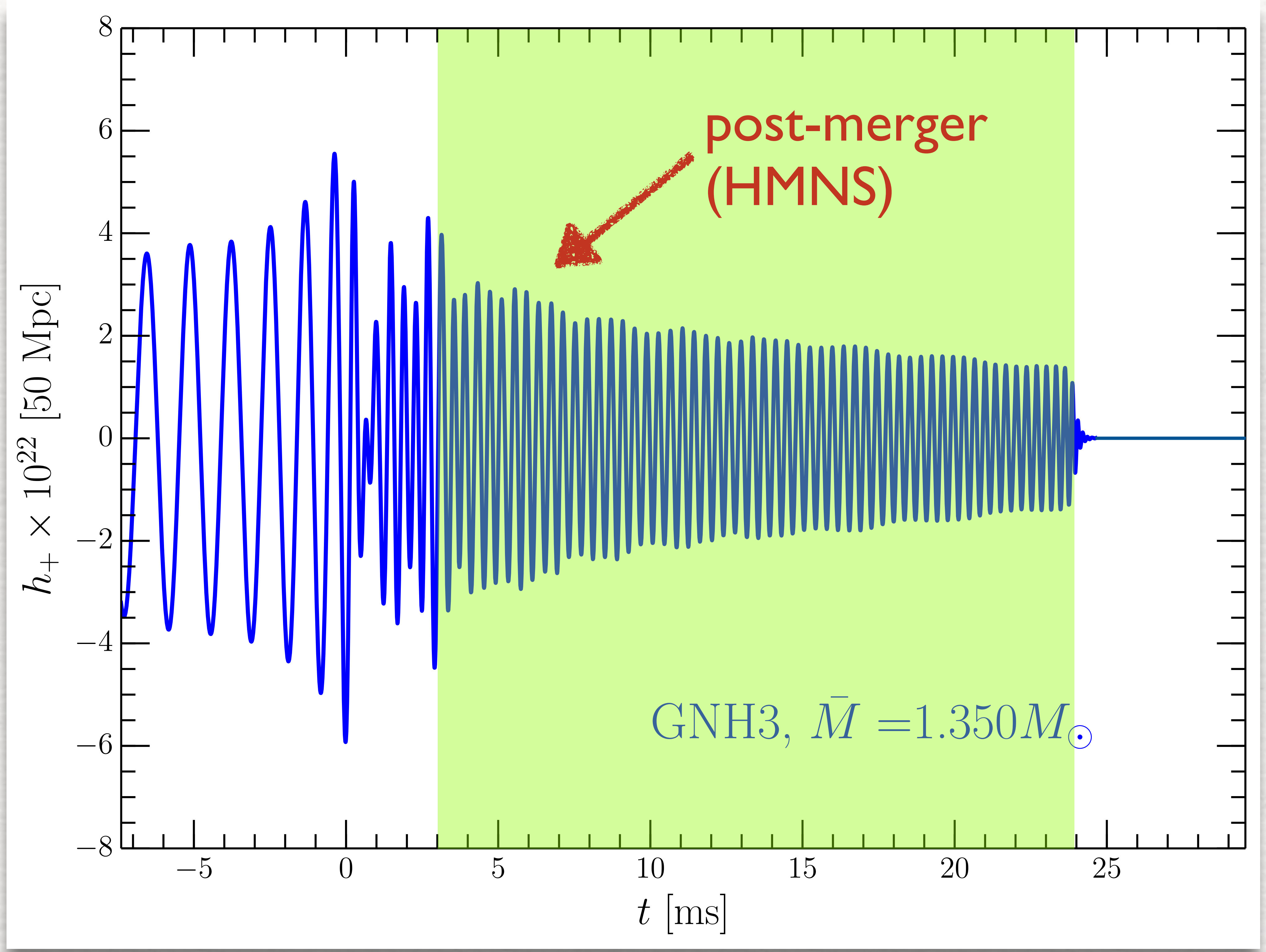
Inspiral: well approximated by PN/EOB; tidal effects important

Anatomy of the GW signal



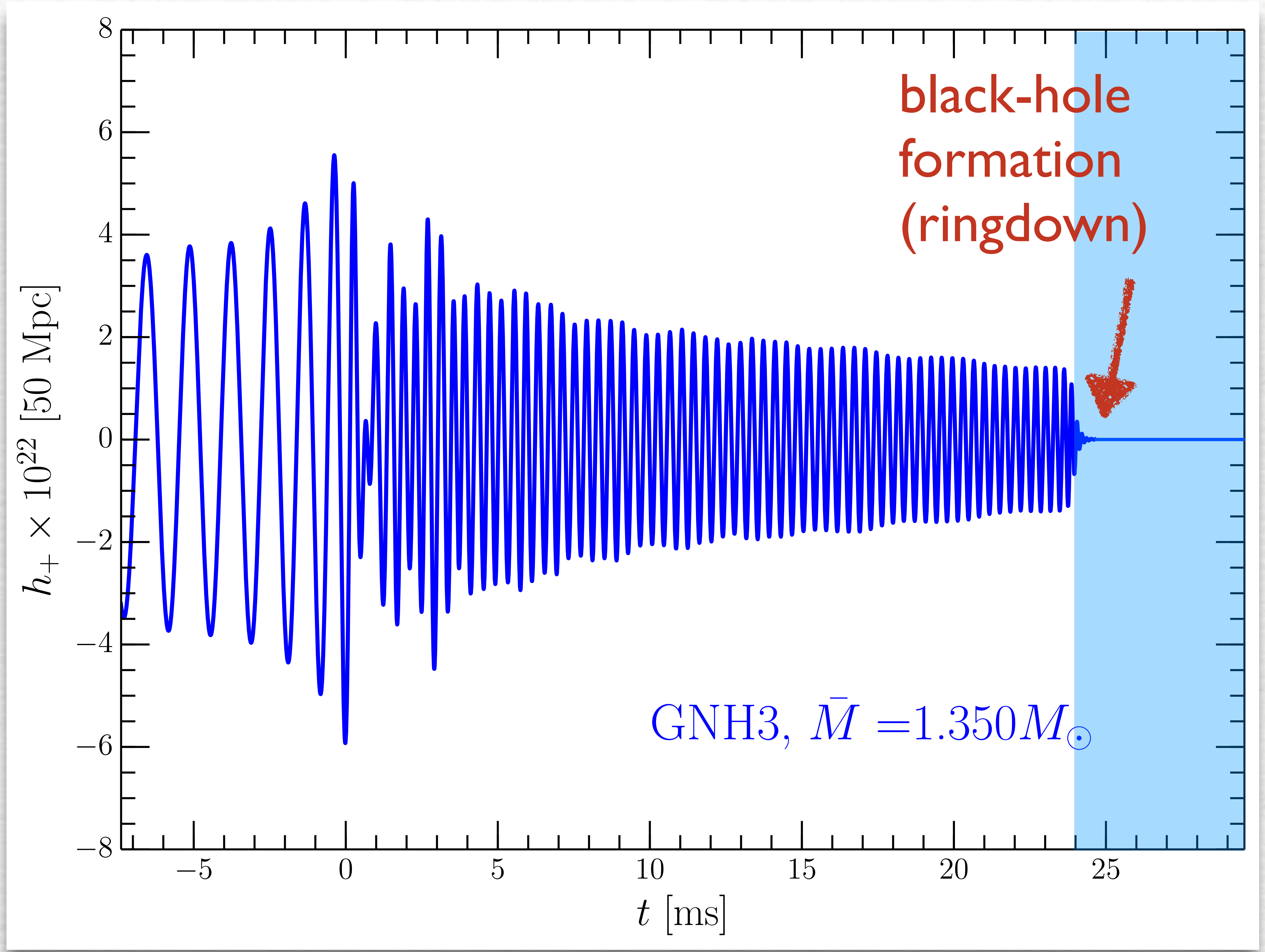
Merger: highly nonlinear but analytic description possible

Anatomy of the GW signal



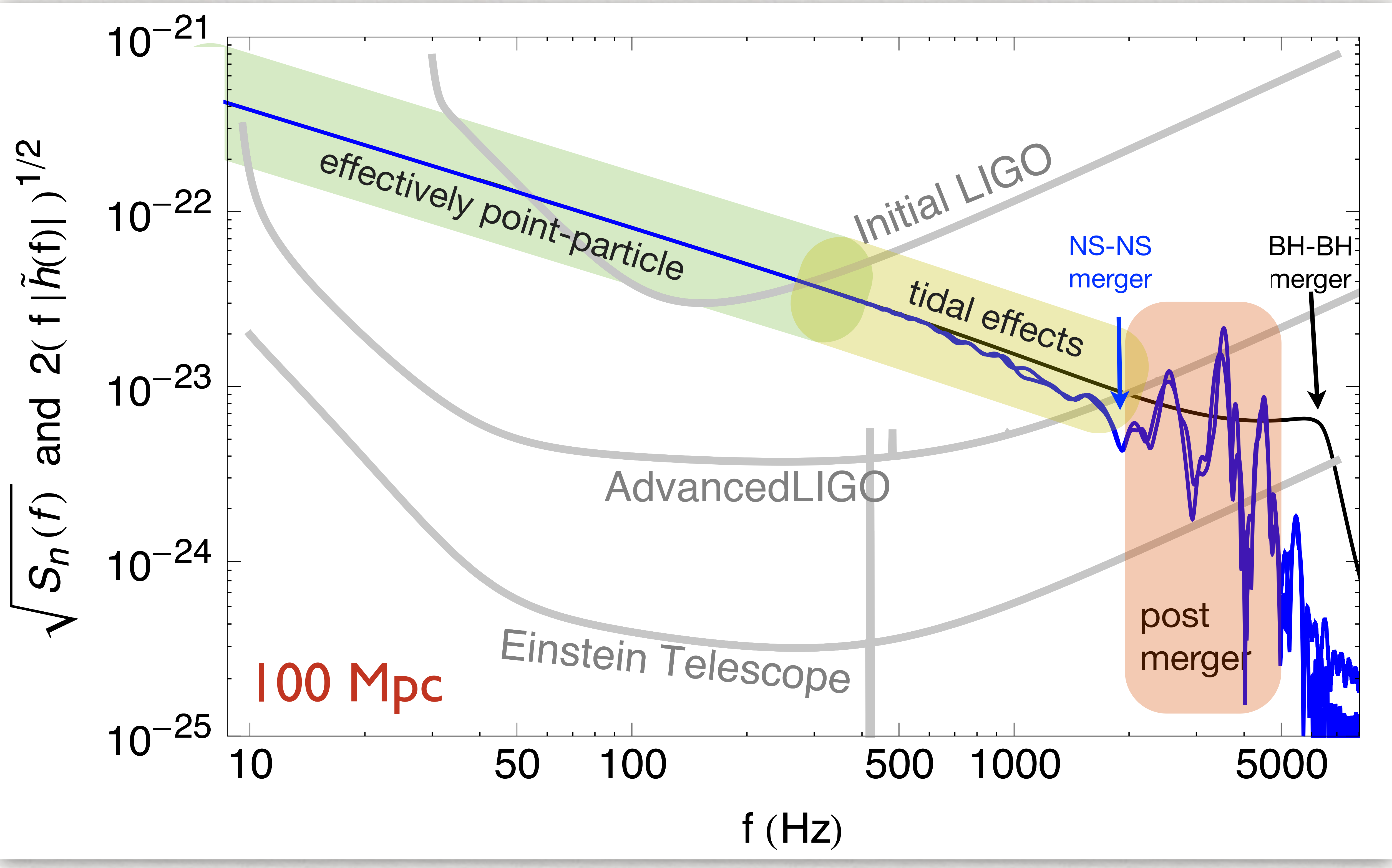
post-merger: quasi-periodic emission of bar-deformed HMNS

Anatomy of the GW signal



Collapse-ringdown: signal essentially shuts off.

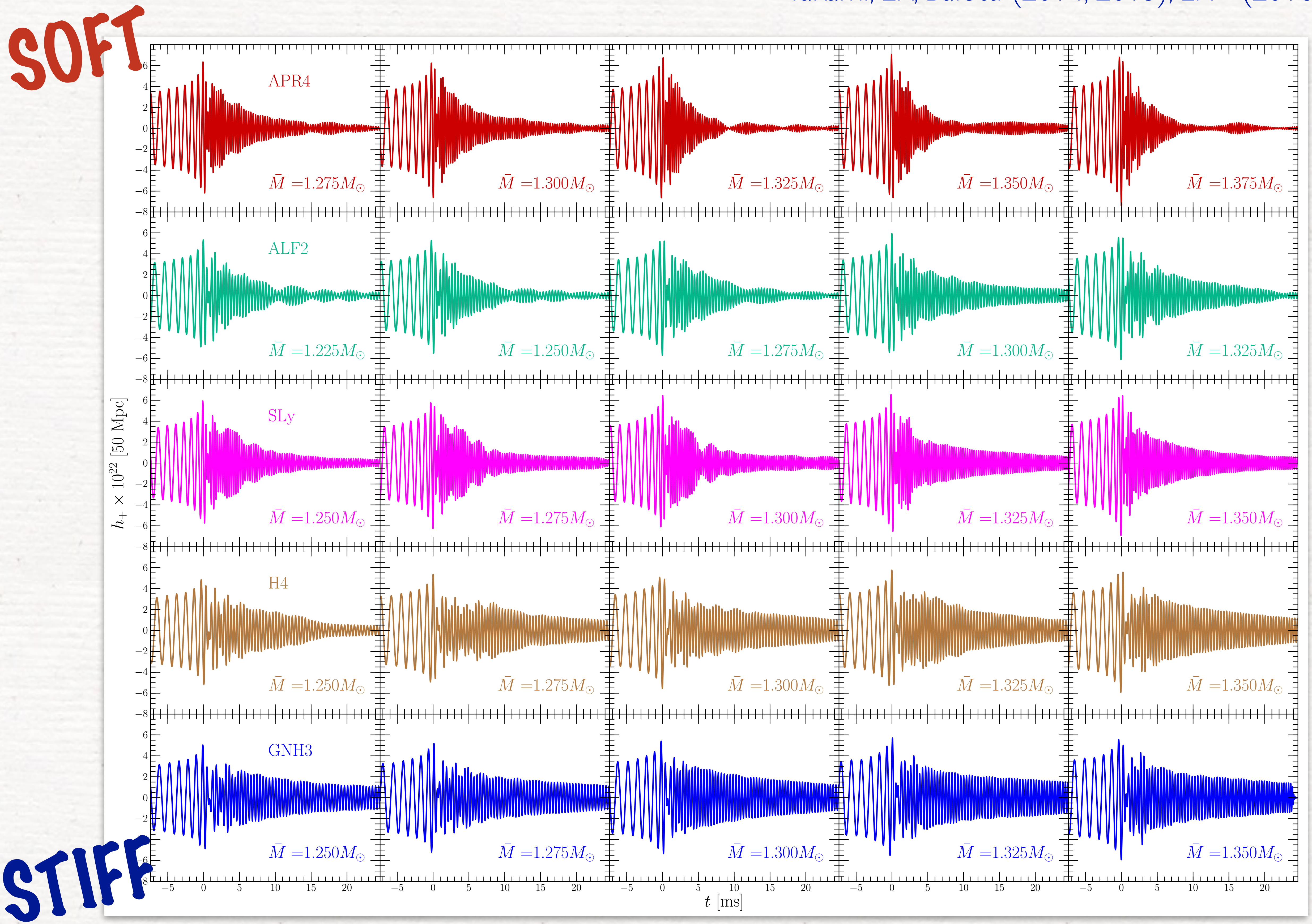
In frequency space



Read et al. (2013)

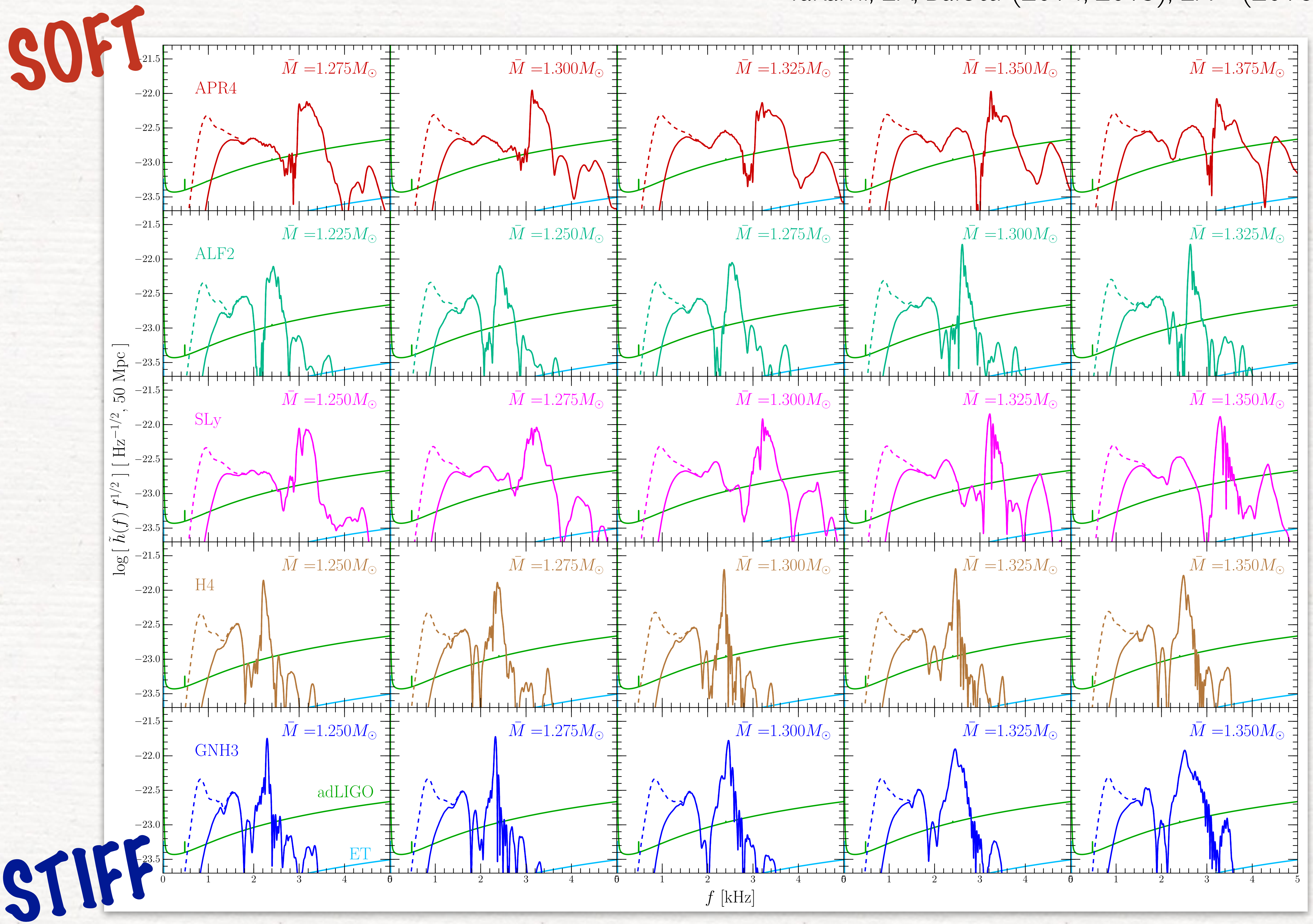
What we can do nowadays

Takami, LR, Baiotti (2014, 2015), LR+ (2016)



Extracting information from the EOS

Takami, LR, Baiotti (2014, 2015), LR+ (2016)



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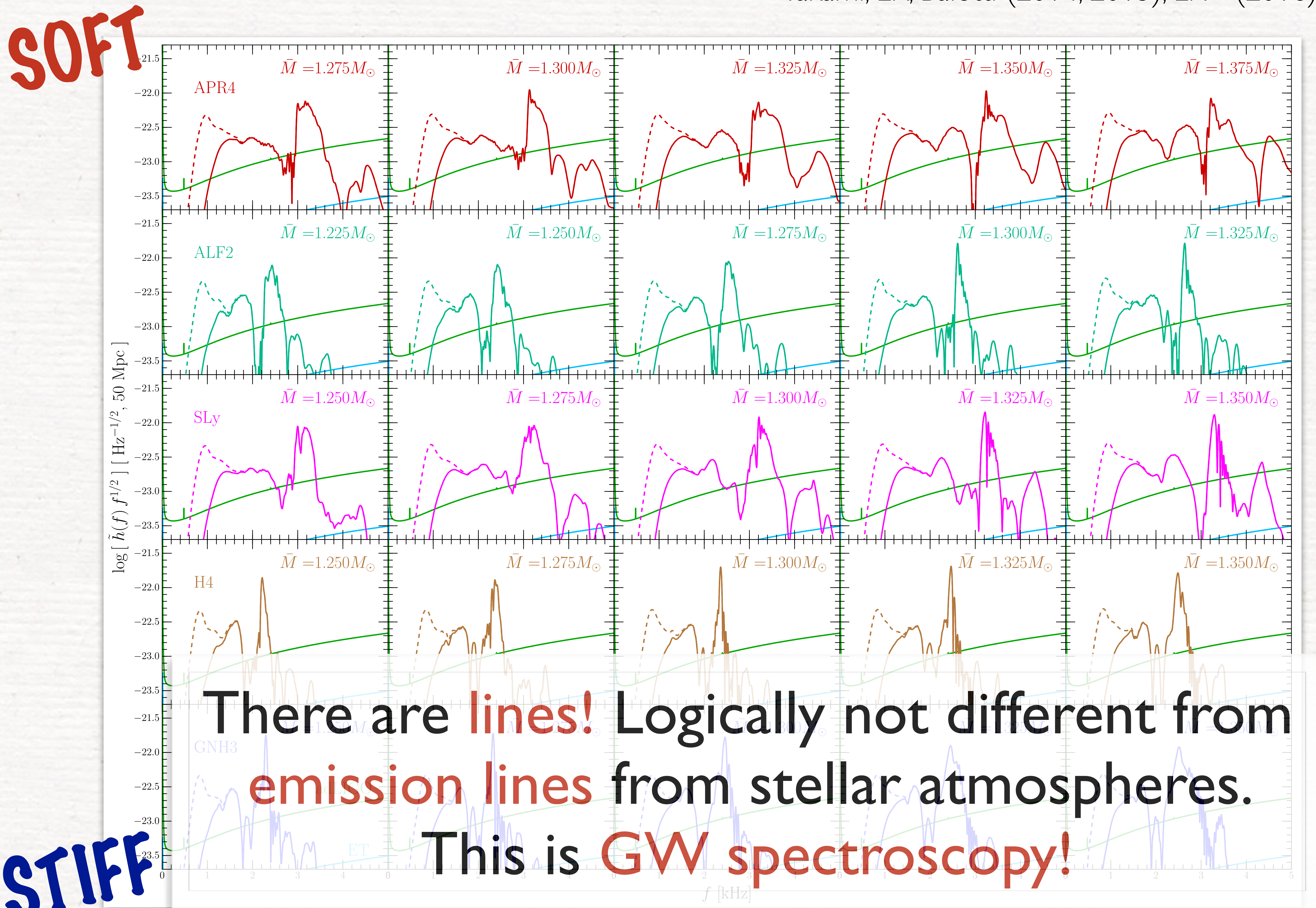
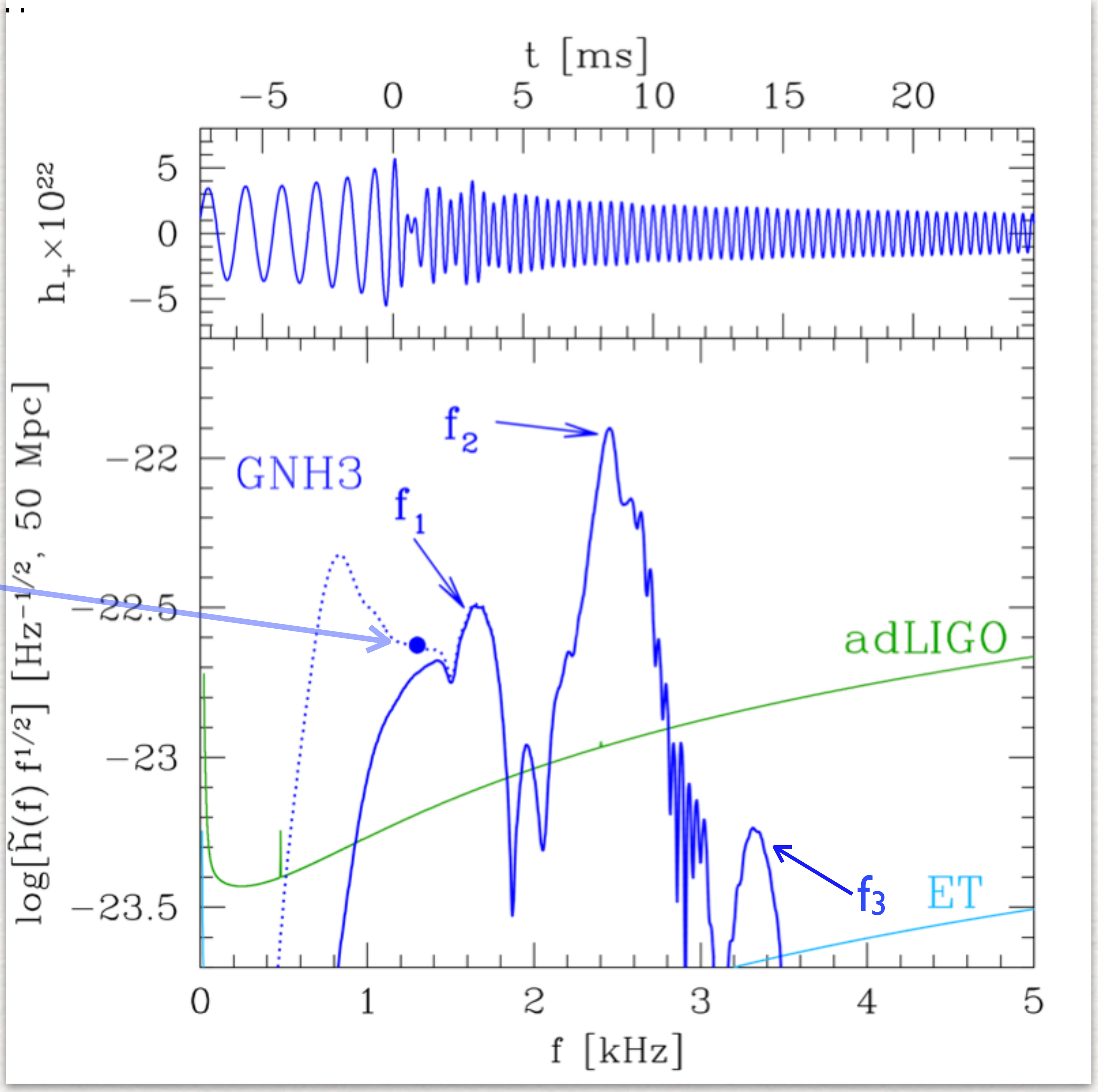


There are lines! Logically not different from emission lines from stellar atmospheres. This is GW spectroscopy!

A spectroscopic approach to the EOS

Oechslin+2007, Baiotti+2008, Bauswein+ 2011, 2012, Stergioulas+ 2011, Hotokezaka+ 2013, Takami 2014, 2015, Bernuzzi 2014, 2015, Bauswein+ 2015, Clark+ 2016, LR+2016, de Pietri+ 2016, Feo+ 2017, Bose+ 2017 …

merger frequency



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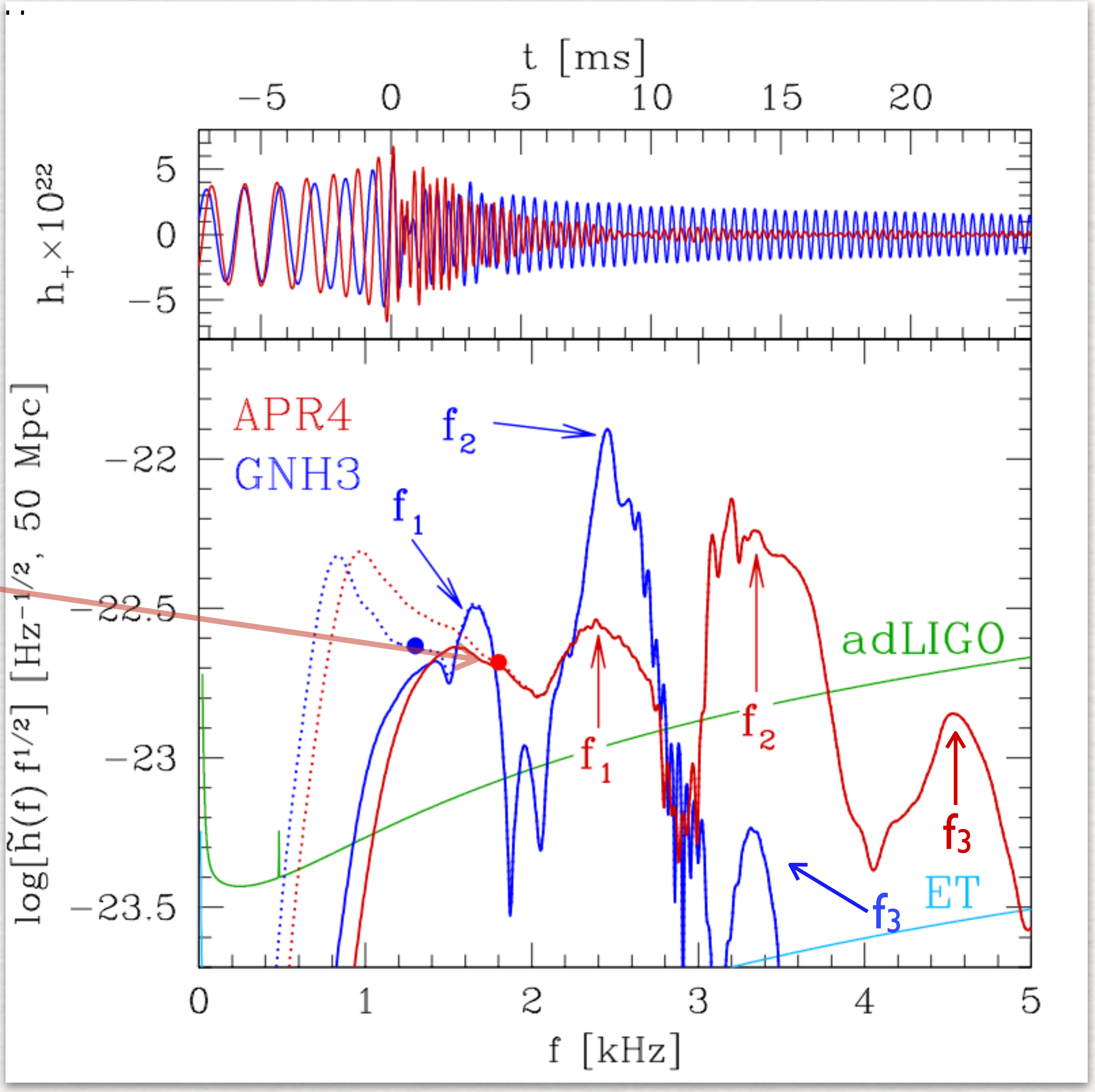
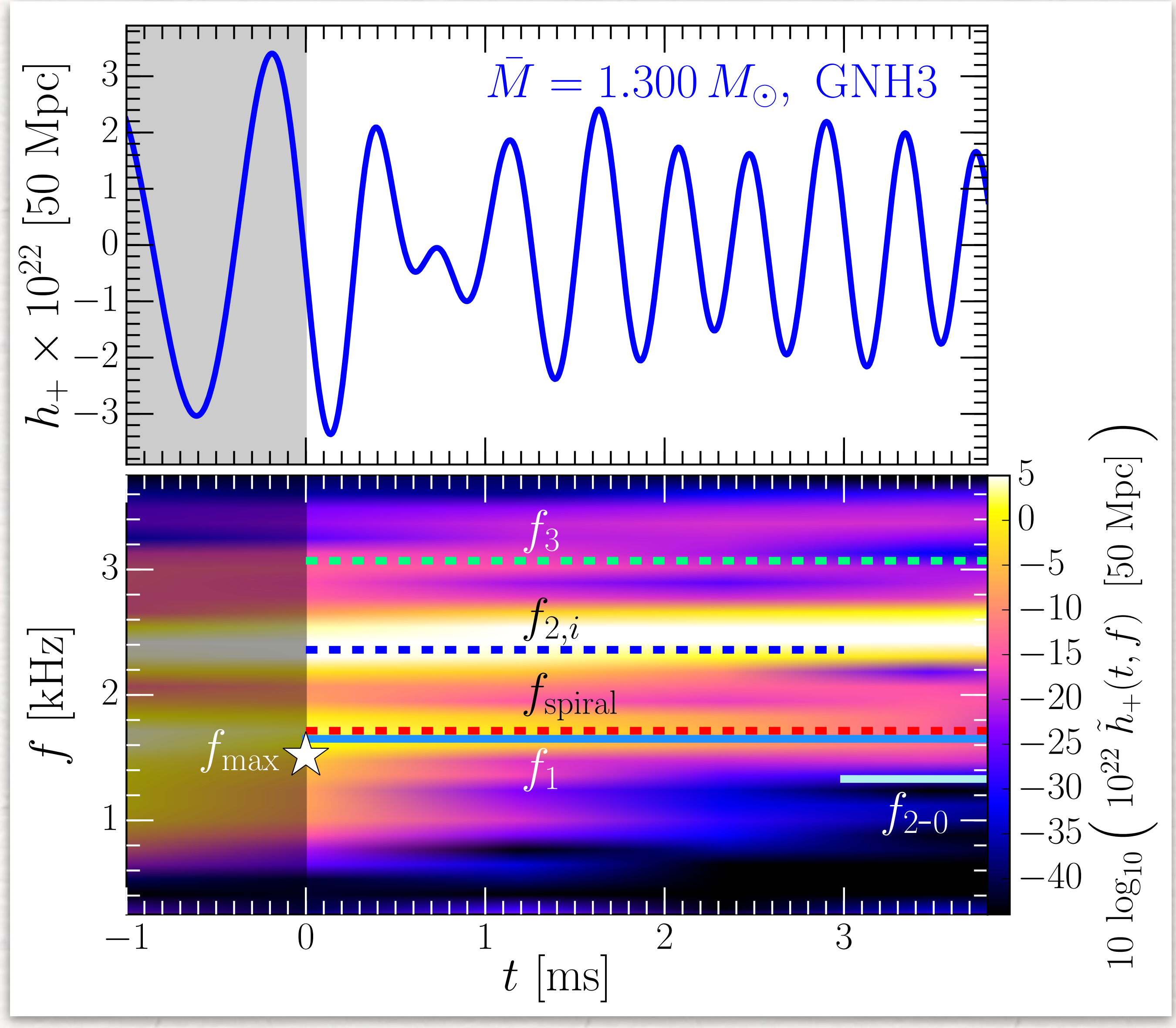
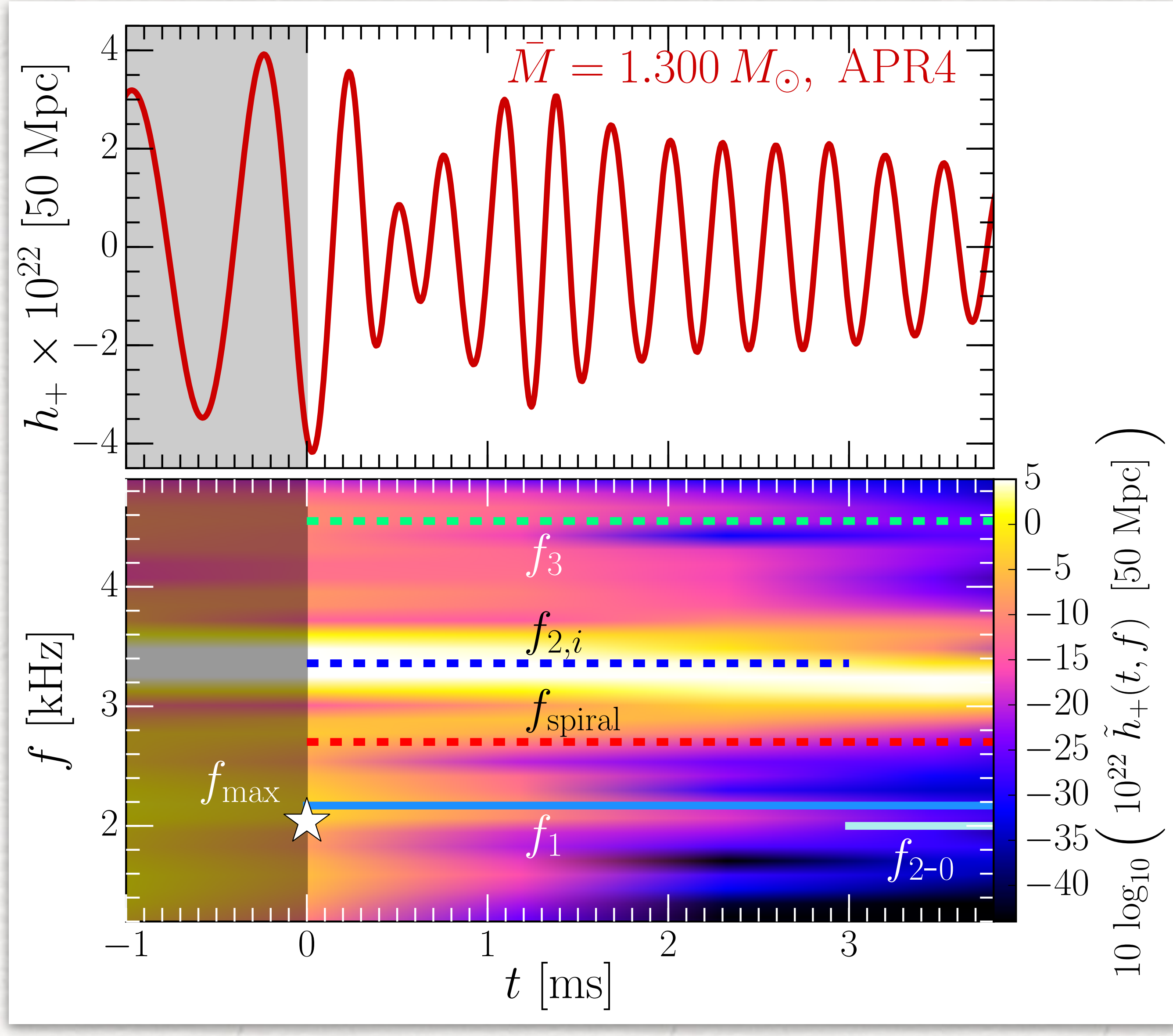


Understanding mode evolution

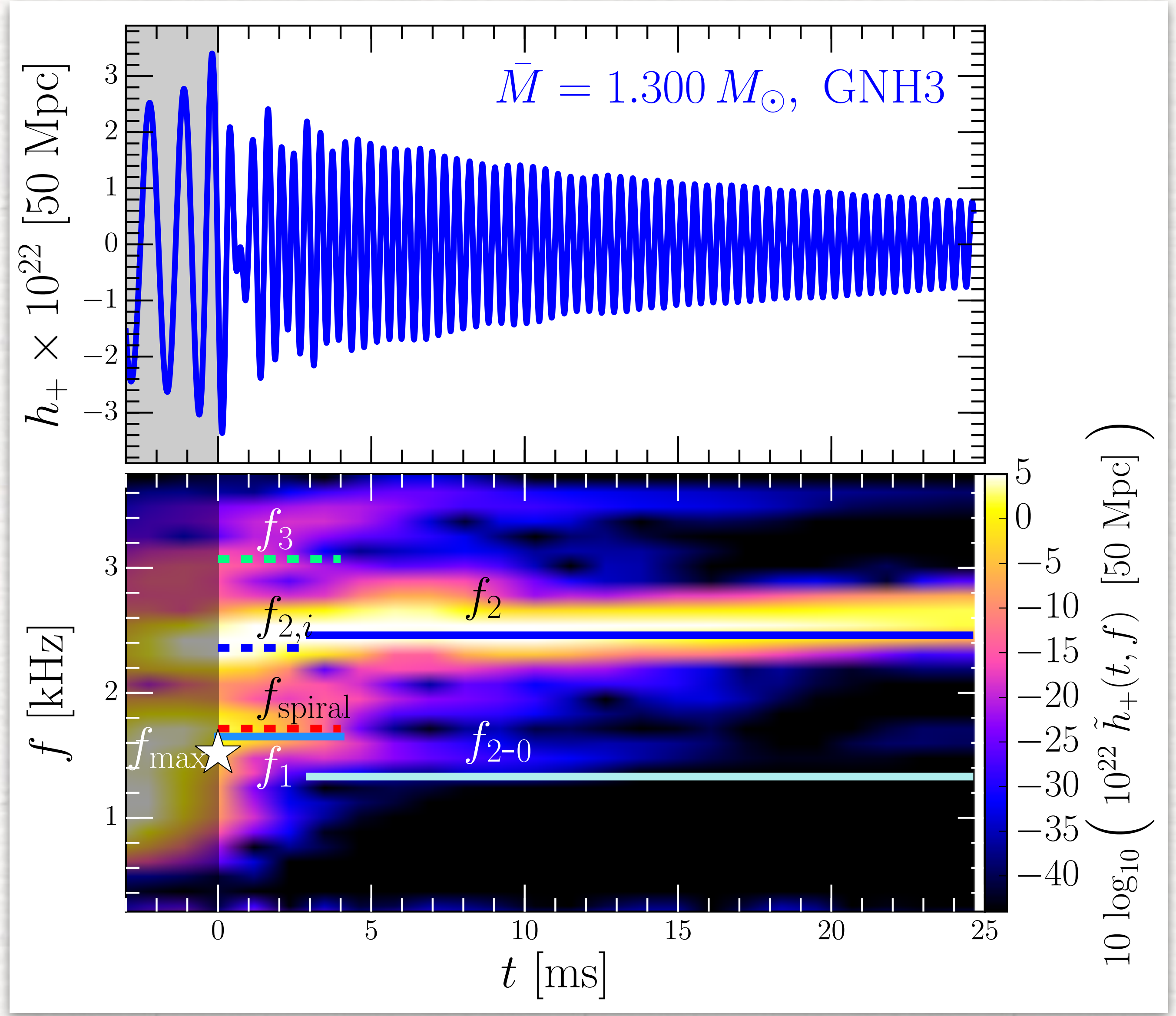
On a **short** timescale after the merger, it is possible to see the emergence of f_1, f_2, and f_3.

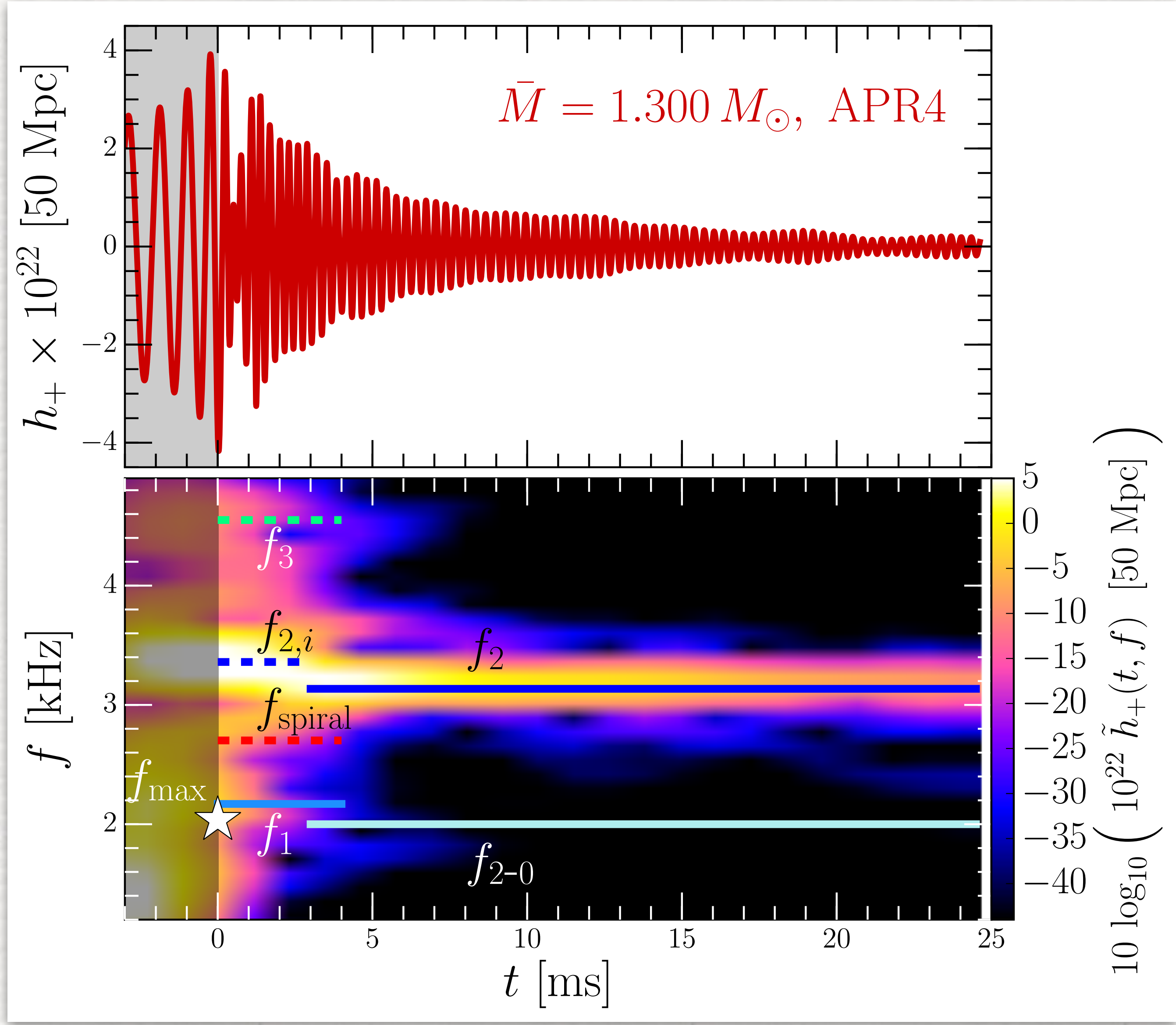




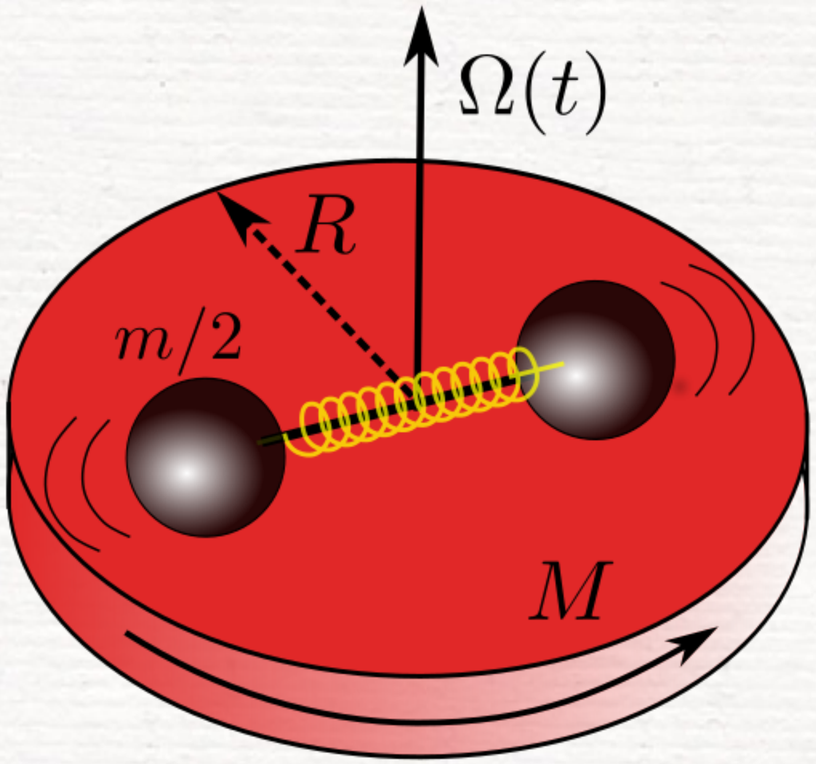
Understanding mode evolution

On a **long** timescale after the merger, only **f_2** survives.

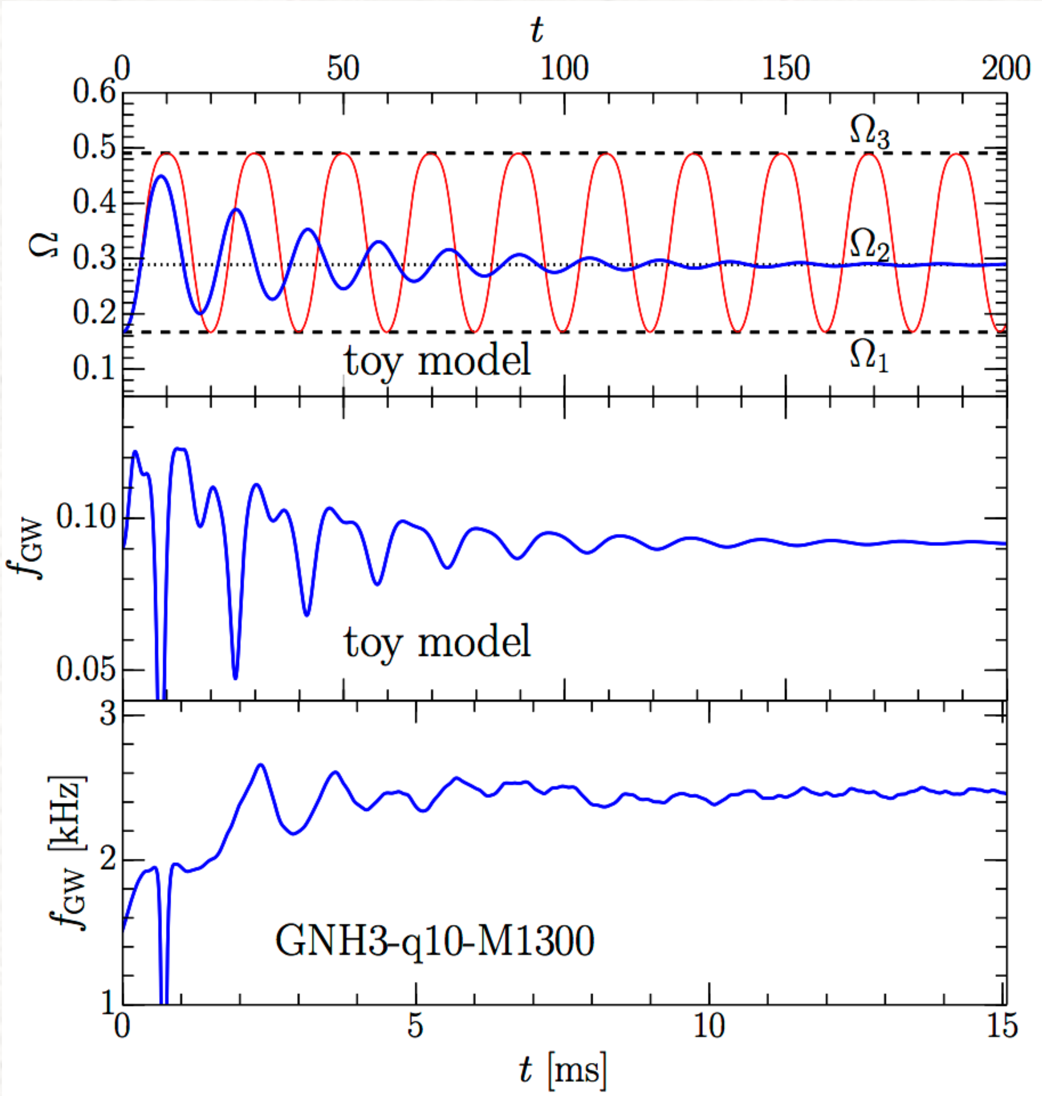




A mechanical toy model for the f_1, f_3 peaks

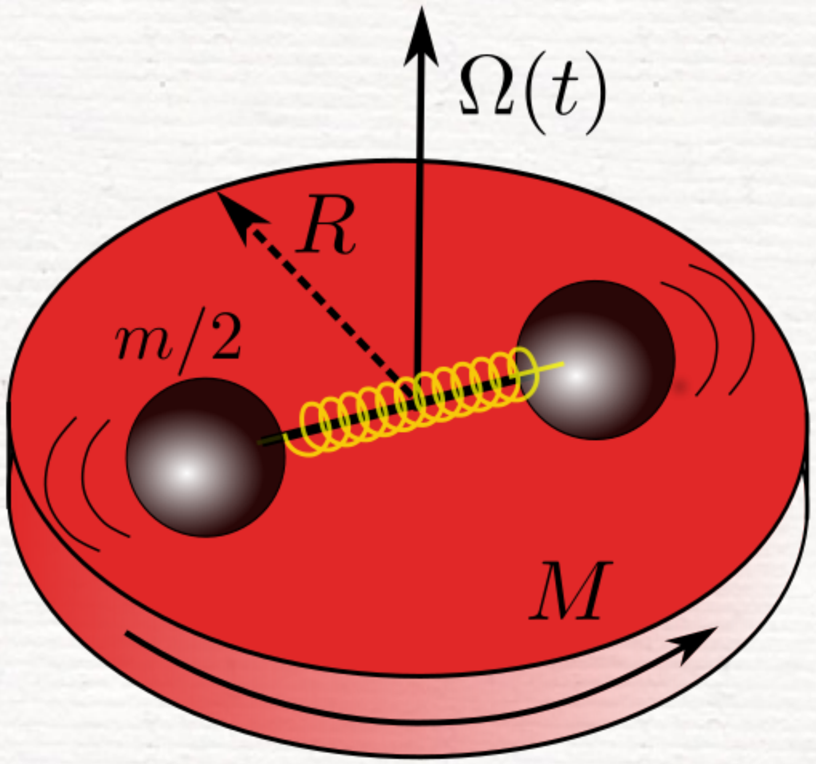


- Consider disk with 2 masses moving along a shaft and connected via a spring ~ HMNS with 2 stellar cores
- Let disk rotate and mass oscillate while conserving angular momentum



- If there is no friction, system will spin between: low freq (**f_1**, masses are far apart) and high (**f_3**, masses are close).
- If friction is present, system will spin asymptotically at **$f_2 \sim (f_1+f_3)/2$**.
- analytic model possible of post merger.

A mechanical toy model for the **f_1, f_3** peaks



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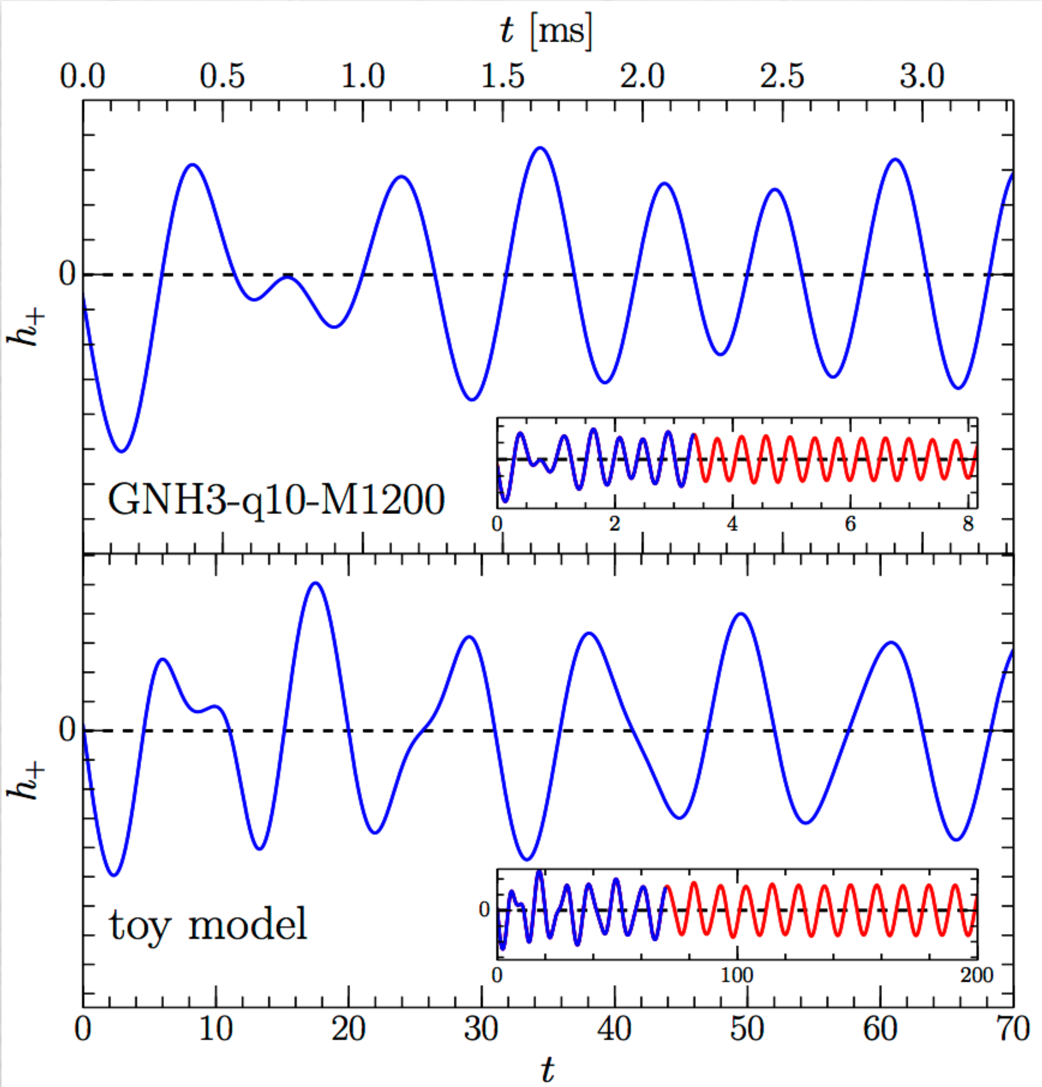

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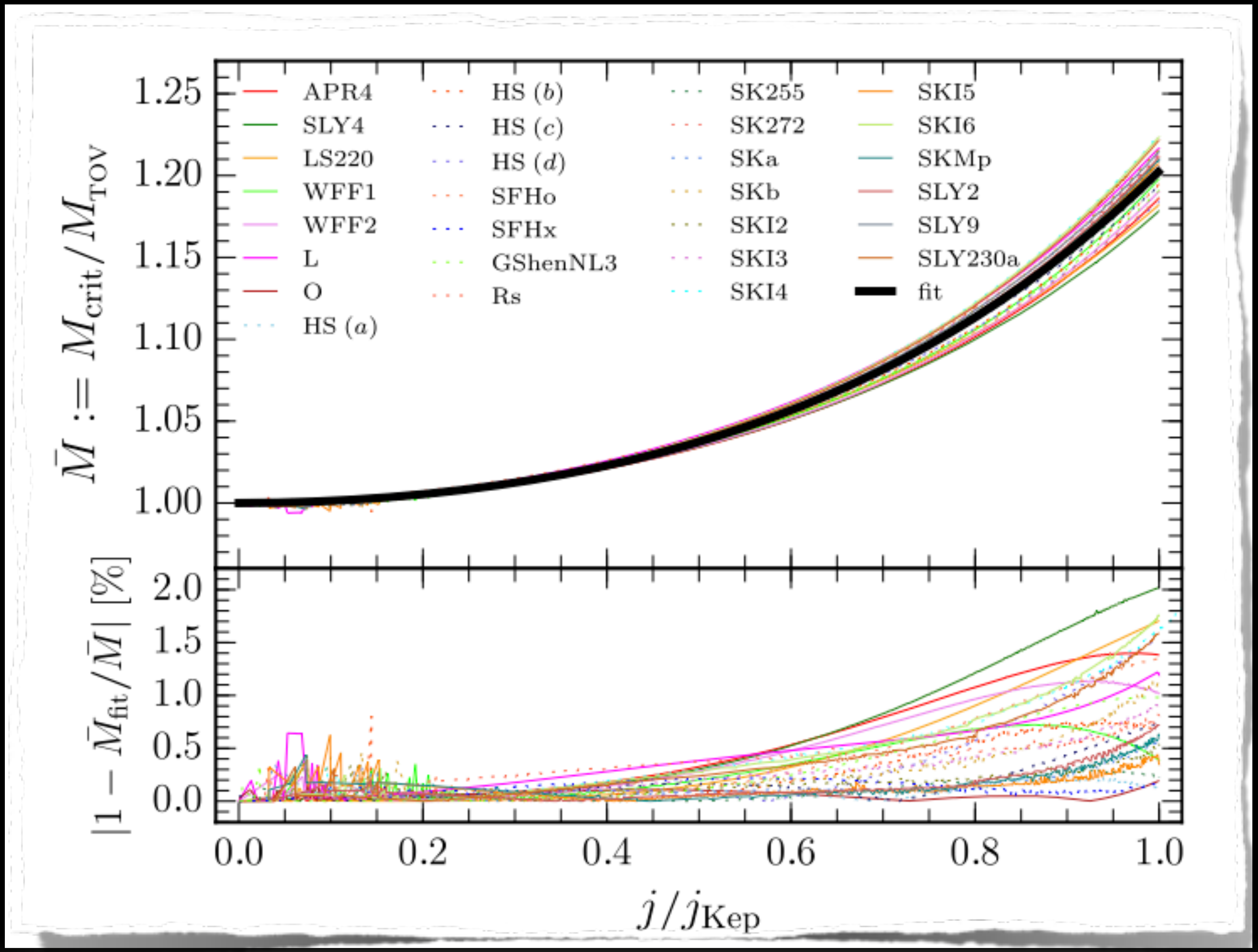
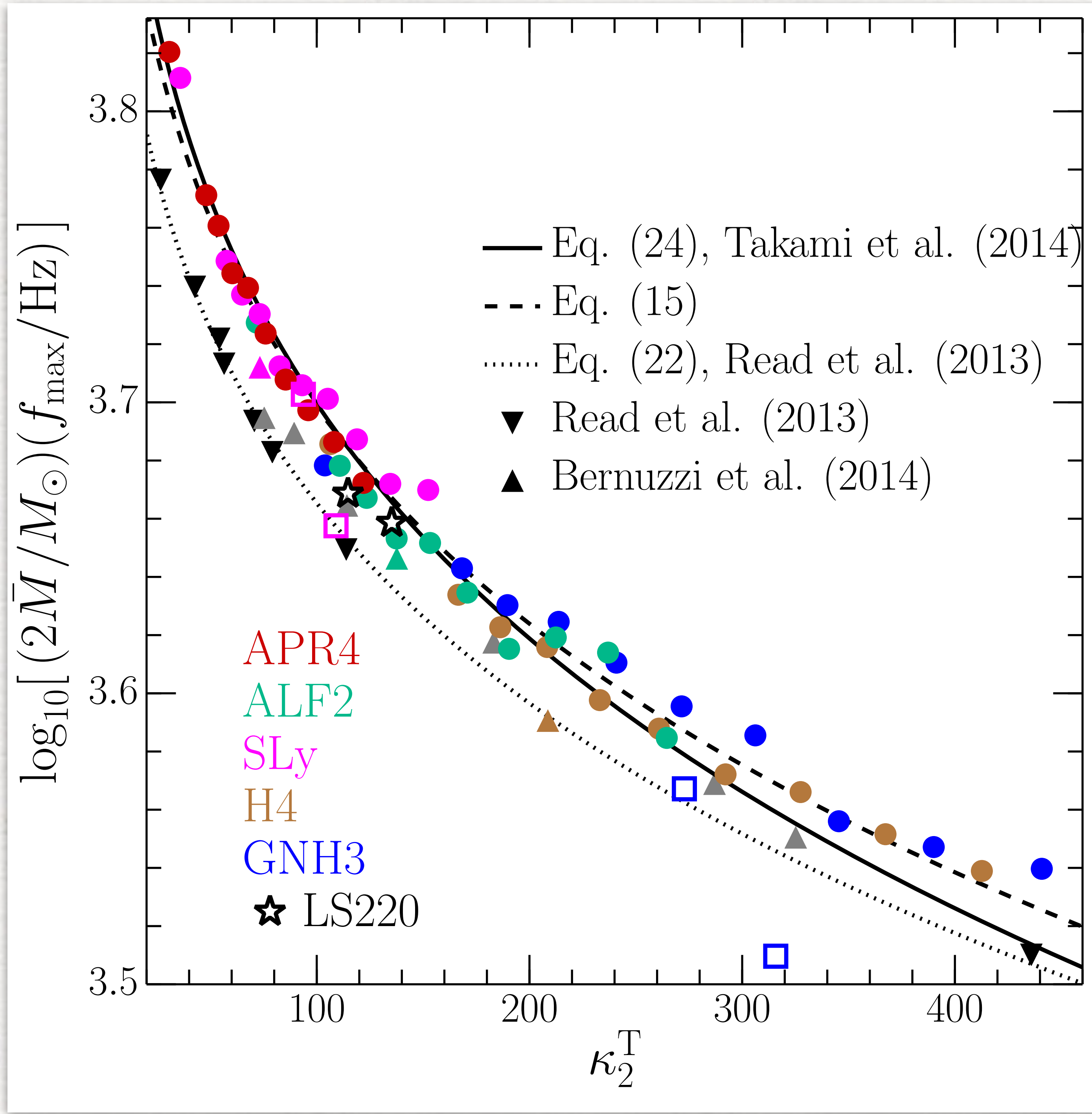
Quasi-universal behaviour



Quasi-universal behaviour: **inspiral**



"surprising" result: quasi-universal behaviour of GW frequency at amplitude peak (Read+2013)

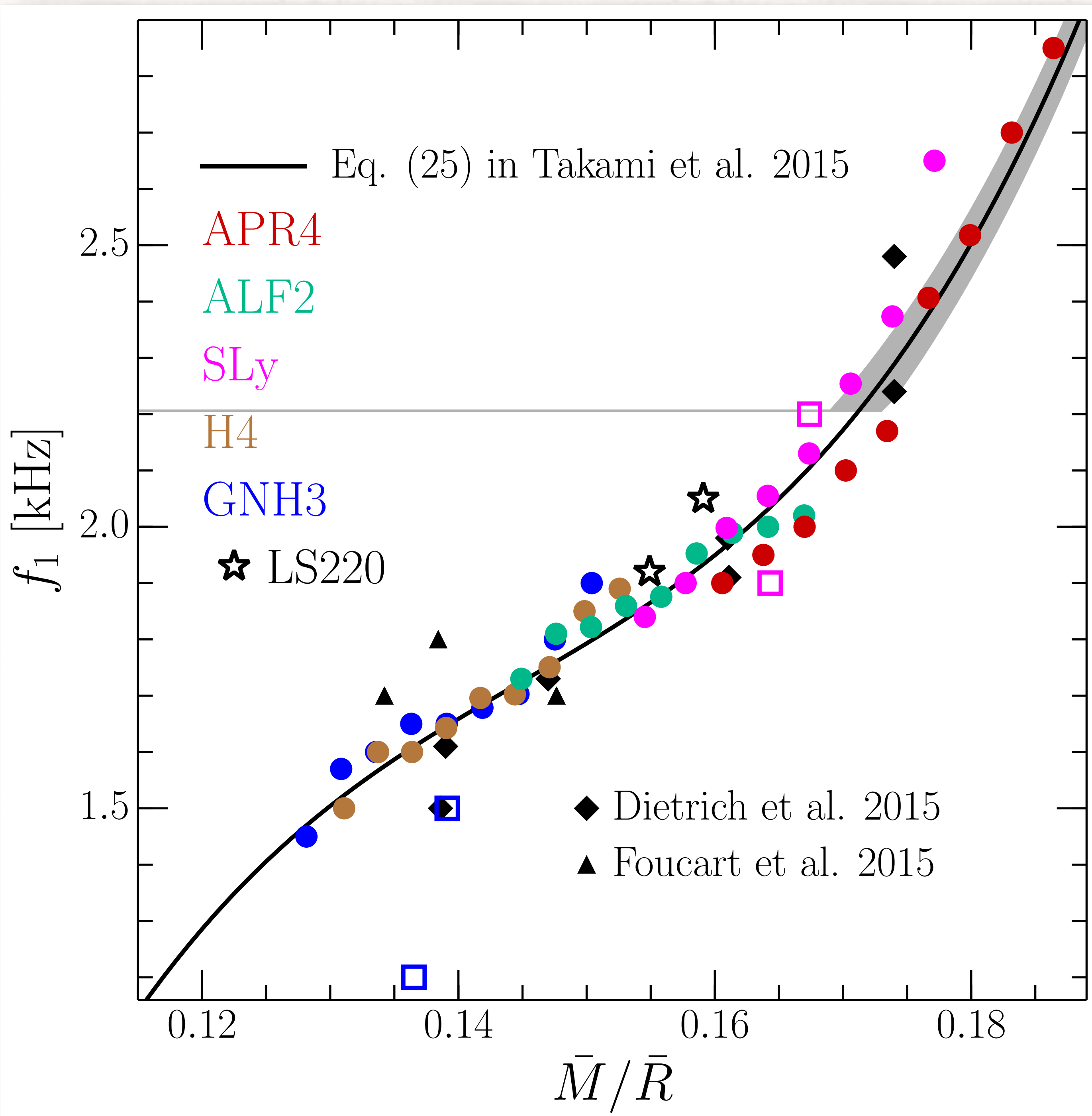
Many other simulations have confirmed this (Bernuzzi+ 2014, Takami+ 2015, LR+2016) .

Quasi-universal behaviour in the **inspiral** implies that once **f_{max}** is measured, so is tidal deformability, hence $I, \ Q, \ M/R$

$$\Lambda = \frac{\lambda}{\bar{M}^5} = \frac{16}{3}\kappa_2^T$$

tidal deformability or Love number

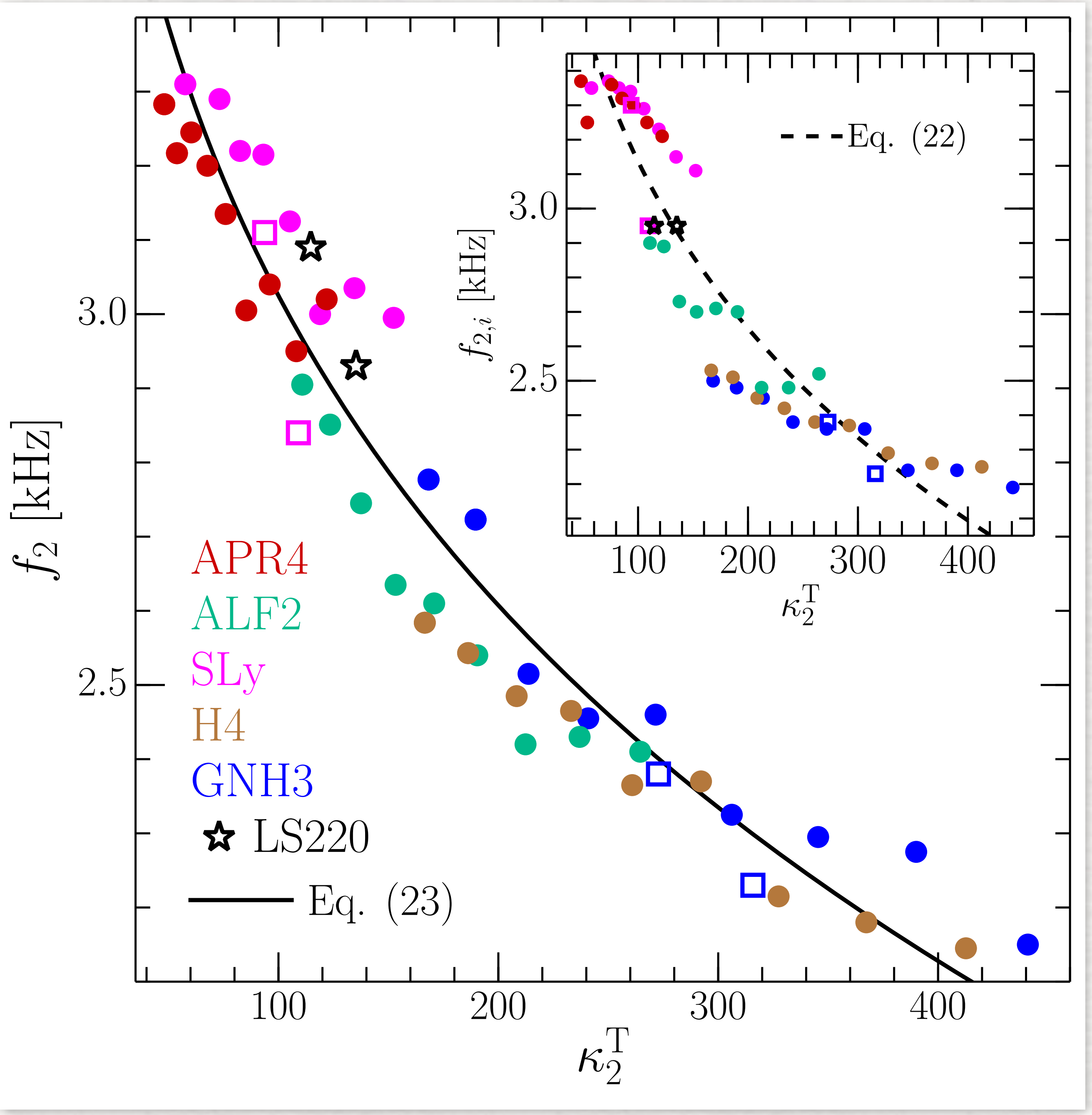
Quasi-universal behaviour: **post-merger**



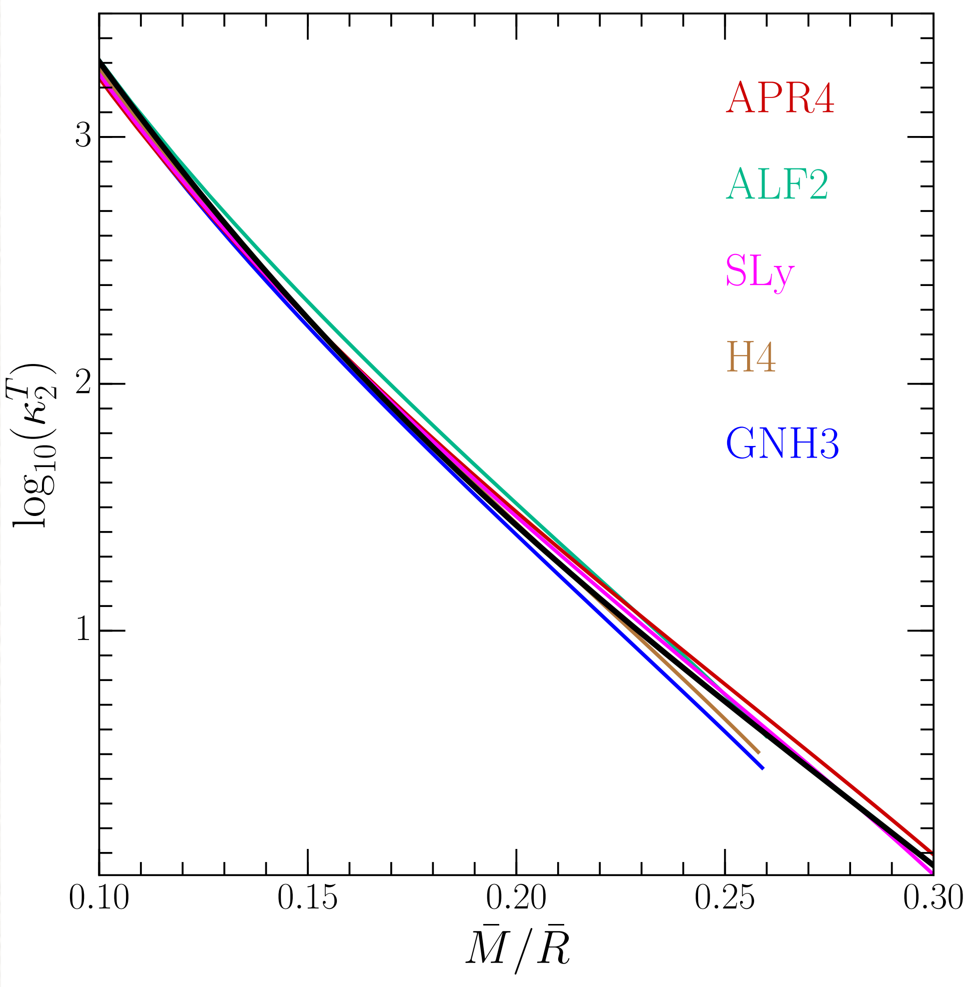
We have found **quasi-universal behaviour:** i.e., the properties of the spectra are only weakly dependent on the EOS.

This has profound implications for the analytical modelling of the GW emission: "what we do for one EOS can be extended to all EOSs."

Quasi-universal behaviour: **post-merger**



- Correlations with Love number found also for high frequency peak f_2.
- This and other correlations are **weaker** but equally useful.



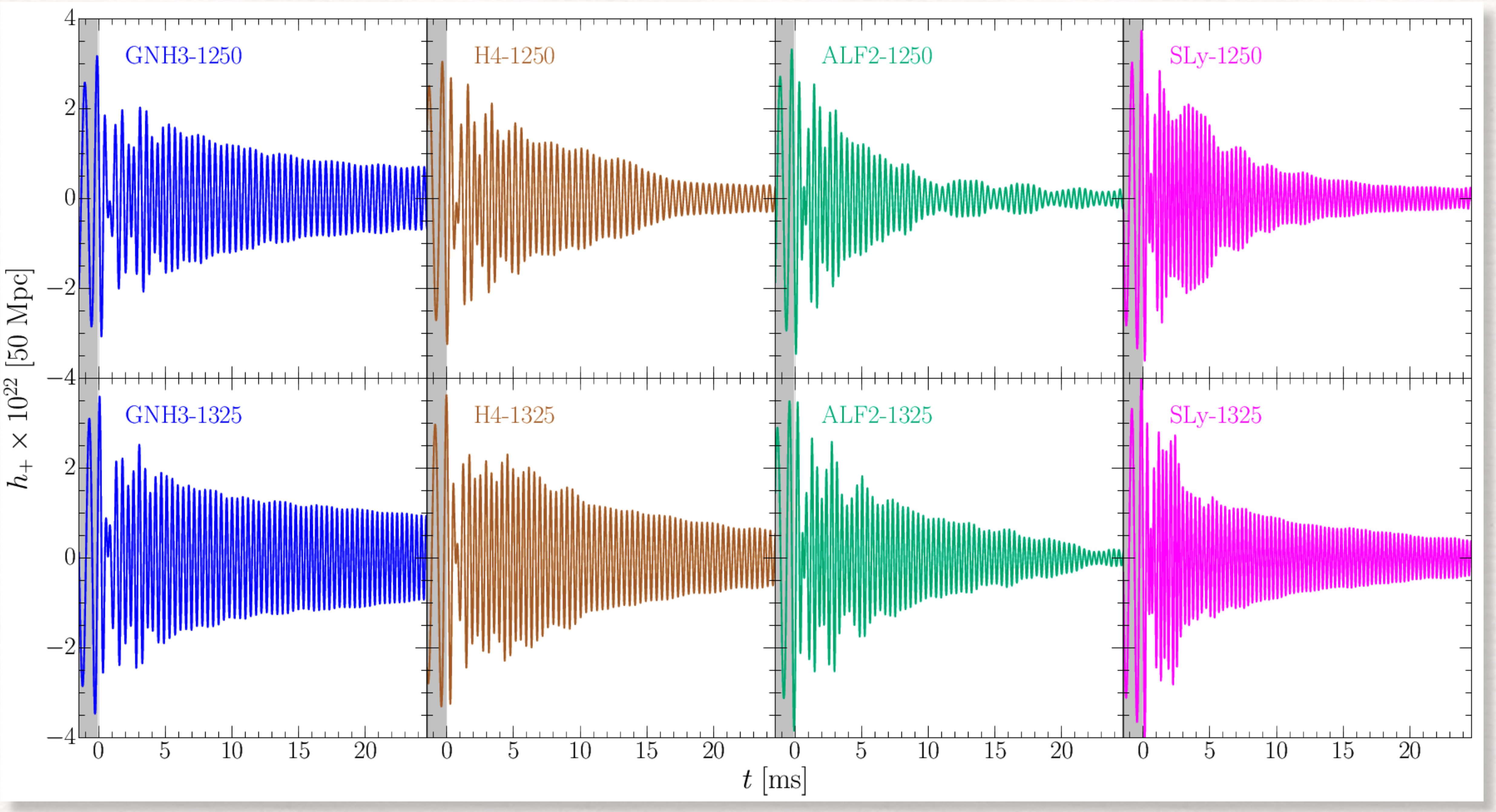
- Important correlation also between **compactness** and **deformability**

Radius estimate from binary population

Bose, Chakravarti, LR, Sathyaprakash, Takami (2017)

Analytical modelling of postmerger waveform

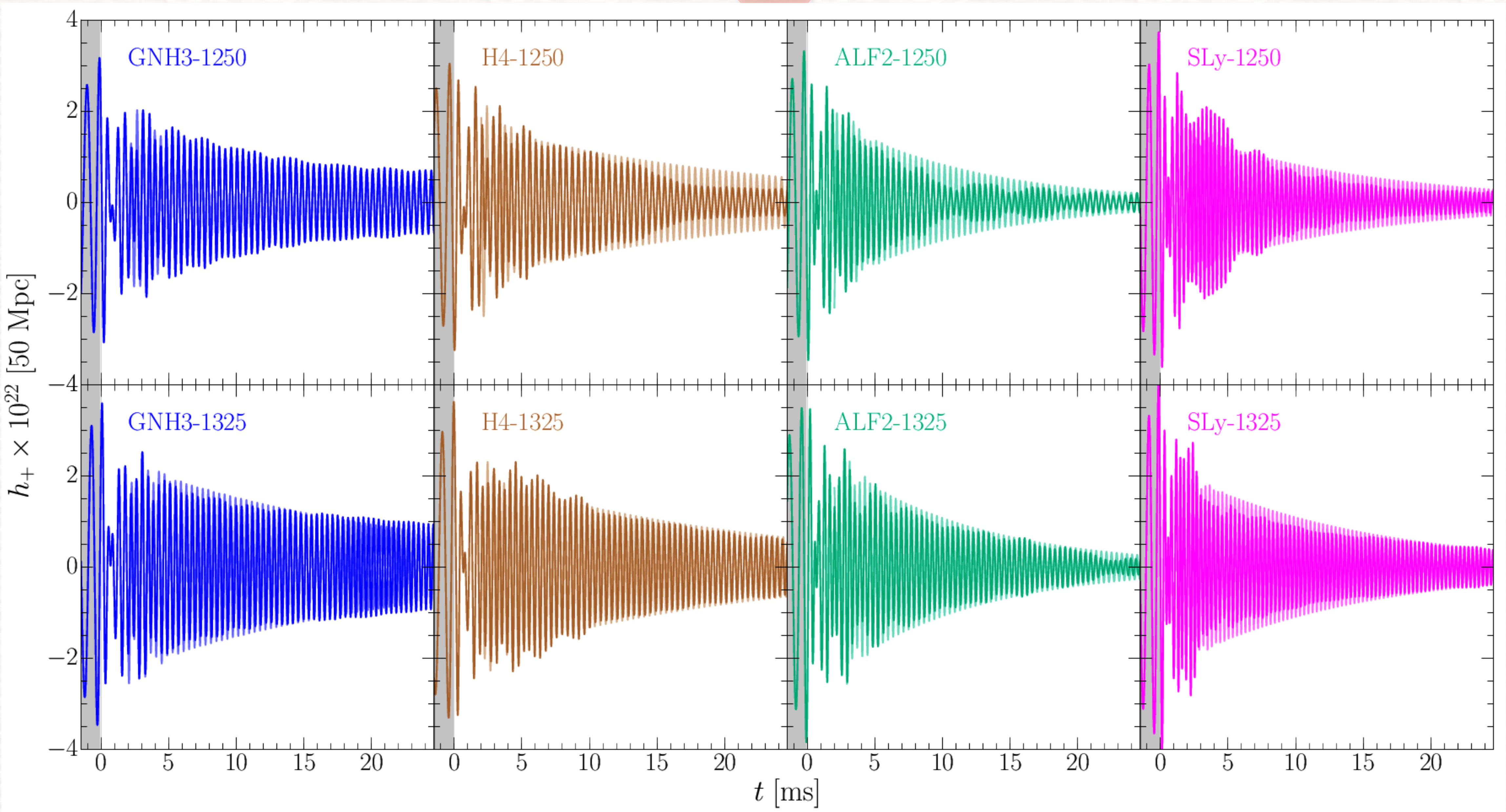
- **Postmerger** appears hopeless but isn't (Clark+14, 16; Bose+17)



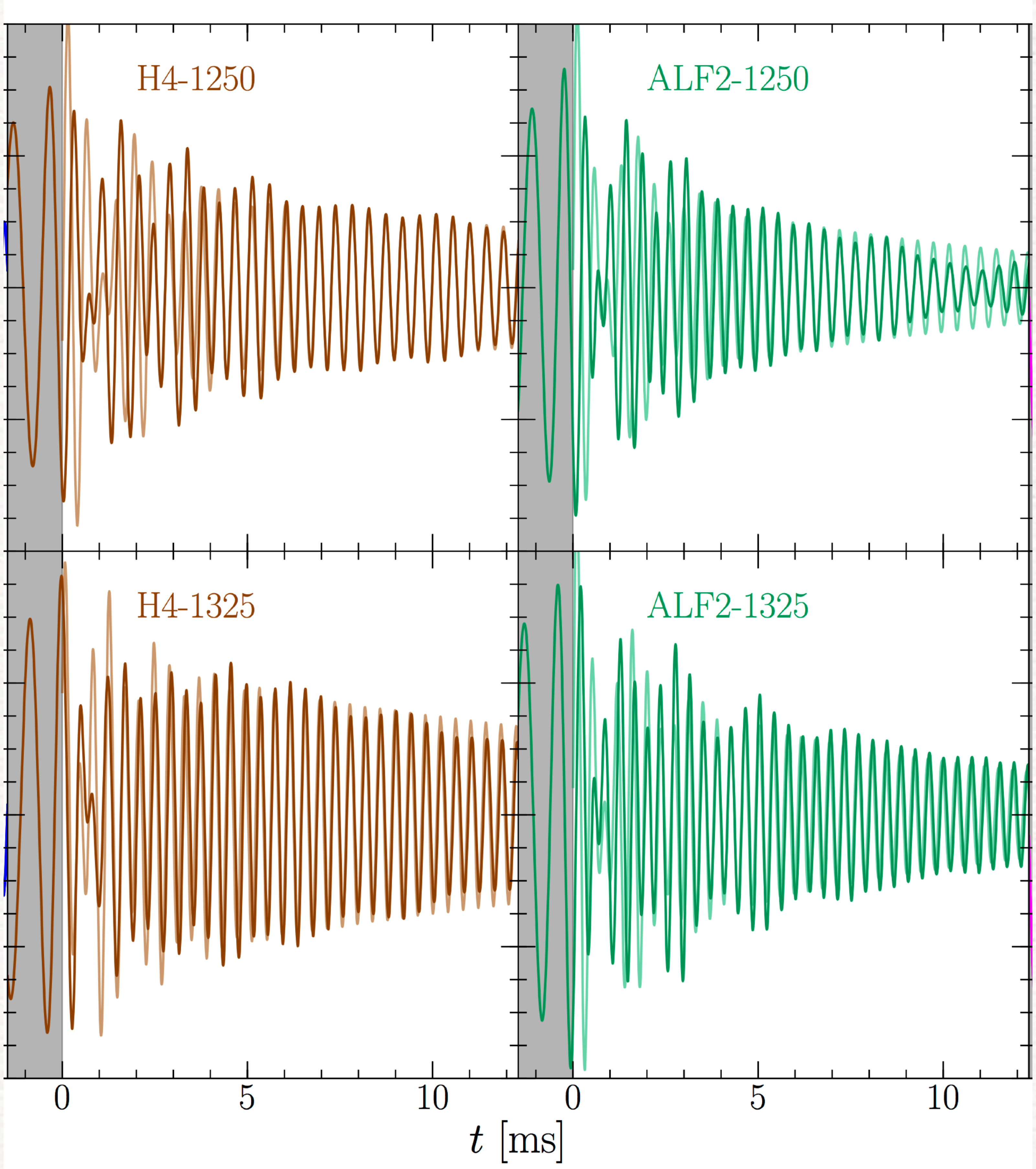
Analytical modelling of postmerger waveform

- Knowledge of spectral properties provides **analytic ansatz**

$$h(t) = \alpha \exp(-t/\tau_1)\left[\sin(2\pi f_1 t) + \sin(2\pi(f_1 - f_{1\epsilon})t) + \sin(2\pi(f_1 + f_{1\epsilon})t)\right] + \exp(-t/\tau_2)\sin(2\pi f_2 t + 2\pi\gamma_2 t^2 + \pi\beta_2)\,.$$

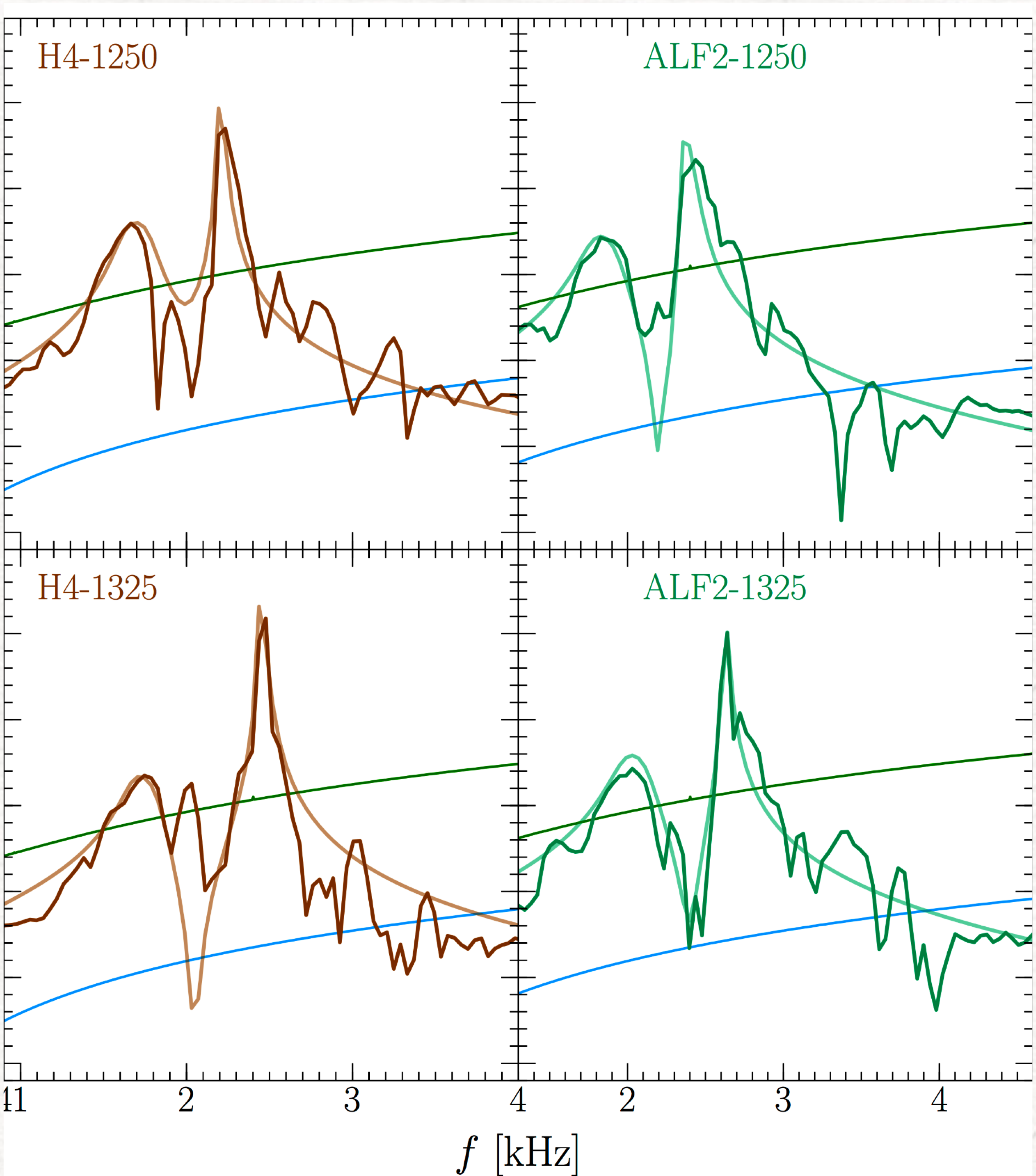


Analytical modelling of postmerger waveform



- Overall pretty decent fit in **phase**
- Fit in **amplitude** is less good but also less important

Analytical modelling of postmerger waveform



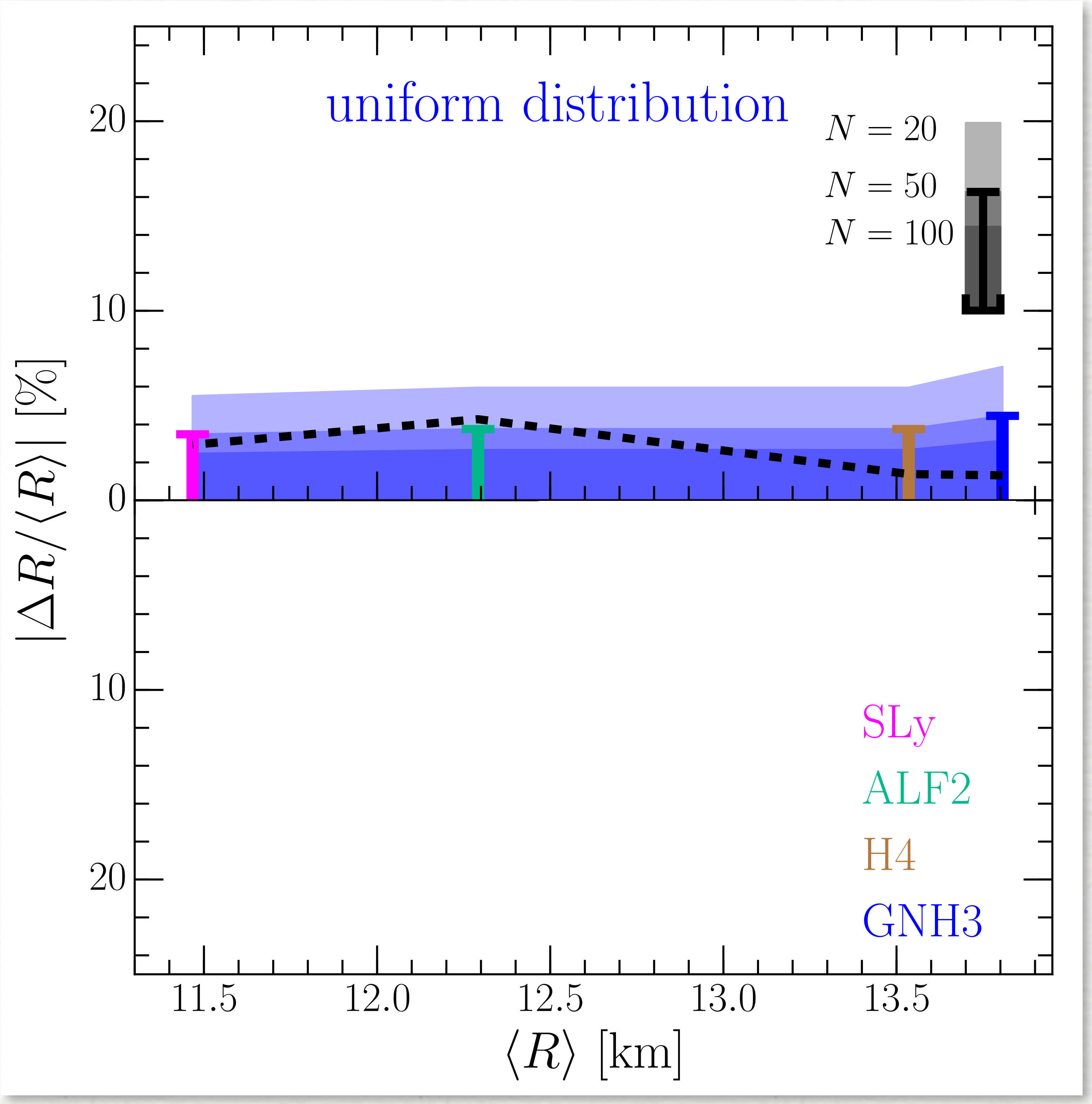
- Good match is clear also in **frequency space**

  In summary: despite the complex signal, an **analytic** description of the **full GW signal** is now possible.

Even a small SNR counts

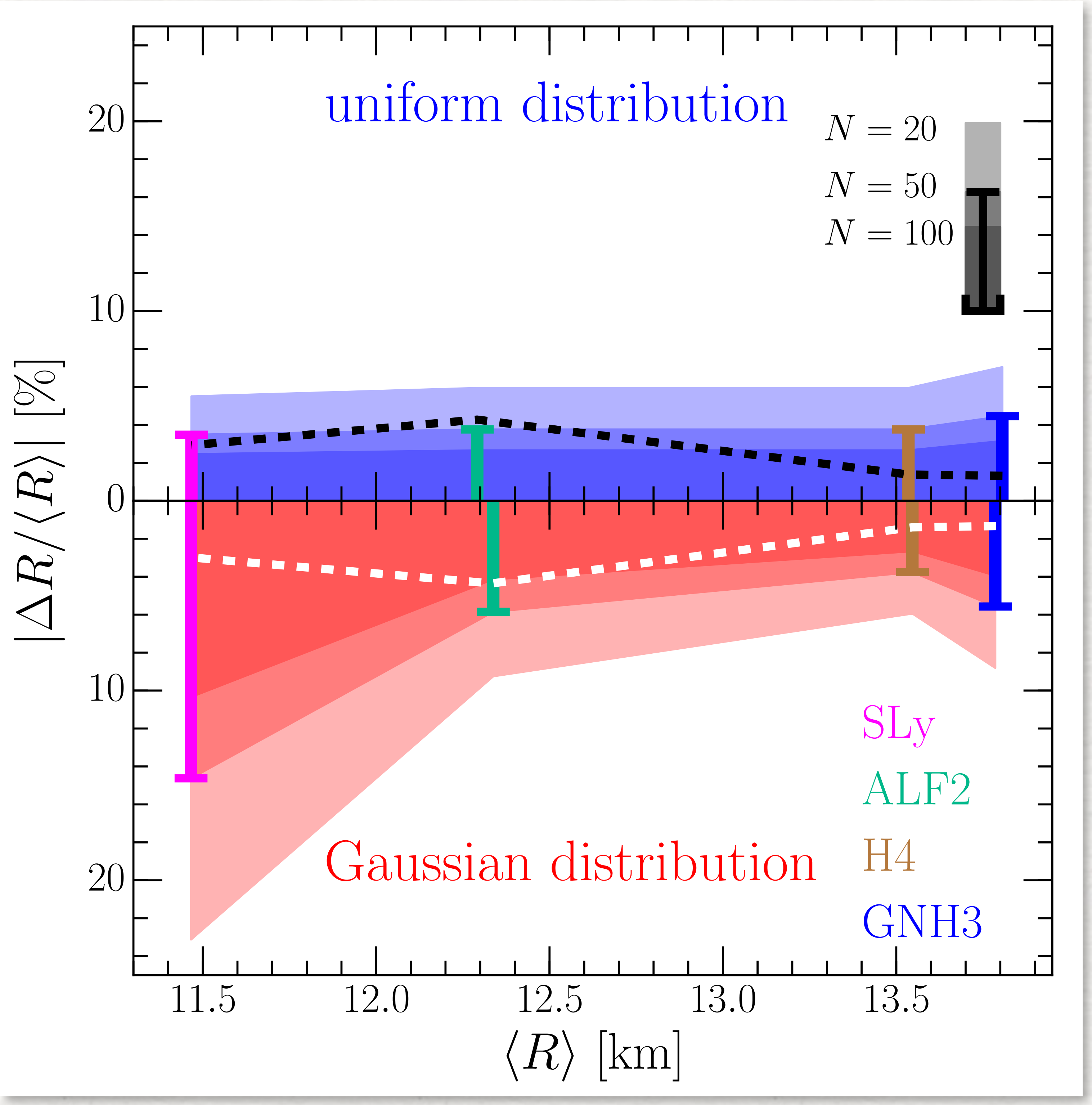
- Using **analytical modelling** performed **Fisher-matrix** analysis of GWs and **Monte-Carlo** simulation.
- Waveforms aligned at frequency, f_2^c. Standard frequency estimation yields value of f_2^c and statistical spread.
- **Quasi-universal relation** between f_2 and compactness, and error-propagation, to deduce the error in radius.
- Employed 100 BNS signals injected in 100 uncorrelated timeseries of Gaussian noise with aLIGO sensitivity.
- Used information on f_1 and chirp **mass** from **inspiral**.
- Repeated over 900 experiments to build statistics.

Constraining the radius: MonteCarlo vs Fisher



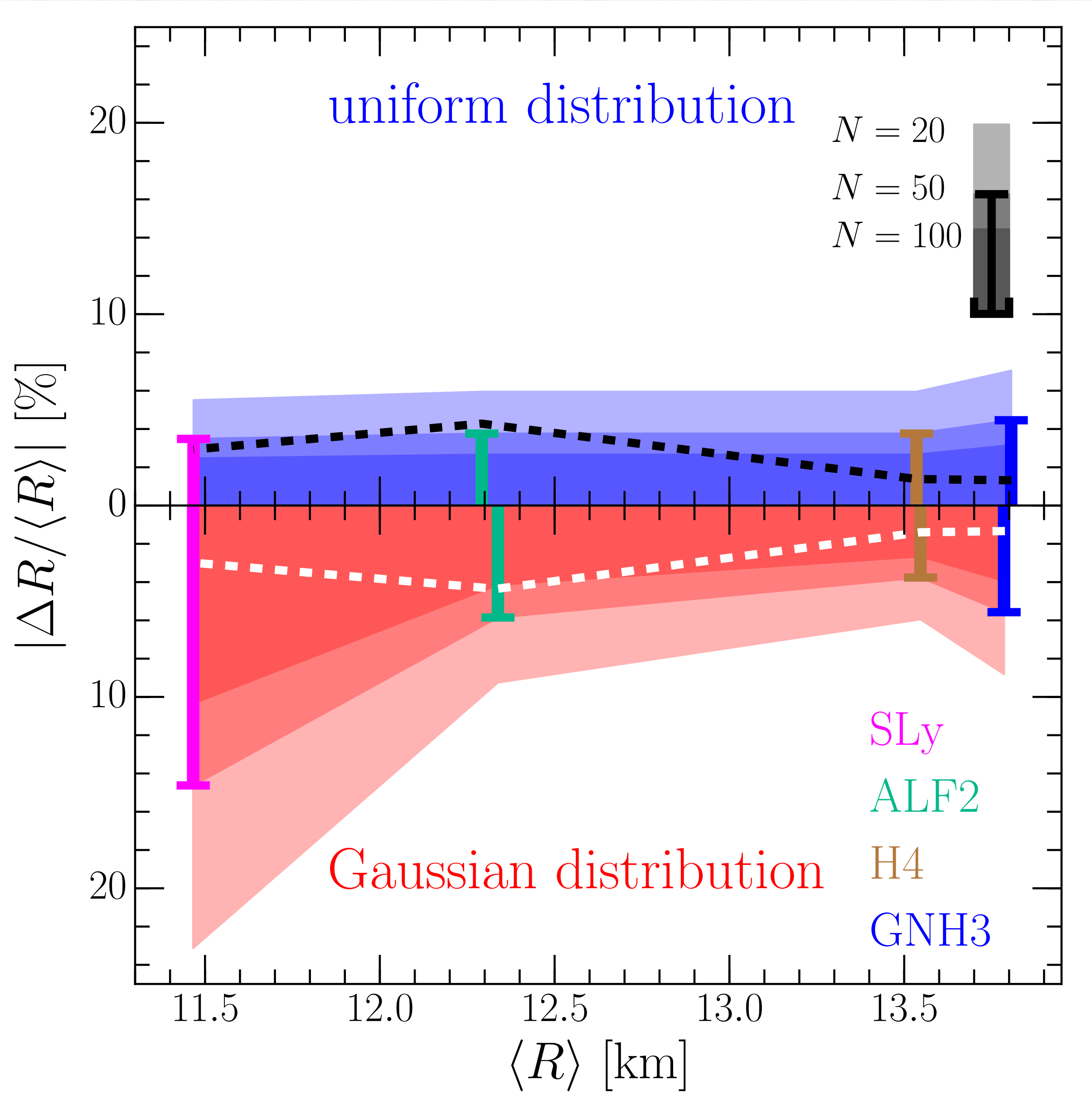
- **uniform** distribution in mass [1.21, 1.38] M⊙ between 100 and 300 Mpc; isotropic distribution in space.
- dashed lines for results of Fisher-matrix analysis with N=50
- errors scale like $\sqrt{N}$

Constraining the radius: MonteCarlo vs Fisher



- **Gaussian** distribution in mass [1.21, 1.38] M⊙ centred at 1.35 M⊙ with variance 0.05 Binaries are between 100 and 300 Mpc; isotropic distribution in space.
- dashed lines for results of Fisher-matrix analysis with N=50
- errors scale like $\sqrt{N}$

All in all



- stiff EOSs: $|\Delta R/\langle R\rangle| < 10\%$ for **N~20**
- soft EOSs: $|\Delta R/\langle R\rangle| \sim 10\%$ for **N~50**
- discriminating stiff/soft EOSs will possible even with moderate N
- discriminating two-stiff /two-soft EOSs will be harder
- very soft EOSs remain a challenge
- golden binary: **SNR ~ 6** at **30 Mpc** $|\Delta R/\langle R\rangle| \lesssim 2\%$ at 90% confidence

Recap (I)

- GW signal from binary neutron stars depends on a number of factors: mass, mass ratio, EOS, magnetic fields, neutrino transport.
- Inspiral part is reasonably well understood and approximated with PN or EOB approaches; post-merger much more complex.
- Spectra of post-merger shows clear **"quasi-universal"** peaks.
- Unless binary very close, peaks have **SNR ~ 1**. However, multiple signals can be stacked and **SNR** will **increase coherently**.
- Fisher-matrix and Monte-Carlo simulations can be performed combining information from inspiral and post-merger:
  - **stiff** EOSs: $|\Delta R/\langle R\rangle| < 10\%$ for ***N~20***
  - **soft** EOSs: $|\Delta R/\langle R\rangle| < 10\%$ for ***N~50***
  - **very soft** EOS will be a challenge for aLIGO-Virgo (ET?)